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Possibility Neutrosophic Vague Soft Expert Set for Decision under Uncertainty

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Abstract. In this paper, we extend the notion of classical soft expert sets to possibility neutrosophic vague soft expert sets by applying the theory of soft expert sets to possibility neutrosophic vague soft sets. The subset, complement, union, intersection, AND and OR operations as well as some related concepts pertaining to this notion are defined.

INTRODUCTION

Most real-life problems involve data with a high level of uncertainty and imprecision. Traditionally, classical mathematical theories such as fuzzy mathematics, probability theory, and interval mathematics are used to deal with uncertainties and fuzziness. However, all these theories have their inherent difficulties and weaknesses. This led to the introduction of the theory of soft sets by Molodtsov [1]. Molodtsov mentioned a soft set as a mathematical way to represent and solve these problems with uncertainties which traditional mathematical tools cannot handle. Alkhazaleh et al. [2] introduced the concept of soft multiset as a generalization of Molodtsov's soft set, followed by possibility fuzzy soft set [3] and multiparameterized soft set [4]. Adam and Hassan expanded further to Q-fuzzy soft sets [5-11]. Vague set theory was proposed by Gau and Buehrer [12] and vague soft set by Xu et al. [13]. Alkhazaleh and Salleh then proceeded to introduce the notion of soft expert set [14] followed by fuzzy soft expert sets [15], while Hassan and Alhazaymeh introduced the theory of vague soft expert sets [16] followed by the mapping on generalized vague soft expert set [17]. They also introduced the concept of possibility vague soft set [18], interval-valued vague soft sets [19, 20] and other variants of vague soft sets [21-26]. Smarandache [27] proposed the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data. Maji [28] introduced neutrosophic soft set which can be viewed as a new path of thinking for engineers, mathematicians, computer scientists and others. Bashir et al. [29] introduced the concept of possibility intuitionistic fuzzy soft set. Selvachandran and Salleh [30] proposed the theory of possibility intuitionistic fuzzy soft expert set as a generalization of possibility fuzzy soft expert set [31] followed by possibility vague soft expert theory [32]. As a combination of neutrosophic set and vague set, Alkhazaleh [33] introduced the concept of neutrosophic vague set, followed by Al-Quran and Hassan [34, 35] on the notion of neutrosophic vague soft expert set theory and fuzzy parameterized single valued neutrosophic soft expert set.

We will further extend the studies on possibility vague soft expert theory [32] and neutrosophic vague set [33] through the establishment of the notion of possibility neutrosophic vague soft expert sets (denoted as PNVSES from now on). PNVSESs can better handle the elements of imprecision and uncertainty compared to the other generalizations of soft expert sets [14] such as fuzzy soft expert sets [15], vague soft expert sets [16], possibility vague soft sets [18] and neutrosophic vague soft expert set [34]. The PNVSES model is also significantly more advantageous compared to possibility fuzzy soft sets [3] and possibility vague soft set [18] as it has the added advantage of allowing the users to know the opinion of all the experts in one model. Moreover, even after performing any operations, the users can still know the opinion of all the experts. In line with this, the purpose of this paper is to extend the classical soft expert set model [14] to the PNVSES model and thereby establish a new generalization of the soft expert set model called the possibility neutrosophic vague soft expert set (PNVSES).

PRELIMINARIES

In this section, we review some basic notions in neutrosophic vague set, possibility fuzzy soft expert set and neutrosophic vague soft expert set. We give a definition of possibility neutrosophic vague soft set and neutrosophic vague soft set. We also give some basic operations on neutrosophic vague soft set .

We begin by recalling the definition of neutrosophic vague set and its basic operations, namely subset, complement, union and intersection.

Definition 1. (See [33]) A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as

$$A_{NV} = \{< x; \widehat{T}_{A_{NV}}(x); \widehat{I}_{A_{NV}}(x); \widehat{F}_{A_{NV}}(x) >; x \in X\},$$

whose truth membership, indeterminacy membership and false membership functions is defined as:

$\widehat{T}_{A_{NV}}(x) = [T^-, T^+]$, $\widehat{I}_{A_{NV}}(x) = [I^-, I^+]$ and $\widehat{F}_{A_{NV}}(x) = [F^-, F^+]$ where,

1. $T^+ = 1 - F^-$,
2. $F^+ = 1 - T^-$, and
3. $^-0 \leq T^- + I^- + F^- \leq 2^+$.

