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New solution of Maxwell's equations for spherical wave in the far zone

Contents

Annotation

It is noted that the known solution for a spherical electromagnetic wave in the far zone does not satisfy the law of conservation of energy (it is retained only on the average), the electric and magnetic intensities of the same name (in coordinate) are in phase, only one of Maxwell's equations is satisfied. A solution is offered that is free from these shortcomings.

1. Introduction

In [1], a cylindrical electromagnetic wave is considered. Below we consider a spherical electromagnetic wave far from the vibrator - in the so-called the far zone, where the longitudinal (radial-directed) electric and magnetic intensities can be neglected. The main drawbacks of the known solution (see Appendix 1) are that

1. the law of conservation of energy is fulfilled only on the average (in time),
2. the magnetic and electrical components are in phase,
3. in the Maxwell equations system, in the known solution, only one equation of eight is satisfied.

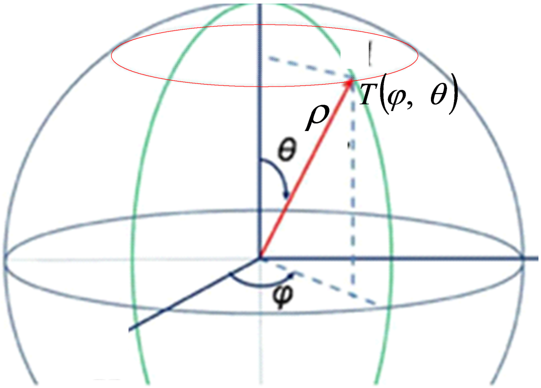

Fig. 1.

2. Solution of the Maxwell's equations

Fig. 1 shows the spherical coordinate system (ρ, θ, φ). Expressions for the rotor and the divergence of vector E in these coordinates are given in Table 1 [2]. The following notation is used:

E - electrical intensities,

H - magnetic intensities,

μ - absolute magnetic permeability,

ε - absolute dielectric constant.

The Maxwell's equations in spherical coordinates in the absence of charges and currents have the form given in Table. 2. Next, we will seek a solution for $E_\rho = 0,\ H_\rho = 0$ and in the form of the functions $E,\ H$ presented in Table 3, where the function $g(\theta)$ and functions of the species $E_{\varphi\rho}(\rho)$ are to be calculated. We assume that the intensities $E,\ H$ do not depend on the argument φ. Under these conditions, we transform Table 1 in Table 3a. Further we substitute functions from Table 3 in Table 3a. Then we get Table 4.

Substituting the expressions for the rotors and divergences from Table 4 into the Maxwell's equations (see Table 2), differentiating with respect to time and reducing the common factors, we obtain a new form of the Maxwell's equations - see Table 5.

Consider the Table 5. From line 2 it follows:

$$\frac{H_{\varphi\rho}}{\rho}+\frac{\partial H_{\varphi\rho}}{\partial\rho}=0, \tag{2}$$

$$\chi H_{\varphi\rho}+\frac{\omega\varepsilon}{c}E_{\theta\rho}=0. \tag{3}$$

Consequently,

$$H_{\varphi\rho}=\frac{h_{\varphi\rho}}{\rho}, \tag{4}$$

$$H_{\varphi\rho}=-\frac{\omega\varepsilon}{\chi c}E_{\theta\rho}, \tag{5}$$

where $h_{\varphi\rho}$ is some constant. Likewise, from lines 3, 5, 5 should be correspondingly:

$$H_{\theta\rho}=\frac{h_{\theta\rho}}{\rho}, \tag{6}$$

$$H_{\theta\rho}=\frac{\omega\varepsilon}{\chi c}E_{\varphi\rho}, \tag{7}$$

$$E_{\varphi\rho}=\frac{e_{\varphi\rho}}{\rho}, \tag{8}$$

$$E_{\varphi\rho}=\frac{\omega\mu}{\chi c}H_{\theta\rho}, \tag{9}$$

$$E_{\theta\rho}=\frac{e_{\theta\rho}}{\rho}, \tag{10}$$

$$E_{\theta\rho}=-\frac{\omega\mu}{\chi c}H_{\varphi\rho}. \tag{11}$$

It follows from (5) that

$$E_{\theta\rho}=-\frac{\chi c}{\omega\varepsilon}H_{\varphi\rho}, \tag{12}$$

and from a comparison of (11) and (12) it follows that

$$\frac{\omega\mu}{\chi c}=\frac{\chi c}{\omega\varepsilon}$$

or

$$\chi=\frac{\omega}{c}\sqrt{\varepsilon\mu}. \tag{13}$$

The same formula follows from a comparison of (7) and (9).
It follows from (5, 13) that

