

Neutrosophic Goal Programming Technique and its Application

Samir Dey¹, Tapan Kumar Roy²

¹Department of Mathematics, Asansol Engineering College
Vivekananda Sarani, Asansol-713305, West Bengal, India.

²Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur.
P.O.-Botanic Garden, Howrah-711103, West Bengal, India.

Abstract — This paper develops a multi-objective Neutrosophic Goal Optimization technique for optimizing the design of truss structure with multiple objectives subject to a specified set of constraints. In this optimum design formulation, the objective functions are weight and the deflection; the design variables are the cross-sections of the bar; the constraints are the stress in member. The classical three bar truss structure is presented here in to demonstrate the efficiency of the neutrosophic goal programming approach. The model is numerically illustrated by Neutrosophic Goal Optimization technique with different aggregation method. The result shows that the Neutrosophic Goal Optimization technique is very efficient in finding the best optimal solutions.

Keywords - Neutrosophic Set, Neutrosophic Goal Programming, Structural Optimization.

I. INTRODUCTION

In present day, problems are there with different types of uncertainties which cannot be solved by classical theory of mathematics. Fuzzy set (FS) theory has long been introduced to deal with inexact and imprecise data by Zadeh[2], Later on the fuzzy set theory was used by Bellman and Zadeh[3] to the decision making problem. A few works have been done as an application of fuzzy set theory on structural design. Several researchers like Wang et al.[5] first applied α -cut method to structural designs where various design levels α were used to solve the non-linear problems. In this regard, a generalized fuzzy number has been used Dey et al.[6] in context of a non-linear structural design optimization. Dey et al.[8] developed parameterized t-norm based fuzzy optimization method for optimum structural design.

In such extension, Intuitionistic fuzzy set which is one of the generalizations of fuzzy set theory and was characterized by a membership, a non-membership and a hesitancy function was first introduced by Atanassov[1]. In fuzzy set theory the degree of acceptance is only considered but in case of IFS it is characterized by degree of membership and non-membership in such a way that their sum is less or equal to one. Dey et al.[7] solved two bar truss non-linear problem by using intuitionistic fuzzy

optimization problem. Again, Dey et al. [9] used intuitionistic fuzzy optimization technique to solve multi objective structural design. Intuitionistic fuzzy sets consider both truth and falsity membership and can only handle incomplete information but not the information which is connected with indeterminacy or inconsistency.

In due course, any generalization of fuzzy set failed to handle problems with indeterminate or inconsistent information. To overcome this, Smarandache [4] proposed a new theory, namely, neutrosophic logic, by adding another independent membership function named as indeterminacy membership $I x$ along with truth membership $T x$ and falsity $F x$ membership functions. Neutrosophic set is a generalization of intuitionistic fuzzy sets. If hesitancy degree $H x$ of intuitionistic fuzzy sets and the indeterminacy membership degree $I x$ of neutrosophic sets are equal, then neutrosophic set will become the intuitionistic fuzzy set. The components of neutrosophic set, namely truth-membership degree, indeterminacy-membership degree and falsity-membership degree, were suitable to represent indeterminacy and inconsistent information.

Goal Programming (GP) models was originally introduced by Charnes and Copper [11] in early 1977. Multiple and conflicting goals can be used in goal programming. Also, GP allows simultaneous solution of a system of complex objectives, and the solution of the problem requires ascertaining among these multiple objectives. In this case, the model must be solved in such a way, that each of the objective to be achieved. Dey et al.[10] proposed intuitionistic goal programming technique on nonlinear structural model. The Neutrosophic approach for goal programming in structural design is rare. This is the first time NSGO technique is in application to multi-objective structural design. The present study investigates computational algorithm for solving multi-objective structural problem by single valued generalized NSGO technique. The results are compared numerically for different aggregation method of NSGO technique. From our numerical result, it has been seen that the best result obtained for geometric aggregation method for

NSGO technique in the perspective of structural optimization technique.

II. MULTI-OBJECTIVE STRUCTURAL MODEL

In the design problem of the structure i.e. lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure. In truss structure system, the basic parameters (including allowable stress etc.) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions.