Definition 2. (See [33]) Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$,
(1) $\widehat{T}_{A_{NV}}(u_i) \leq \widehat{T}_{B_{NV}}(u_i)$, (2) $\widehat{I}_{A_{NV}}(u_i) \geq \widehat{I}_{B_{NV}}(u_i)$ and (3) $\widehat{F}_{A_{NV}}(u_i) \geq \widehat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \leq i \leq n$.

Definition 3. (See [33]) The complement of a NVS A_{NV} is denoted by A^c and is defined by $\widehat{T}^c_{A_{NV}}(x) = [1 - T^+, 1 - T^-]$, $\widehat{I}^c_{A_{NV}}(x) = [1 - I^+, 1 - I^-]$ and $\widehat{F}^c_{A_{NV}}(x) = [1 - F^+, 1 - F^-]$.

Definition 4. (See [33]) The union of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned}
\widehat{T}_{H_{NV}}(x) &= \left[\max\left(T^-_{A_{NV}x}, T^-_{B_{NV}x}\right), \max\left(T^+_{A_{NV}x}, T^+_{B_{NV}x}\right)\right], \\
\widehat{I}_{H_{NV}}(x) &= \left[\min\left(I^-_{A_{NV}x}, I^-_{B_{NV}x}\right), \min\left(I^+_{A_{NV}x}, I^+_{B_{NV}x}\right)\right] \text{ and} \\
\widehat{F}_{H_{NV}}(x) &= \left[\min\left(F^-_{A_{NV}x}, F^-_{B_{NV}x}\right), \min\left(F^+_{A_{NV}x}, F^+_{B_{NV}x}\right)\right].
\end{aligned}$$

Definition 5. (See [33]) The intersection of two NVSs A_{NV} and B_{NV} is a NVS C_{NV}, written as $H_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$$\begin{aligned}
\widehat{T}_{H_{NV}}(x) &= \left[\min\left(T^-_{A_{NV}x}, T^-_{B_{NV}x}\right), \min\left(T^+_{A_{NV}x}, T^+_{B_{NV}x}\right)\right], \\
\widehat{I}_{H_{NV}}(x) &= \left[\max\left(I^-_{A_{NV}x}, I^-_{B_{NV}x}\right), \max\left(I^+_{A_{NV}x}, I^+_{B_{NV}x}\right)\right] \text{ and} \\
\widehat{F}_{H_{NV}}(x) &= \left[\max\left(F^-_{A_{NV}x}, F^-_{B_{NV}x}\right), \max\left(F^+_{A_{NV}x}, F^+_{B_{NV}x}\right)\right].
\end{aligned}$$

Al-Quran and Hassan [34] extended the concept of neutrosophic vague set to neutrosophic vague soft expert set below.

Definition 6. (See [34]) Let U be a universe, E a set of parameters, X a set of experts (agents), and $O = \{1 = agree, 0 = disagree\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$.

A pair (F, A) is called a neutrosophic vague soft expert set over U, where F is a mapping given by

$$F : A \rightarrow NV^U,$$

where NV^U denotes the power neutrosophic vague set of U.

Bashir and Salleh [31] conceptualized possibility fuzzy soft expert set and gave the basic operations on subset, complement, union and intersection as in the following five definitions.

Definition 7. (See [31]) Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements, $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters, X be a set of experts, and $O = \{1 = agree, 0 = disagree\}$ a set of opinions . Let $Z = E \times X \times O$ and $A \subseteq Z$. Let $F : Z \to I^U$ and μ be a fuzzy subset of Z , i.e. $\mu : Z \to I^U$, where I^U is the collection of all fuzzy subsets of U . Let $F_\mu : Z \to I^U \times I^U$ be a function defined as fellows:

$$F_\mu(z) = (F(z)(u),\ \mu(z)(u)), \forall u \in U.$$

Then F_μ is called a possibility fuzzy soft expert set (PFSES in short) over the soft universe (U,Z).