$$H_{\varphi\rho} = -\sqrt{\frac{\varepsilon}{\mu}} E_{\theta\rho}, \tag{14}$$

and it follows from (14, 4, 11, 12) that

$$h_{\varphi\rho} = -e_{\theta\rho}\sqrt{\frac{\varepsilon}{\mu}}, \tag{15}$$

Similarly, it follows from (7, 13) that

$$H_{\theta\rho} = -\sqrt{\frac{\varepsilon}{\mu}} E_{\varphi\rho}, \tag{16}$$

and it follows from (16, 6, 8, 12) that

$$h_{\theta\rho} = -e_{\varphi\rho}\sqrt{\frac{\varepsilon}{\mu}}. \tag{17}$$

From a comparison of (15) and (17) it follows that

$$\frac{h_{\varphi\rho}}{h_{\theta\rho}} = \frac{e_{\theta\rho}}{e_{\varphi\rho}} = q, \tag{18}$$

$$\frac{h_{\varphi\rho}}{e_{\theta\rho}} = \frac{h_{\theta\rho}}{e_{\varphi\rho}} = -\sqrt{\frac{\varepsilon}{\mu}}. \tag{19}$$

Further we notice that lines 1, 4, 7 and 8 coincide, from which it follows that the function $g(\theta)$ is a solution of the differential equation

$$\frac{g(\theta)}{\mathrm{tg}(\theta)} + \frac{\partial(g(\theta))}{\partial\theta} = 0. \tag{20}$$

We note that in the well-known solution $g(\theta) = \sin(\theta)$ - see Appendix 1. It is easy to see that such a function does not satisfy equation (20). Consequently,

> **in the known solution 4 Maxwell's equations with expressions $\mathrm{rot}_\rho(E)$, $\mathrm{rot}_\rho(H)$, $\mathrm{div}(E)$, $\mathrm{div}(H)$ are not satisfied.**

Thus, the solution of the Maxwell's equations for a spherical wave in the far zone has the form of the intensities presented in Table 3, where

$$H_{\varphi\rho} = \frac{h_{\varphi\rho}}{\rho},\ H_{\theta\rho} = \frac{h_{\theta\rho}}{\rho},\ E_{\varphi\rho} = \frac{e_{\varphi\rho}}{\rho},\ E_{\theta\rho} = \frac{e_{\theta\rho}}{\rho} \tag{21}$$

$$\chi = \frac{\omega}{c}\sqrt{\varepsilon\mu} \quad \text{(see 13)}, \quad \frac{g(\theta)}{\mathrm{tg}(\theta)} + \frac{\partial(g(\theta))}{\partial\theta} = 0 \quad \text{(see 20)}$$

and the constants $h_{\varphi\rho}$, $h_{\theta\rho}$, $e_{\theta\rho}$, $e_{\varphi\rho}$ satisfy conditions

$$\frac{h_{\varphi\rho}}{h_{\theta\rho}}=\frac{e_{\theta\rho}}{e_{\varphi\rho}}=q \quad \text{(см. 18)}, \quad \frac{h_{\varphi\rho}}{e_{\theta\rho}}=\frac{h_{\theta\rho}}{e_{\varphi\rho}}=-\sqrt{\frac{\varepsilon}{\mu}}. \quad \text{(см. 19)}$$

From Table. 3 it follows that

> **the same (with respect to the coordinates φ and θ) electric and magnetic intensities are shifted in phase by a quarter of the period.**

This corresponds to experimental electrical engineering. In Fig. 2 shows the intensities vectors in a spherical coordinate system.

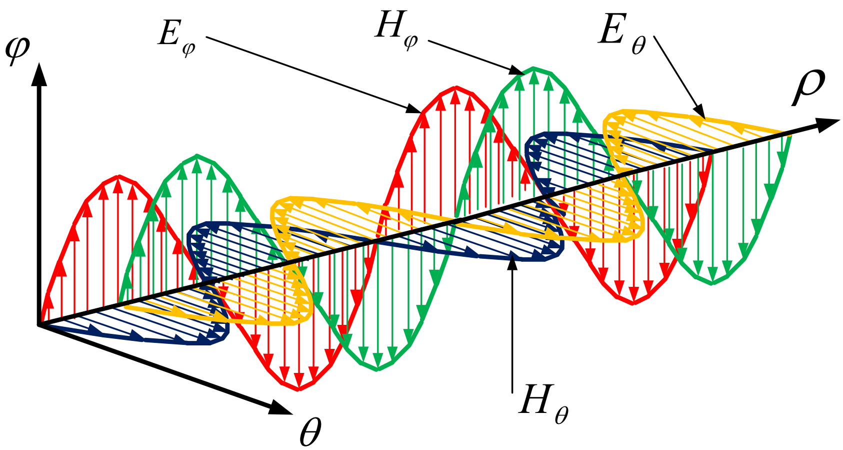

Fig. 2.