The multi-objective structural model can be expressed as

$$\text{Minimize } WT \ A \tag{1}$$

$$\text{minimize } \delta \ A$$

$$\text{subject to } \sigma \ A \leq \sigma$$

$$A^{\min} \leq A \leq A^{\max}$$

where $A = [A_1, A_2, \dots, A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar, $WT \ A = \sum_{i=1}^n \rho_i A_i L_i$ is the total weight of the structure, $\delta \ A$ is the deflection of the loaded joint, where L_i, A_i and ρ_i are the bar length, cross section area and density of the i^{th} group bars respectively. $\sigma \ A$ is the stress constraint and σ is allowable stress of the group bars under various conditions, A^{\min} and A^{\max} are the lower and upper bounds of cross section area A respectively.

III. MATHEMATICAL PRELIMINARIES

A. Fuzzy Set

Let X be a fixed set. A fuzzy set A set of X is an object having the form $\tilde{A} = \langle x, T_A \ x \rangle : x \in X$ where the function $T_A : X \rightarrow [0, 1]$ defined the truth membership of the element $x \in X$ to the set A.

B. Intuitionistic Fuzzy Set

Let a set X be fixed. An intuitionistic fuzzy set or IFS \tilde{A} in X is an object of the form $\tilde{A} = \langle X, T_A \ x, F_A \ x \rangle : x \in X$ where $T_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$ define the truth membership and falsity membership respectively, for every element of $x \in X$, $0 \leq T_A \ x + F_A \ x \leq 1$.

C. Neutrosophic Set

Let a set X be a space of points (objects) and $x \in X$. A neutrosophic set \tilde{A} in X is defined by a truth membership function $T_A \ x$, an indeterminacy-membership function $I_A \ x$ and a falsity membership function $F_A \ x$ and having the form $\tilde{A} = \langle x, T_A \ x, I_A \ x, F_A \ x \rangle : x \in X$. $T_A \ x, I_A \ x$ and $F_A \ x$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is

$$T_A \ x : X \rightarrow]0^-, 1^+[$$

$$I_A \ x : X \rightarrow]0^-, 1^+[$$

$$F_A \ x : X \rightarrow]0^-, 1^+[$$

There is no restriction on the sum of $T_A \ x, I_A \ x$ and $F_A \ x$ so

$$0^- \leq \sup T_A \ x \leq \sup I_A \ x \leq \sup F_A \ x \leq 3^+.$$

D. Single Valued Neutrosophic Set

Let a set X be the universe of discourse. A single valued neutrosophic set \tilde{A} over X is an object having the form $\tilde{A} = \langle x, T_A \ x, I_A \ x, F_A \ x \rangle : x \in X$ where $T_A : X \rightarrow [0, 1], I_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$ with $0 \leq T_A \ x + I_A \ x + F_A \ x \leq 3$ for all $x \in X$.

E. Single Valued Generalized Neutrosophic Set

Let a set X be the universe of discourse. A single valued neutrosophic set \tilde{A} over X is an object having the form $\tilde{A} = \langle x, T_A \ x, I_A \ x, F_A \ x \rangle : x \in X$ where $T_A : X \rightarrow [0, w_1], I_A : X \rightarrow [0, w_2]$ and $F_A : X \rightarrow [0, w_3]$ with $0 \leq T_A \ x + I_A \ x + F_A \ x \leq w_1 + w_2 + w_3$ where $w_1, w_2, w_3 \in [0, 1]$ for all $x \in X$.

F. Complement of Neutrosophic Set

Complement of a single valued neutrosophic set A is denoted by $c \ A$ and is defined by

$$T_{c \ A} \ x = F_A \ x$$

$$I_{c \ A} \ x = 1 - F_A \ x$$

$$F_{c \ A} \ x = T_A \ x$$

G. Union of Neutrosophic Sets

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$, whose truth membership,

indeterminacy-membership and falsity-membership functions are given by

$$T_{C \ A} \ x = \max T_A \ x , T_B \ x$$

$$I_{C \ A} \ x = \max I_A \ x , I_B \ x$$

$$F_{C \ A} \ x = \min F_A \ x , F_B \ x \quad \text{for all } x \in X$$

H. Intersection of Neutrosophic Sets

The intersection of two single valued neutrosophic sets *A* and *B* is a single valued neutrosophic set *C* , written as $C = A \cap B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$$T_{C \ A} \ x = \min T_A \ x , T_B \ x$$