Definition 8. (See [31]) Let (F_μ, A) and (G_δ, B) be two PFSESs over (U,Z). Then (F_μ, A) is said to be a possibility fuzzy soft expert subset (PFSE subset) of (G_δ, B) and we write $(F_\mu, A) \subseteq (G_\delta, B)$, if $A \subseteq B$ and $\forall \varepsilon \in A$,

1. $\mu(\varepsilon)$ is a fuzzy subset of $\delta(\varepsilon)$,
2. $F(\varepsilon)$ is a a fuzzy subset of $G(\varepsilon)$.

Definition 9. (See [31]) Let (F_μ, A) be a PFSES over (U,Z) . Then the complement of (F_μ, A) denoted by $(F_\mu, A)^c$ is defined by

$$(F_\mu, A)^c = (\widetilde{c}(F(\neg\alpha)), c(\mu(\neg\alpha))) \ , \forall \alpha \in \neg A,$$

where $\widetilde{c}$ is a fuzzy soft expert complement and c is a fuzzy complement.

Definition 10. (See [31]) Union of two PFSESs (F_μ, A) and (G_δ, B) over U , denoted by $(F_\mu, A)\widetilde{\cup}(G_\delta, B)$, is a PFSES (H_υ, C) where $C = A \cup B$, is defined by

$$\upsilon(\alpha) = s(\mu(\alpha), \delta(\alpha)),\ \forall \alpha \in C,$$

and

$$H(\alpha) = F(\alpha)\widetilde{\cup}G(\alpha),\ \forall \alpha \in C,$$

where S is an S-norm and $\widetilde{\cup}$ is a fuzzy soft expert union.

Definition 11. (See [31]) Intersection of two PFSESs (F_μ, A) and (G_δ, B) over U , denoted by $(F_\mu, A)\widetilde{\cap}(G_\delta, B)$, is a PFSES (H_υ, C) where $C = A \cup B$, is defined by

$$\upsilon(\alpha) = t(\mu(e), \delta(e)),\ \forall e \in C,$$

and

$$H(\alpha) = F(e)\widetilde{\cap}G(e),\ \forall e \in C,$$

where t is an t-norm and $\widetilde{\cap}$ is a fuzzy soft expert intersection.

We will now propose a definition of neutrosophic vague soft set and its basic operations of union and intersection, which will then lead to the concept of possibility neutrosophic vague soft set.

Definition 12. Let U be an initial universal set and let E be a set of parameters. Let $NV(U)$ denote the power set of all neutrosophic vague subsets of U and let $A \subseteq E$. A collection of pairs $(\widehat{F}, E)$ is called a neutrosophic vague soft set (NVSS) over U where $\widehat{F}$ is a mapping given by

$$\widehat{F} : A \to NV(U).$$

Definition 13. The union of two NVSSs $(\widehat{F},A)$ and $(\widehat{G},B)$ over a universe U, is a NVSS $(\widehat{H},C)$, denoted by $(\widehat{F},A)\widetilde{\cup}(\widehat{G},B)$, such that $C = A\cup B$ and $\forall e \in C$,

$$(\widehat{H},C) = \begin{cases} \widehat{F}(e), & \text{if } e \in A-B\ ; \\ \widehat{G}(e), & \text{if } e \in B-A\ ; \\ \widehat{F}(e)\widetilde{\cup}\widehat{G}(e), & \text{if } e \in A\cap B\ , \end{cases}$$

where $\widetilde{\cup}$ denotes the neutrosophic vague set union.