3. Energy Flows

Also, as in [1], the flow density of electromagnetic energy - the Poynting vector is

$$S=\eta E\times H, \tag{1}$$

where

$$\eta=c/4\pi. \tag{2}$$

In spherical coordinates φ, θ, ρ the flow density of electromagnetic energy has three components $S_\varphi, S_\theta, S_\rho$ directed along the radius, along the circumference, along the axis, respectively. They are determined by the formula

$$S=\begin{bmatrix} S_\varphi \\ S_\theta \\ S_\rho \end{bmatrix}=\eta(E\times H)=\eta\begin{bmatrix} E_\theta H_\rho - E_\rho H_\theta \\ E_\rho H_\varphi - E_\varphi H_\rho \\ E_\varphi H_\theta - E_\theta H_\varphi \end{bmatrix}. \tag{4}$$

From here and from Table 3 it follows that

$$S_\varphi = 0$$
$$S_\theta = 0 \qquad . \tag{5}$$
$$S_\rho = \eta \begin{pmatrix} E_{\varphi\rho} H_{\theta\rho} \left(g(\theta) \sin(\chi\rho + \omega t) \right)^2 - \\ - E_{\theta\rho} H_{\varphi\rho} \left(g(\theta) \cos(\chi\rho + \omega t) \right)^2 \end{pmatrix}$$

It follows from (2.9, 2.11) that

$$E_{\varphi\rho} H_{\theta\rho} = \frac{\omega\mu}{\chi c} \left(H_{\theta\rho} \right)^2, \tag{6}$$

$$E_{\theta\rho} H_{\varphi\rho} = -\frac{\omega\mu}{\chi c} \left(H_{\varphi\rho} \right)^2. \tag{7}$$

Further from (6, 7, 2.4, 2.6) it follows that

$$E_{\varphi\rho} H_{\theta\rho} = \frac{\omega\mu}{\chi c} \left(h_{\theta\rho} \right)^2 \frac{1}{\rho^2}, \tag{8}$$

$$E_{\theta\rho} H_{\varphi\rho} = -\frac{\omega\mu}{\chi c} \left(h_{\varphi\rho} \right)^2 \frac{1}{\rho^2}. \tag{9}$$

From (5, 8, 9) we obtain:

$$S_\rho = \eta \cdot g^2(\theta) \frac{\omega\mu}{\chi c} \frac{1}{\rho^2} \begin{pmatrix} \left(h_{\theta\rho} \right)^2 \left(\sin(\chi\rho + \omega t) \right)^2 + \\ + \left(h_{\varphi\rho} \right)^2 \left(\cos(\chi\rho + \omega t) \right)^2 \end{pmatrix}. \tag{9}$$

Further from (9, 2.13, 2.18) it follows that

$$S_\rho = \eta \cdot g^2(\theta) \omega \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\rho^2} \begin{pmatrix} \left(h_{\theta\rho} \right)^2 \left(\sin(\chi\rho + \omega t) \right)^2 + \\ + \left(q h_{\theta\rho} \right)^2 \left(\cos(\chi\rho + \omega t) \right)^2 \end{pmatrix}. \tag{10}$$

where q is a previously undefined constant. If we take

$$q = 1, \tag{10a}$$

then we get

$$S_\rho = \eta \cdot g^2(\theta) \omega \sqrt{\frac{\mu}{\varepsilon}} \frac{h_{\theta\rho}^2}{\rho^2}. \tag{11}$$

We also note that the surface area of a sphere with a radius ρ is equal to $4\pi\rho^2$. Then the flow of energy passing through a sphere with a radius ρ is

$$\overline{S_\rho} = 4\pi\eta\omega \cdot g^2(\theta) h_{\theta\rho}^2 \sqrt{\frac{\mu}{\varepsilon}}. \tag{12}$$

It follows from (12) that

> **in a spherical electromagnetic wave, the energy flux passing through the spheres along the radius remains**

constant with increasing radius and does not change with time.

This strictly corresponds to the law of conservation of energy.

It follows from (12) that the energy flow density varies along the meridian in accordance with the law $g^2(\theta)$, since function can not be a constant - see equation (20). The same conclusion follows from the well-known solution, where $g(\theta)=\sin(\theta)$.