$$I_{C \ A} \ x = \min I_A \ x , I_B \ x$$

$$F_{C \ A} \ x = \max F_A \ x , F_B \ x \quad \text{for all } x \in X$$

IV. MATHEMATICAL ANALYSIS

I. Neutrosophic Goal Programming

Goal programming can be written as

$$\text{Find } x = x_1, x_2, \dots, x_n^T$$

to achieve:

$$z_i = t_i \quad i = 1, 2, \dots, k$$

Subject to $x \in X$ where t_i are scalars and represent the target achievement levels of the objective functions that the decision maker wishes to attain provided, *X* is feasible set of constraints.

The nonlinear goal programming problem can be written as

$$\text{Find } x = x_1, x_2, \dots, x_n^T$$

So as to

Minimize z_i with target value t_i ,acceptance tolerance a_i ,indeterminacy tolerance d_i rejection tolerance c_i

(2)

$$x \in X$$

$$g_j \ x \leq b_j , \ j = 1, 2, \dots, m$$

$x_i \geq 0, \ i = 1, 2, \dots, n$ with truth-membership, indeterminacy-membership and falsity-membership functions

$$T_{z_i \ z_i} = \begin{cases} 1 & \text{if } z_i \leq t_i \\ \left(\frac{t_i + a_i - z_i}{a_i} \right) & \text{if } t_i \leq z_i \leq t_i + a_i \\ 0 & \text{if } z_i \geq t_i + a_i \end{cases}$$

$$I_{z_i \ z_i} = \begin{cases} 0 & \text{if } z_i \leq t_i \\ \left(\frac{z_i - t_i}{d_i} \right) & \text{if } t_i \leq z_i \leq t_i + a_i \\ \left(\frac{t_i + a_i - z_i}{a_i - d_i} \right) & \text{if } t_i + d_i \leq z_i \leq t_i + a_i \\ 0 & \text{if } z_i \geq t_i + a_i \end{cases}$$

$$F_{z_i \ z_i} = \begin{cases} 0 & \text{if } z_i \leq t_i \\ \left(\frac{z_i - t_i}{c_i} \right) & \text{if } t_i \leq z_i \leq t_i + c_i \\ 1 & \text{if } z_i \geq t_i + c_i \end{cases}$$

To maximize the degree of acceptance and indeterminacy of nonlinear goal programming (NGP) objectives and constraints also to minimize degree of rejection of NGP objectives and constraints ,

$$\text{Maximize } T_{z_i \ z_i} , \ i = 1, 2, \dots, k \tag{3}$$

$$\text{Maximize } I_{z_i \ z_i} , \ i = 1, 2, \dots, k$$

$$\text{Minimize } F_{z_i \ z_i} , \ i = 1, 2, \dots, k$$

Subject to

$$0 \leq T_{z_i \ z_i} + I_{z_i \ z_i} + F_{z_i \ z_i} \leq 3, \ i = 1, 2, \dots, k$$

$$T_{z_i \ z_i} \geq 0, I_{z_i \ z_i} \geq 0, F_{z_i \ z_i} \geq 0, \ i = 1, 2, \dots, k$$

$$T_{z_i \ z_i} \geq I_{z_i \ z_i} , \ i = 1, 2, \dots, k$$

$$T_{z_i \ z_i} \geq F_{z_i \ z_i} , \ i = 1, 2, \dots, k$$

$$g_j \ x \leq b_j , \ j = 1, 2, \dots, m$$

$$x_i \geq 0, \ i = 1, 2, \dots, n$$

where $T_{z_i \ z_i}$, $I_{z_i \ z_i}$ and $F_{z_i \ z_i}$ are truth membership function ,indeterminacy membership function ,falsity membership function of neutrosophic decision set respectively.