Definition 14. The intersection of two NVSSs $(\widehat{F},A)$ and $(\widehat{G},B)$ over a universe U, is a NVSS $(\widehat{K},C)$, denoted by $(\widehat{F},A)\widetilde{\cap}(\widehat{G},B)$, such that $C = A\cup B$ and $\forall e \in C$,

$$(\widehat{H},C) = \begin{cases} \widehat{F}(e), & \text{if } e \in A-B\ ; \\ \widehat{G}(e), & \text{if } e \in B-A\ ; \\ \widehat{F}(e)\tilde{\cap}\widehat{G}(e), & \text{if } e \in A\cap B\ , \end{cases}$$

where $\widetilde{\cap}$ denotes the neutrosophic vague set intersection.

We will now give a definition of possibility neutrosophic vague soft set below, by extending the concept of neutrosophic vague soft set defined earlier.

Definition 15. Let $U = \{x_1, x_2, ..., x_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. Let $F : E \to NV(U)$, where $NV(U)$ is the collection of all neutrosophic vague subsets of U and μ be a fuzzy subset of E, that is $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to NV(U)\times I^U$ be a function defined as follows:

$$F_\mu(e) = (F(e)(x),\ \mu(e)(x)), \forall x \in U.$$

Then F_μ is called a possibility neutrosophic vague soft set (PNVSS in short) over the soft universe (U,E).

POSSIBILITY NEUTROSOPHIC VAGUE SOFT EXPERT SET

In this section we introduce the concept of possibility neutrosophic vague soft expert set and define some operations of subset, equality, null, absolute, complement, union, intersection, AND and OR.

Now we propose the definition of a possibility neutrosophic vague soft expert set.

Definition 16. Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters, X be a set of experts, and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Z = \{E\times X\times O\}$ and $A \subseteq Z$. Let $F : Z \to NV(U)$, where $NV(U)$ is the collection of all neutrosophic vague subsets of U and μ be a fuzzy subset of Z, that is $\mu : Z \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : Z \to NV(U)\times I^U$ be a function defined as follows:

$$F_\mu(z) = (F(z)(u),\ \mu(z)(u)), \forall u \in U.$$

Then F_μ is called a possibility neutrosophic vague soft expert set (PNVSES in short) over the soft universe (U,Z).

For each parameter z_i , $F_\mu(z_i) = (F(z_i)(u)\ ,\mu(z_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(z_i)$, but also the degree of possibility of belongingness of the elements of U in $F(z_i)$, which is represented by $\mu(z_i)$. Thus we can write $F_\mu(z_i)$ as follows:

$$F_\mu(z_i) = \left\{\left(\tfrac{u_i}{F(z_i)(u_i)}, \mu(z_i)(u_i)\right)\right\},\ \text{for } i = 1,2,3,...,$$

where $F(z_i)(u_i) = \langle\ [T^-_{F(Z_i)}(u_i), T^+_{F(Z_i)}(u_i)], [I^-_{F(z_i)}(u_i), I^+_{F(z_i)}(u_i)], [F^-_{F(Z_i)}(u_i), F^+_{F(Z_i)}(u_i)]\ \rangle$ and $T^+_{F(Z_i)}(u_i) = 1-F^-_{F(Z_i)}(u_i),\ F^+_{F(Z_i)}(u_i) = 1-T^-_{F(Z_i)}(u_i)$ with $[T^-_{F(Z_i)}(u_i), T^+_{F(Z_i)}(u_i)], [I^-_{F(z_i)}(u_i), I^+_{F(z_i)}(u_i)]$ and $[F^-_{F(Z_i)}(u_i), F^+_{F(Z_i)}(u_i)]$ representing the truth-membership function, indeterminacy-membership function and falsity-membership function of each of the elements $u_i \in U$, respectively .

We can write F_μ as $\left(F_\mu, Z\right)$. If $A \subseteq Z$ we can also have a PNVSES $\left(F_\mu, A\right)$.

In the following, we introduce the concept of the subset of two PNVSESs .

Definition 17. Let $\left(F_\mu, A\right)$ and (G_δ, B) be two PNVSESs over (U, Z). $\left(F_\mu, A\right)$ is said to be a possibility neutrosophic vague soft expert subset of (G_δ, B) if $A \subseteq B$ and for all $\varepsilon \in A$, the following conditions are satisfied:

1. $\mu(\varepsilon)$ is a fuzzy subset of $\delta(\varepsilon)$,
2. $F(\varepsilon)$ is a neutrosophic vague subset of $G(\varepsilon)$.

