Bell (1964) revisited for high-school STEM students

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Abstract: Bringing an elementary knowledge of vectors to Bell (1964), we eliminate 13 false or unnecessary expressions and negate Bell's famous inequality. We hope this interesting result will move STEM students to study one of the most famous—and strangest—works in the history of physics: for who else but famous Bell uses the flawed approximation of an unnecessary experiment to invalidate their flawed use of a mathematical fact? And then rejects the fact?

1 Preamble

1.1. Bell (1964) is freely available online (see References). To facilitate discussion, please do this: after Bell 1964:(14), identify Bell's unnumbered math-style expressions as (14a)-(14c), (15a), (21a)-(21e), (23). We here eliminate all those after (15): which is Bell's famous (but false-in-our-view) inequality.

1.2. An extended preamble is provided in Watson (2017d); freely available on line (see References). There, at ¶¶2.1-2.7, we introduce EPR-Bohm (EPRB): the experiment discussed in Bell (1964). [For teachers: The key to our analysis there is this: *we simply reject inferences that are false in quantum settings*. Our doctrine is true local realism: the union of true locality (after Einstein) and true (non-naive) realism (after Bohr). All our results are consequently in full accord with QM and experiment.]

1.3. NB: Bell uses P to designate an expectation (an average) whereas we, reserving P for probabilities, denote expectations via $\langle\cdot\rangle$. Here, however, to make this analysis easier to follow wrt Bell (1964), we compromise and replace Bell's P with an E: E for expectation. Our equivalent to Bell's original $P(\vec{a},\vec{b})$ is thus $E(a,b)$: since our a and b are also unit vectors.

2 Analysis

2.1. We begin with Bell's famous inequality—Bell 1964:(15)—in Bell's terms. Our replacement follows: hereafter referred to as 'our (2)' to distinguish it from Bell's 1964:(2) which Bell rejects and we defend.

$$|\,P(\vec{a},\vec{b})-P(\vec{a},\vec{c})\,| \;\leq\; 1+P(\vec{b},\vec{c})\,; \tag{1}$$

$$\text{ie: } |\,E(a,b)-E(a,c)\,| \;\leq\; 1+E(b,c). \tag{2}$$

2.2. Following his (15), Bell introduces approximations: ϵ in his (16), δ in his (17). In our analysis here, having no interest in approximations: both δ and ϵ are zero and unnecessary. Thus, comparing Bell's 1964:(15) with his 1964:(21d)-(21e)—now with $\epsilon+\delta=0$—we see that our (2) may be written:

$$|a\cdot c-a\cdot b| \;\leq\; 1-b\cdot c\,; \tag{3}$$

$$\text{ie: } |\,a\cdot c-a\cdot b\,| -1+b\cdot c \;\leq\; 0. \tag{4}$$

2.3. (3) shows Bell's insertion of QM values into his inequality; though it could have been done after (15). We now evaluate (4), a convenient form of (3), using Bell's example: see above his (23). To wit:

$$a\cdot c=0;\; a\cdot b=b\cdot c=\tfrac{1}{\sqrt{2}}. \tag{5}$$

$$\text{So, from (5) and LHS (4): } \;|\,a\cdot c-a\cdot b\,| -1+b\cdot c=\sqrt{2}-1\not\leq 0.\,\blacktriangle \tag{6}$$

2.4. Thus freed from approximations: Bell's famous inequality is absurd and false under QM. ■

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2.5. So, en route to developing the correct equality—via a not-too-difficult particle-by-particle analysis of EPRB—Watson 2017d:(34)-(39) shows that (3) should read (for all a, b, c; not a limited sample):

$$|a \cdot c - a \cdot b| \leq 1 - (a \cdot b)(a \cdot c); \tag{7}$$

$$\text{ie: } |a \cdot c - a \cdot b| - 1 + (a \cdot b)(a \cdot c) \leq 0; \tag{8}$$

$$\text{which is a mathematical fact since: } |a \cdot c - a \cdot b| - 1 + (a \cdot b)(a \cdot c) \leq \tfrac{1}{2}. \blacksquare \tag{9}$$

2.6. After his (15a)-(23), a more surprising fact is Bell's famous—though in-our-view false—claim: the QM expectation for EPRB cannot be represented, either accurately or arbitrarily, in the form

$$E(a, b) = \int d\lambda \, \rho(\boldsymbol{\lambda}) A(a, \boldsymbol{\lambda}) B(b, \boldsymbol{\lambda}) \neq -a \cdot b. \tag{10}$$

2.7. For further reading and analysis: Bell's erroneous claim—(10)—is refuted at Watson (2017d:17).

3 Conclusions

3.1. The principal defect in Bell's 1964:(15a)-(23) is this: under idealization—eg, the reduction of δ and ϵ to zero; via improved technology if you like—the analysis should return to his (15), his starting-point. That is so in (21e) when $\delta + \epsilon = 0$, but not (23). This failure should have triggered the rejection of his 'approximate' approach and reinforced this next: his example—see (5)-(6) above—should see him reject [as we do] the analysis that brought him to his (15).

3.2. Widely regarded as a leading mathematician in UK, du Sautoy (2016:170) may well say: 'Bell's theorem is as mathematically robust as they come.' But Bell's math is conducted in the context of EPRB, and in that context Bell's math fails due to flawed analysis: not due to LHS (10) above. See the correct approach at Watson (2017d:¶2.22-2.28).

3.3. Well may we say: Bell uses the flawed approximation of an unnecessary experiment to invalidate his flawed use of a mathematical fact; and then rejects the fact. But surely—better known, in the company of facts—his 'don't be a cissy' have-a-go attitude will encourage more students to a life in science.

3.4. Students should be encouraged to locate the interesting errors in Bell 1964:(19) and (21e). The deeper errors in (21a)-(21d)—via those in (14a)-(15)—are discussed at Watson 2017d:(34)-(40): in a particle-by-particle analysis that is more elementary (and much less daunting) than it sounds. For the key to our analysis is this: *we simply reject inferences that are false in quantum settings*.

4 Acknowledgment

It's a great pleasure to acknowledge continuing collaboration with Roger Mc Murtrie (Canberra).

5 References [DA = date accessed]

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3. Watson, G. (2017d). "Bell's dilemma resolved, nonlocality negated, QM demystified, etc." http://vixra.org/abs/1707.0322 [DA20171107]