Now the neutrosophic goal programming (NSGP) in model (3) can be represented by crisp programming model using truth membership, indeterminacy membership and falsity membership functions as

$$\text{Maximize } \alpha, \text{Maximize } \gamma, \text{Minimize } \beta \tag{4}$$

$$T_{z_i \ z_i} \geq \alpha, \ i = 1, 2, \dots, k$$

$$I_{z_i \ z_i} \geq \gamma, \ i = 1, 2, \dots, k$$

$$F_{z_i \ z_i} \leq \beta, \ i = 1, 2, \dots, k$$

$$z_i \leq t_i, \ i = 1, 2, \dots, k$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha, \gamma \geq 0, \beta \leq 1;$$

$$g_j \ x \leq b_j , \ j = 1, 2, \dots, m$$

$$x_i \geq 0, \ i = 1, 2, \dots, n$$

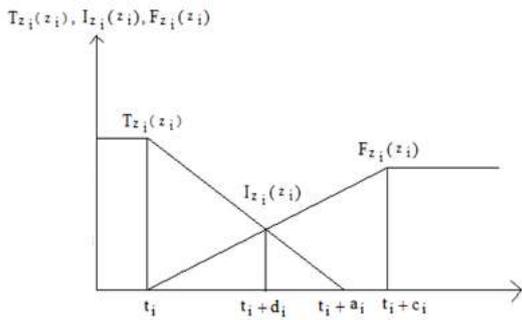


Fig.1.Truth membership, Indeterminacy membership and Falsity membership function of z_i

With the help of truth, indeterminacy, falsity membership function the neutrosophic goal programming problem (4) based on arithmetic aggregation operator can be formulated as

$$\text{Minimize } \left\{ \frac{1-\alpha + \beta + 1-\gamma}{3} \right\} \quad (5)$$

$$z_i \leq t_i + a_i \quad 1-\alpha, i=1,2,\dots,k$$

$$z_i \geq t_i + d_i \gamma, i=1,2,\dots,k$$

$$z_i \leq t_i + a_i - \gamma \quad a_i - d_i, i=1,2,\dots,k$$

$$z_i \leq t_i + c_i \beta, i=1,2,\dots,k$$

$$z_i \leq t_i, i=1,2,\dots,k$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha \in 0,1, \beta \in 0,1, \gamma \in 0,1 ;$$

$$g_j \leq b_j, j=1,2,\dots,m;$$

$$x_i \geq 0, i=1,2,\dots,n$$

With the help of truth, indeterminacy, falsity membership function the neutrosophic goal programming problem (4) based on geometric aggregation operator can be formulated as

$$\text{Minimize } \sqrt[3]{1-\alpha \beta 1-\gamma} \quad (6)$$

$$z_i \leq t_i + a_i \quad 1-\alpha, i=1,2,\dots,k$$

$$z_i \geq t_i + d_i \gamma, i=1,2,\dots,k$$

$$z_i \leq t_i + a_i - \gamma \quad a_i - d_i, i=1,2,\dots,k$$

$$z_i \leq t_i + c_i \beta, i=1,2,\dots,k$$

$$z_i \leq t_i, i=1,2,\dots,k$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha \in 0,1, \beta \in 0,1, \gamma \in 0,1 ;$$

$$g_j \leq b_j, j=1,2,\dots,m;$$

$$x_i \geq 0, i=1,2,\dots,n$$

Now these non-linear programming problems (5) and (6) can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (2) by neutrosophic goal optimization approach.

V. SOLUTION OF MULTI-OBJECTIVE STRUCTURAL OPTIMIZATION PROBLEM (MOSOP) BY NEUTROSOPHIC GOAL OPTIMIZATION TECHNIQUE

The multi-objective neutrosophic fuzzy structural model can be expressed as

Minimize $WT A$ with target value WT_0 , truth tolerance a_{WT} , indeterminacy tolerance d_{WT} and rejection tolerance c_{WT} (7)

minimize δA with target value δ_0 , truth tolerance a_{δ_0} , indeterminacy tolerance d_{δ_0} and rejection tolerance c_{δ_0}

subject to $\sigma A \leq \sigma$

$$A^{\min} \leq A \leq A^{\max}$$

where $A = [A_1, A_2, \dots, A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar.