This relationship is denoted as $\left(F_\mu, A\right) \subseteq (G_\delta, B)$. In this case, (G_δ, B) is called a possibility neutrosophic vague soft expert superset (PNVSE superset) of $\left(F_\mu, A\right)$.

In the following, we propose the concept of the equality of two PNVSESs.

Definition 18. Let $\left(F_\mu, A\right)$ and (G_δ, B) be two PNVSESs over (U, Z). Then $\left(F_\mu, A\right)$ and (G_δ, B) are said to be equal and we write $\left(F_\mu, A\right) = (G_\delta, B)$ if $\left(F_\mu, A\right)$ is a PNVSE subset of (G_δ, B) and (G_δ, B) is a PNVSE subset of $\left(F_\mu, A\right)$. In other words, $\left(F_\mu, A\right) = (G_\delta, B)$ if for all $\varepsilon \in E$, the following conditions are satisfied:

1. $\mu(\varepsilon)$ is equal to $\delta(\varepsilon)$,
2. $F(\varepsilon)$ is equal to $G(\varepsilon)$.

In the following, we define the possibility null neutrosophic vague soft expert set and the possibility absolute neutrosophic vague soft expert set.

Definition 19. A PNVSES $\left(F_\mu, A\right)$ is said to be a possibility null neutrosophic vague soft expert set, denoted by $(\tilde{\phi_\mu}, A)$ and defined as

$$(\tilde{\phi_\mu}, A) = (F(\alpha), \mu(\alpha)), \forall \alpha \in Z,$$

where $F(\alpha) = ([0,0],[1,1],[1,1])$ and $\mu(\alpha) = 0, \forall \alpha \in Z$.

Definition 20. A PNVSES $\left(F_\mu, A\right)$ is said to be a possibility absolute neutrosophic vague soft expert set, denoted by $\left(F_\mu, A\right)_{abs}$ and defined as:

$$\left(F_\mu, A\right)_{abs} = (F(\alpha), \mu(\alpha)), \forall \alpha \in Z,$$

where $F(\alpha) = ([1,1],[0,0],[0,0])$ and $\mu(\alpha) = 1, \forall \alpha \in Z$.

Now, we put forward the definition of an agree- possibility neutrosophic vague soft expert set and the definition of a disagree- possibility neutrosophic vague soft expert set.

Definition 21. Let $\left(F_\mu, A\right)$ be a PNVSES over a soft universe (U, Z). An agree- possibility neutrosophic vague soft expert set $(F, A)_1$ over U is a possibility neutrosophic vague soft expert subset of (F, A) defined as follows:

$$(F, A)_1 = (F(\alpha), \mu(\alpha)), \quad \text{where } \alpha \in E \times X \times \{1\}.$$

Definition 22. Let $\left(F_\mu, A\right)$ be a PNVSES over a soft universe (U, Z). A disagree- possibility neutrosophic vague soft expert set $(F, A)_0$ over U is a possibility neutrosophic vague soft expert subset of (F, A) defined as follows:

$$(F, A)_0 = (F(\alpha), \mu(\alpha)), \quad \text{where } \alpha \in E \times X \times \{0\}.$$

In the following, we propose the definition of the complement of a PNVSES.

Definition 23. Let $\left(F_\mu, A\right)$ be a PNVSES over (U, Z). Then the complement of $\left(F_\mu, A\right)$, denoted by $\left(F_\mu, A\right)^c$ is defined by

$$\left(F_\mu, A\right)^c = (\tilde{c}(F(\alpha)), c(\mu(\alpha))) \quad , \forall \alpha \in A,$$

where $\tilde{c}$ is a neutrosophic vague complement and c is a fuzzy complement.

In the following, we introduce the definition of the union of two PNVSESs .