Appendix 1

The known solution has the form [3]:

$$E_\theta = e_\theta \frac{1}{\rho}\sin(\theta)\sin(\omega t - \chi\rho), \tag{1}$$

$$H_\varphi = h_\varphi \frac{1}{\rho}\sin(\theta)\sin(\omega t - \chi\rho), \tag{2}$$

$k_{e\theta} = \dfrac{\chi^2 lI}{4\pi\omega\varepsilon\varepsilon_o}$, $k_{h\varphi} = \dfrac{\chi lI}{4\pi}$, where l, I - length and current of the vibrator. We notice, that

$$\frac{e_\theta}{h_\varphi} = \frac{\chi}{\omega\varepsilon} \tag{3}$$

It should be noted that these tensions <u>are in phase</u>, which contradicts practical electrical engineering.

Let us consider how equations (1, 2) relate to Maxwell's system of equations - see Table 2. The intensities (1, 2) enter only in equation (6) from Table 2, which has the form

$$\mathrm{rot}_\varphi E + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0 \tag{4}$$

or

$$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial\rho} + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0. \tag{5}$$

We substitute (1, 2) into (5) and obtain:

$$\begin{aligned} &- e_\theta \frac{\chi}{\rho}\sin(\theta)\cos(\omega t - \chi\rho) - \\ &- h_\varphi \frac{\chi}{\rho}\frac{\mu}{c}\sin(\theta)\cos(\omega t - \chi\rho) = 0 \end{aligned} \tag{6}$$

or

$$\frac{e_\theta}{h_\varphi} + \frac{\mu}{c} = 0. \tag{7}$$

From a comparison of (3) and (7) it follows that the intensities (1, 2) satisfy equation (4). The remaining 7 Maxwell equations are violated. In the equations (2, 3, 5) from Table 2 one of the terms differs from zero, and the other is equal to zero. The violation of equations (1, 4, 7, 8) from Table. 2 is shown above in Section 2. So,

> **the known solution does not satisfy Maxwell's system of equations.**

Tables

Table 1.

1	**2**	**3**
1	$\mathrm{rot}_\rho(E)$	$\frac{E_\varphi}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_\varphi}{\rho \partial \theta} - \frac{\partial E_\theta}{\rho \sin(\theta) \partial \varphi}$
2	$\mathrm{rot}_\theta(E)$	$\frac{\partial E_\rho}{\rho \sin(\theta) \partial \varphi} - \frac{E_\varphi}{\rho} - \frac{\partial E_\varphi}{\partial \rho}$
3	$\mathrm{rot}_\varphi(E)$	$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial \rho} - \frac{\partial E_\rho}{\rho \partial \varphi}$
4	$\mathrm{div}(E)$	$\frac{E_\rho}{\rho} + \frac{\partial E_\rho}{\partial \rho} + \frac{E_\theta}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_\theta}{\rho \partial \theta} + \frac{\partial E_\varphi}{\rho \sin(\theta) \partial \varphi}$

Table 2.

1	**2**
1.	$\mathrm{rot}_\rho H - \frac{\varepsilon}{c}\frac{\partial E_\rho}{\partial t} = 0$
2.	$\mathrm{rot}_\theta H - \frac{\varepsilon}{c}\frac{\partial E_\theta}{\partial t} = 0$
3.	$\mathrm{rot}_\varphi H - \frac{\varepsilon}{c}\frac{\partial E_\varphi}{\partial t} = 0$
4.	$\mathrm{rot}_\rho E + \frac{\mu}{c}\frac{\partial H_\rho}{\partial t} = 0$
5.	$\mathrm{rot}_\theta E + \frac{\mu}{c}\frac{\partial H_\theta}{\partial t} = 0$

6.	$\mathrm{rot}_\varphi E + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0$
7.	$\mathrm{div}(E) = 0$
8.	$\mathrm{div}(H) = 0$

Table 3.

1	**2**
	$E_\theta = E_{\theta\rho}(\rho)g(\theta)\cos(\chi\rho + \omega t)$
	$E_\varphi = E_{\varphi\rho}(\rho)g(\theta)\sin(\chi\rho + \omega t)$
	$E_\rho = 0$
	$H_\theta = H_{\theta\rho}(\rho)g(\theta)\sin(\chi\rho + \omega t)$
	$H_\varphi = H_{\varphi\rho}(\rho)g(\theta)\cos(\chi\rho + \omega t)$
	$H_\rho = 0$

Table 3a.