To solve this problem we first calculate truth, indeterminacy and falsity membership function of objective as follows

$$T_{WT(A)} WT A = \begin{cases} 1 & \text{if } WT A \leq WT_0 \\ \left(\frac{WT_0 + a_{WT} - WT A}{a_{WT}} \right) & \text{if } WT_0 \leq WT A \leq WT_0 + a_{WT} \\ 0 & \text{if } WT A \geq WT_0 + a_{WT} \end{cases}$$

$$I_{WT(A)} WT A = \begin{cases} 0 & \text{if } WT A \leq WT_0 \\ \left(\frac{WT A - WT_0}{d_{WT}} \right) & \text{if } WT_0 \leq WT A \leq WT_0 + a_{WT} \\ \left(\frac{WT_0 + a_{WT} - WT A}{a_{WT} - d_{WT}} \right) & \text{if } WT_0 + d_{WT} \leq WT A \leq WT_0 + a_{WT} \\ 0 & \text{if } WT A \geq WT_0 + a_{WT} \end{cases}$$

$$\text{where } d_{WT} = \frac{1}{\frac{1}{a_{WT}} + \frac{1}{c_{WT}}}$$

$$F_{WT(A)} WT A = \begin{cases} 0 & \text{if } WT A \leq WT_0 \\ \left(\frac{WT A - WT_0}{c_{WT}} \right) & \text{if } WT_0 \leq WT A \leq WT_0 + c_{WT} \\ 1 & \text{if } WT A \geq WT_0 + c_{WT} \end{cases}$$

and

$$T_{\delta(A)} \delta A = \begin{cases} 1 & \text{if } \delta A \leq \delta_0 \\ \left(\frac{\delta_0 + a_{\delta_0} - \delta A}{a_{\delta_0}} \right) & \text{if } \delta_0 \leq \delta A \leq \delta_0 + a_{\delta_0} \\ 0 & \text{if } \delta A \geq \delta_0 + a_{\delta_0} \end{cases}$$

$$I_{\delta(A)} \delta A = \begin{cases} 0 & \text{if } \delta A \leq \delta_0 \\ \left(\frac{\delta A - \delta_0}{d_{\delta_0}} \right) & \text{if } \delta_0 \leq \delta A \leq \delta_0 + a_{\delta_0} \\ \left(\frac{\delta_0 + a_{\delta_0} - \delta A}{a_{\delta_0} - d_{\delta_0}} \right) & \text{if } \delta_0 + d_{\delta_0} \leq \delta A \leq \delta_0 + a_{\delta_0} \\ 0 & \text{if } \delta A \geq \delta_0 + a_{\delta_0} \end{cases}$$

where $d_\delta = \frac{1}{\frac{1}{a_\delta} + \frac{1}{c_\delta}}$

$$F_{\delta A} = \begin{cases} 0 & \text{if } \delta A \leq \delta_0 \\ \left(\frac{\delta A - \delta_0}{c_\delta} \right) & \text{if } \delta_0 \leq \delta A \leq \delta_0 + c_\delta \\ 1 & \text{if } \delta A \geq \delta_0 + c_\delta \end{cases}$$

According to neutrosophic goal optimization technique using truth, indeterminacy and falsity membership function, MOSOP (7) can be formulated as

Maximize α , Maximize γ , Minimize β (8)

$WT A \leq WT_0 + a_{WT} (1 - \alpha)$,

$WT A \geq WT_0 + d_{WT} \gamma$,

$WT A \leq WT_0 + a_{WT} - \gamma (a_{WT} - d_{WT})$,

$WT A \leq WT_0 + c_{WT} \beta$,

$WT A \leq WT_0$,

$\delta A \leq \delta_0 + a_\delta (1 - \alpha)$,

$\delta A \geq \delta_0 + d_\delta \gamma$,

$\delta A \leq \delta_0 + a_\delta - \gamma (a_\delta - d_\delta)$,

$\delta A \leq \delta_0 + c_\delta \beta$,

$\delta A \leq \delta_0$,

$0 \leq \alpha + \beta + \gamma \leq 3$;