Definition 24. Let (F_μ, A) and (G_δ, B) be any two PNVSESs over a set universe (U, Z) . Then the union of (F_μ, A) and (G_δ, B) denoted by $(F_\mu, A) \tilde{\cup} (G_\delta, B)$ is a PNVSES defined as $(F_\mu, A) \tilde{\cup} (G_\delta, B) = (H_\gamma, C)$, where $C = A \cup B$ and

$$\gamma(\alpha) = max\,(\mu(\alpha), \delta(\alpha)),\ H(\alpha) = F(\alpha) \tilde{\cup} G(\alpha),$$

$\forall \alpha \in C$, where

$$H(\alpha) = \begin{cases} F(\alpha), & \text{if } \alpha \in A - B\,; \\ G(\alpha), & \text{if } \alpha \in B - A\,; \\ F(\alpha) \tilde{\cup} G(\alpha), & \text{if } e \in A \cap B\,, \end{cases}$$

and $\tilde{\cup}$ denotes the neutrosophic vague set union.

In the following, we introduce the definition of the intersection of two PNVSESs .

Definition 25. Let (F_μ, A) and (G_δ, B) be any two PNVSESs over a set universe (U, Z) . Then the intersection of (F_μ, A) and (G_δ, B) denoted by $(F_\mu, A) \tilde{\cap} (G_\delta, B)$ is a PNVSES defined as $(F_\mu, A) \tilde{\cap} (G_\delta, B) = (H_\gamma, C)$, where $C = A \cup B$ and

$$\gamma(\alpha) = min\,(\mu(\alpha), \delta(\alpha)),\ H(\alpha) = F(\alpha) \tilde{\cap} G(\alpha),$$

$\forall \alpha \in C$, where

$$H(\alpha) = \begin{cases} F(\alpha), & \text{if } \alpha \in A - B\,; \\ G(\alpha), & \text{if } \alpha \in B - A\,; \\ F(\alpha) \tilde{\cap} G(\alpha), & \text{if } e \in A \cap B\,, \end{cases}$$

and $\tilde{\cap}$ denotes the neutrosophic vague set intersection.

In the following , we introduce the definition of AND and OR operations for PNVSES.

Definition 26. Let (F_μ, A) and (G_δ, B) are two PNVSESs over a soft universe (U, Z). Then "(F_μ, A) AND (G_δ, B)", denoted by $(F_\mu, A)\ \tilde{\wedge} (G_\delta, B)$ is defined by

$$(F_\mu, A) \tilde{\wedge} (G_\delta, B) = (H_\lambda, A \times B),$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta), \lambda(\alpha, \beta))$, $\forall (\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = F(\alpha) \tilde{\cap} G(\beta)$ and $\lambda(\alpha, \beta) = \min(\mu(\alpha), \delta(\beta))$, $\forall (\alpha, \beta) \in A \times B$ and $\tilde{\cap}$ represents the basic neutrosophic vague intersection.

Definition 27. Let (F_μ, A) and (G_δ, B) are two PNVSESs over a soft universe (U, Z). Then "(F_μ, A) OR (G_δ, B)", denoted by $(F_\mu, A)\ \tilde{\vee} (G_\delta, B)$ is defined by

$$(F_\mu, A) \tilde{\vee} (G_\delta, B) = (H_\lambda, A \times B),$$

where $H_\lambda(\alpha, \beta) = (H(\alpha, \beta), \lambda(\alpha, \beta))$, $\forall (\alpha, \beta) \in A \times B$, such that $H(\alpha, \beta) = F(\alpha) \tilde{\cup} G(\beta)$ and $\lambda(\alpha, \beta) = \max(\mu(\alpha), \delta(\beta))$, $\forall (\alpha, \beta) \in A \times B$ and $\tilde{\cup}$ represents the basic neutrosophic vague union.

CONCLUSION

We established the concept of possibility neutrosophic vague soft expert set by applying the theory of soft expert sets to possibility neutrosophic vague soft sets. The basic operations on possibility neutrosophic vague soft expert set, namely complement, union, intersection, AND and OR operations were defined. This new extension will provide a significant addition to existing theories for handling indeterminacy and spurs more developments of further research and pertinent applications.

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