

**Conjecture that any square of a prime can be obtained through an unusual operation on the numbers  $360k+72$**

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**Abstract.** In this paper I make the following conjecture: The square of any odd prime can be obtained from the numbers of the form  $360k + 72$  in the following way: let  $d_1, d_2, \dots, d_n$  be the (not distinct) prime factors of the number  $360k + 72$ ; than for any square of a prime  $p^2$  there exist  $k$  such that  $(d_1 - 1) \cdot (d_2 - 1) \cdot \dots \cdot (d_n - 1) + 1 = p^2$ . Example: for  $p^2 = 13^2 = 169$  there exist  $k = 17$  such that from  $360 \cdot 17 + 72 = 6192 = 2^4 \cdot 3^2 \cdot 43$  is obtained  $1^4 \cdot 2^2 \cdot 42 + 1 = 169$ . I also conjecture that any absolute Fermat pseudoprime (Carmichael number) can be obtained through the presented formula, which attests again the special relation that I have often highlighted between the nature of Carmichael numbers and the nature of squares of primes.

**Conjecture:**

The square of any odd prime can be obtained from the numbers of the form  $360k + 72$  in the following way: let  $d_1, d_2, \dots, d_n$  be the (not distinct) prime factors of the number  $360k + 72$ ; than for any square of a prime  $p^2$  there exist  $k$  such that  $(d_1 - 1) \cdot (d_2 - 1) \cdot \dots \cdot (d_n - 1) + 1 = p^2$ .

**The less  $k$  for ten squares of odd primes:**

(obtained for  $k$  up to 100)

- :  $p^2 = 3^2 = 9$  is obtained for  $k = 1$  because from  $360 \cdot 1 + 72 = 432 = 2^4 \cdot 3^3$  is obtained  $1^4 \cdot 2^3 + 1 = 9$ ;
- :  $p^2 = 5^2 = 25$  is obtained for  $k = 11$  because from  $360 \cdot 11 + 72 = 4032 = 2^6 \cdot 3^2 \cdot 7$  is obtained  $1^6 \cdot 2^2 \cdot 6 + 1 = 25$ ;
- :  $p^2 = 7^2 = 49$  is obtained for  $k = 4$  because from  $360 \cdot 4 + 72 = 1512 = 2^3 \cdot 3^3 \cdot 7$  is obtained  $1^3 \cdot 2^3 \cdot 6 + 1 = 49$ ;
- :  $p^2 = 11^2 = 121$  is obtained for  $k = 6$  because from  $360 \cdot 6 + 72 = 2232 = 2^3 \cdot 3^2 \cdot 31$  is obtained  $1^3 \cdot 2^2 \cdot 30 + 1 = 121$ ;

- :  $p^2 = 13^2 = 169$  is obtained for  $k = 17$  because from  $360*17 + 72 = 6192 = 2^4*3^2*43$  is obtained  $1^4*2^2*42 + 1 = 169$ ;
- :  $p^2 = 17^2 = 289$  is obtained for  $k = 18$  because from  $360*18 + 72 = 6552 = 2^3*3^2*7*13$  is obtained  $1^3*2^2*6*12 + 1 = 289$ ;
- :  $p^2 = 23^2 = 529$  is obtained for  $k = 40$  because from  $360*40 + 72 = 14472 = 2^3*3^3*67$  is obtained  $1^3*2^3*66 + 1 = 529$ ;
- :  $p^2 = 29^2 = 841$  is obtained for  $k = 42$  because from  $360*42 + 72 = 15192 = 2^3*3^2*211$  is obtained  $1^3*2^2*210 + 1 = 841$ ;
- :  $p^2 = 31^2 = 961$  is obtained for  $k = 48$  because from  $360*48 + 72 = 17352 = 2^3*3^2*241$  is obtained  $1^3*2^2*240 + 1 = 961$ ;
- :  $p^2 = 41^2 = 1681$  is obtained for  $k = 84$  because from  $360*84 + 72 = 30312 = 2^3*3^2*421$  is obtained  $1^3*2^2*420 + 1 = 1681$ .

**Conjecture:**

Any Carmichael number can be obtained from the numbers of the form  $360*k + 72$  in the following way: let  $d_1, d_2, \dots, d_n$  be the (not distinct) prime factors of the number  $360*k + 72$ ; than for any Carmichael number  $C$  there exist  $k$  such that  $(d_1 - 1)*(d_2 - 1)*\dots*(d_n - 1) + 1 = C$ .

**The less k for two Carmichael numbers:**

(obtained for  $k$  up to 100)

- :  $C = 561$  is obtained for  $k = 85$  because from  $360*85 + 72 = 30672 = 2^4*3^3*71$  is obtained  $1^4*2^3*70 + 1 = 561$ ;
- :  $C = 1729$  is obtained for  $k = 96$  because from  $360*96 + 72 = 34632 = 2^3*3^2*13*37$  is obtained  $1^3*2^2*12*36 + 1 = 1729$ .