$\alpha \in 0,1$, $\gamma \in 0,1$, $\beta \in 0,1$;

$g_j x \leq b_j, j=1,2,\dots,m$

$x_j \geq 0, j=1,2,\dots,n$

With the help of truth, indeterminacy, falsity membership function the neutrosophic goal programming problem (8) based on arithmetic aggregation operator can be formulated as

Model –I

Minimize $\left\{ \frac{1 - \alpha + \beta + 1 - \gamma}{3} \right\}$ (9)

$WT A \leq WT_0 + a_{WT} (1 - \alpha)$,

$WT A \geq WT_0 + d_{WT} \gamma$,

$WT A \leq WT_0 + a_{WT} - \gamma (a_{WT} - d_{WT})$,

$WT A \leq WT_0 + c_{WT} \beta$,

$WT A \leq WT_0$,

$\delta A \leq \delta_0 + a_\delta (1 - \alpha)$,

$\delta A \geq \delta_0 + d_\delta \gamma$,

$\delta A \leq \delta_0 + a_\delta - \gamma (a_\delta - d_\delta)$,

$\delta A \leq \delta_0 + c_\delta \beta$,

$\delta A \leq \delta_0$,

$0 \leq \alpha + \beta + \gamma \leq 3$;

$\alpha \in 0,1$, $\beta \in 0,1$, $\gamma \in 0,1$;

$g_j \leq b_j, j=1,2,\dots,m$;

$x_i \geq 0, i=1,2,\dots,n$

With the help of truth, indeterminacy, falsity membership function the neutrosophic goal programming problem (8) based on geometric aggregation operator can be formulated as

Model –II

Minimize $\sqrt[3]{1 - \alpha \beta (1 - \gamma)}$ (10)

$WT A \leq WT_0 + a_{WT} (1 - \alpha)$,

$WT A \geq WT_0 + d_{WT} \gamma$,

$WT A \leq WT_0 + a_{WT} - \gamma (a_{WT} - d_{WT})$,

$WT A \leq WT_0 + c_{WT} \beta$,

$WT A \leq WT_0$,

$\delta A \leq \delta_0 + a_\delta (1 - \alpha)$,

$\delta A \geq \delta_0 + d_\delta \gamma$,

$\delta A \leq \delta_0 + a_\delta - \gamma (a_\delta - d_\delta)$,

$\delta A \leq \delta_0 + c_\delta \beta$,

$\delta A \leq \delta_0$,

$0 \leq \alpha + \beta + \gamma \leq 3$;

$\alpha \in 0,1$, $\beta \in 0,1$, $\gamma \in 0,1$;

$g_j \leq b_j, j=1,2,\dots,m$;

$x_i \geq 0, i=1,2,\dots,n$

Now these non-linear programming Model-I, II can be easily solved through an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (7) by neutrosophic goal optimization approach.

VI. NUMERICAL ILLUSTRATION

A well-known three bar planer truss is considered to minimize weight of the structure $WT A_1, A_2$ and minimize the deflection $\delta A_1, A_2$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members

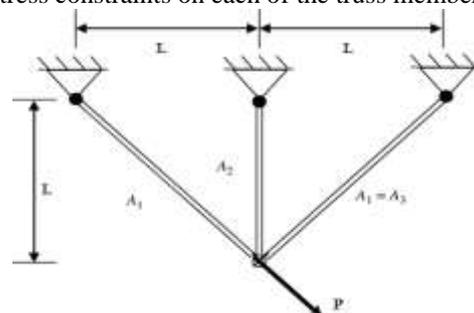


Fig 2. Design of three bar planer truss

The multi-objective optimization problem can be stated as follows

$$\text{Minimize } WT \ A_1, A_2 = \rho L \ 2\sqrt{2}A_1 + A_2 \quad (11)$$

$$\text{Minimize } \delta \ A_1, A_2 = \frac{PL}{E \ A_1 + \sqrt{2}A_2}$$

Subject to

$$\sigma_1 \ A_1, A_2 \equiv \frac{P \ \sqrt{2}A_1 + A_2}{2A_1^2 + 2A_1A_2} \leq [\sigma_1^T];$$