1	**2**	**3**
1	$\mathrm{rot}_\rho(E)$	$\frac{E_\varphi}{\rho\,\mathrm{tg}(\theta)} + \frac{\partial E_\varphi}{\rho\partial\theta}$
2	$\mathrm{rot}_\theta(E)$	$-\frac{E_\varphi}{\rho} - \frac{\partial E_\varphi}{\partial\rho}$
3	$\mathrm{rot}_\varphi(E)$	$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial\rho}$
4	$\mathrm{div}(E)$	$\frac{E_\theta}{\rho\,\mathrm{tg}(\theta)} + \frac{\partial E_\theta}{\rho\partial\theta}$

Table 4.

1	**2**	**3**
1	$\mathrm{rot}_\rho(E)$	$\frac{E_\varphi}{\rho\,\mathrm{tg}(\theta)} + \frac{\partial E_\varphi}{\rho\partial\theta}$
2	$\mathrm{rot}_\theta(E)$	$-\left(\frac{E_{\varphi\rho}}{\rho}\sin(\ldots) + \frac{\partial E_{\varphi\rho}}{\partial\rho}\sin(\ldots) + \chi E_{\varphi\rho}\cos(\ldots)\right)g(\theta)$

3	$\mathrm{rot}_{\varphi}(E)$	$\left(\frac{E_{\theta\rho}}{\rho}\cos(...)+\frac{\partial E_{\theta\rho}}{\partial\rho}\cos(...)-\chi E_{\theta\rho}\sin(...)\right)g(\theta)$
4	$\mathrm{div}(E)$	$\frac{E_{\theta}}{\rho\mathrm{tg}(\theta)}+\frac{\partial E_{\theta}}{\rho\partial\theta}$
5	$\mathrm{rot}_{\rho}(H)$	$\frac{H_{\varphi}}{\rho\mathrm{tg}(\theta)}+\frac{\partial H_{\varphi}}{\rho\partial\theta}$
6	$\mathrm{rot}_{\theta}(H)$	$-\left(\frac{H_{\varphi\rho}}{\rho}\cos(...)+\frac{\partial H_{\varphi\rho}}{\partial\rho}\cos(...)-\chi H_{\varphi\rho}\sin(...)\right)g(\theta)$
7	$\mathrm{rot}_{\varphi}H$	$\left(\frac{H_{\theta\rho}}{\rho}\sin(...)+\frac{\partial H_{\theta\rho}}{\partial\rho}\sin(...)+\chi H_{\theta\rho}\cos(...)\right)g(\theta)$
8	$\mathrm{div}(H)$	$\frac{H_{\theta}}{\rho\mathrm{tg}(\theta)}+\frac{\partial H_{\theta}}{\rho\partial\theta}$

Table 5.

1	**2**
1.	$\frac{g(\theta)}{\mathrm{tg}(\theta)}+\frac{\partial(g(\theta))}{\partial\theta}=0$
2.	$-\frac{H_{\varphi\rho}}{\rho}\cos(...)-\frac{\partial H_{\varphi\rho}}{\partial\rho}\cos(...)+\chi H_{\varphi\rho}\sin(...)+\frac{\omega\varepsilon}{c}E_{\theta\rho}\sin(...)=0$
3.	$\frac{H_{\theta\rho}}{\rho}\sin(...)+\frac{\partial H_{\theta\rho}}{\partial\rho}\sin(...)+\chi H_{\theta\rho}\cos(...)-\frac{\omega\varepsilon}{c}E_{\varphi\rho}\cos(...)=0$
4.	$\frac{g(\theta)}{\mathrm{tg}(\theta)}+\frac{\partial(g(\theta))}{\partial\theta}=0$
5.	$-\frac{E_{\varphi\rho}}{\rho}\sin(...)-\frac{\partial E_{\varphi\rho}}{\partial\rho}\sin(...)-\chi E_{\varphi\rho}\cos(...)+\frac{\omega\mu}{c}H_{\theta\rho}\sin(...)=0$
6.	$\frac{E_{\theta\rho}}{\rho}\cos(...)+\frac{\partial E_{\theta\rho}}{\partial\rho}\cos(...)-\chi E_{\theta\rho}\sin(...)-\frac{\omega\mu}{c}H_{\varphi\rho}\sin(...)=0$
7.	$\frac{g(\theta)}{\mathrm{tg}(\theta)}+\frac{\partial(g(\theta))}{\partial\theta}=0$
8.	$\frac{g(\theta)}{\mathrm{tg}(\theta)}+\frac{\partial(g(\theta))}{\partial\theta}=0$

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