$$\sigma_2 \ A_1, A_2 \equiv \frac{P}{A_1 + \sqrt{2}A_2} \leq [\sigma_2^T];$$

$$\sigma_3 \ A_1, A_2 \equiv \frac{PA_2}{2A_1^2 + 2A_1A_2} \leq [\sigma_3^C];$$

$$A_i^{\min} \leq A_i \leq A_i^{\max} \quad i = 1, 2$$

where $P =$ applied load ; $\rho =$ material density ; $L =$ length ; $E =$ Young's modulus ; $A_1 =$ Cross section of bar-1 and bar-3; $A_2 =$ Cross section of bar-2; δ is deflection of loaded joint. $[\sigma_1^T]$ and $[\sigma_2^T]$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, $[\sigma_3^C]$ is maximum allowable compressive stress for bar 3.

This multi objective structural model can be expressed as neutrosophic fuzzy model as

$$\text{Minimize } WT \ A_1, A_2 = \rho L \ 2\sqrt{2}A_1 + A_2 \quad \text{with target value } 4 \times 10^2 \text{ KN, truth tolerance } 2 \times 10^2 \text{ KN, indeterminacy tolerance } \frac{1}{0.5+0.22} \times 10^2 \text{ KN and rejection tolerance } 4.5 \times 10^2 \text{ KN} \quad (12)$$

$$\text{Minimize } \delta \ A_1, A_2 = \frac{PL}{E \ A_1 + \sqrt{2}A_2} \quad \text{with target value } 2.5 \times 10^{-7} \text{ m, truth tolerance } 2.5 \times 10^{-7} \text{ m, indeterminacy tolerance } \frac{1}{0.4+0.22} \times 10^{-7} \text{ m and rejection tolerance } 4.5 \times 10^{-7} \text{ m}$$

Subject to

$$\sigma_1 \ A_1, A_2 \equiv \frac{P \ \sqrt{2}A_1 + A_2}{2A_1^2 + 2A_1A_2} \leq [\sigma_1^T];$$

$$\sigma_2 \ A_1, A_2 \equiv \frac{P}{A_1 + \sqrt{2}A_2} \leq [\sigma_2^T];$$

$$\sigma_3 \ A_1, A_2 \equiv \frac{PA_2}{2A_1^2 + 2A_1A_2} \leq [\sigma_3^C];$$

$$A_i^{\min} \leq A_i \leq A_i^{\max} \quad i = 1, 2$$

According to neutrosophic goal optimization technique using truth, indeterminacy and falsity membership function ,MOSOP (12) can be formulated as

$$\text{Maximize } \alpha, \text{Maximize } \gamma, \text{Minimize } \beta \quad (13)$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2 \ 1 - \alpha \ ,$$

$$2\sqrt{2}A_1 + A_2 \geq 4 + \frac{1}{0.5+0.22} \gamma,$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2 - \gamma \left(2 - \frac{1}{0.5+0.22} \right),$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 4.5\beta,$$

$$2\sqrt{2}A_1 + A_2 \leq 4,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5 \ 1 - \alpha \ ,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \geq 2.5 + \frac{1}{0.4+0.22} \gamma,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5 - \gamma \left(2.5 - \frac{1}{0.4+0.22} \right),$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 4.5\beta,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5,$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha \in 0, 1 \ , \gamma \in 0, 1 \ , \beta \in 0, 1 \ ;$$

$$\frac{20 \ \sqrt{2}A_1 + A_2}{2A_1^2 + 2A_1A_2} \leq 20;$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 20;$$

$$\frac{20A_2}{2A_1^2 + 2A_1A_2} \leq 15;$$

$$0.1 \leq A_i \leq 5 \quad i = 1, 2$$

With the help of truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming problem (13) based on arithmetic aggregation operator can be formulated as

Model -I

$$\text{Minimize } \left\{ \frac{1 - \alpha + \beta + 1 - \gamma}{3} \right\}$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2 \ 1 - \alpha \ ,$$

$$2\sqrt{2}A_1 + A_2 \geq 4 + \frac{1}{0.5+0.22} \gamma,$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2 - \gamma \left(2 - \frac{1}{0.5 + 0.22} \right),$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 4.5\beta,$$

$$2\sqrt{2}A_1 + A_2 \leq 4,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5(1 - \alpha),$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \geq 2.5 + \frac{1}{0.4 + 0.22}\gamma,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5 - \gamma \left(2.5 - \frac{1}{0.4 + 0.22} \right),$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 4.5\beta,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5,$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha \in [0, 1], \beta \in [0, 1], \gamma \in [0, 1];$$

$$\frac{20\sqrt{2}A_1 + A_2}{2A_1^2 + 2A_1A_2} \leq 20;$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 20;$$

$$\frac{20A_2}{2A_1^2 + 2A_1A_2} \leq 15;$$

$$0.1 \leq A_i \leq 5 \quad i = 1, 2$$

With the help of truth, indeterminacy, falsity membership function the neutrosophic goal programming problem (13) based on geometric aggregation operator can be formulated as

Model –II

$$\text{Minimize } \sqrt[3]{1 - \alpha - \beta - \gamma} \tag{14}$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2(1 - \alpha),$$

$$2\sqrt{2}A_1 + A_2 \geq 4 + \frac{1}{0.5 + 0.22}\gamma,$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 2 - \gamma \left(2 - \frac{1}{0.5 + 0.22} \right),$$

$$2\sqrt{2}A_1 + A_2 \leq 4 + 4.5\beta,$$

$$2\sqrt{2}A_1 + A_2 \leq 4,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5(1 - \alpha),$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \geq 2.5 + \frac{1}{0.4 + 0.22}\gamma,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + 2.5 - \gamma \left(2.5 - \frac{1}{0.4 + 0.22} \right),$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5 + \frac{4.5}{w_3}\beta,$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 2.5,$$

$$0 \leq \alpha + \beta + \gamma \leq 3;$$

$$\alpha \in [0, 1], \beta \in [0, 1], \gamma \in [0, 1];$$

$$\frac{20\sqrt{2}A_1 + A_2}{2A_1^2 + 2A_1A_2} \leq 20;$$

$$\frac{20}{A_1 + \sqrt{2}A_2} \leq 20;$$

$$\frac{20A_2}{2A_1^2 + 2A_1A_2} \leq 15;$$

$$0.1 \leq A_i \leq 5 \quad i = 1, 2$$

Now these non-linear programming problems Model-I,II can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (12) by generalized neutrosophic goal optimization approach and the results are shown in the table 1 as follows. Here we get best solution in geometric aggregation method for objective functions.

Table 1: Input data for crisp model (11)

Applied load P KN	Volume density ρ KN/m^3	Length L m	Maximum allowable tensile stress $[\sigma^T]$ KN/m^2	Maximum allowable compressive stress $[\sigma^C]$ KN/m^2	Young's modulus E KN/m^2	A_i^{\min} and A_i^{\max} of cross section of bars $10^{-4}m^2$
20	100	1	20	15	2×10^7	$A_i^{\min} = 0.1$ $A_i^{\max} = 5, i = 1, 2$

Table 2: Comparison solution of MOSOP (2) based on different Aggregation

Methods	A_1 $\times 10^{-4} m^2$	A_2 $\times 10^{-4} m^2$	WT A_1, A_2 $\times 10^3 KN$	$\delta A_1, A_2$ $\times 10^{-7} m$
Neutrosophic optimization (NSGP) based on Arithmetic Aggregation	5	0.4321468	4.904282	3.564333
Neutrosophic optimization (NSGP) based on Geometric Aggregation	5	1.109954	4.462428	3.044273

VII. CONCLUSIONS

The research study investigates that neutrosophic goal programming can be utilized to optimize a nonlinear structural problem. The results obtained for different aggregation method of the undertaken problem show that the best result is achieved using geometric aggregation method. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. As we have considered a non-linear three bar truss design problem and find out minimum weight of the structure as well as minimum deflection of loaded joint, the results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in different field.

Conflict of interests: The authors declare that there is no conflict of interests.

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