# The relativistic motion of RLC circuit

**Miroslav Pardy**
Department of Physical Electronics
Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

**Abstract**

The special theory of relativity formula for the photon Doppler effect is applied to the frequency of the $RLC$ circuit moving vith velocity $v$ relativly to the rest system. The relativistic transformation of the $RLC$ circuit components is derived in case the components are all in series with the voltage source $(R-L-C-v)$. The generalization to the more complex situation was not considered. It is not excluded that the article is the preamble for the future investigation of electronic physics and will be integral part of such instituions as Bell Laboratories, NASA, CERN and so on.

## 1 Introduction

An RLC circuit is an electrical circuit consisting of a resistor ($R$), an inductor ($L$), and a capacitor ($C$), connected in series or in parallel. The circuit forms a harmonic oscillator for current.

The three circuit elements, $R, L$ and $C$ can be combined in a number of different topologies. All three elements in series or all three elements in parallel are the simplest in concept and the most straightforward to analyse. There are, however, other arrangements, some with practical importance in real circuits. One issue often encountered is the need to take into account inductor resistance. Inductors are typically constructed from coils of wire, the resistance of which is not usually desirable, but it often has a significant effect on the circuit.

## 2 Series $RLC$ circuit

In the situation where we consider series $RLC$ circuit, the three components are all in series with the voltage source ($R-L-C-v$). The governing differential equation can be found by substituting into Kirchhoffs voltage law (KVL) the constitutive equation for each of the three elements. From KVL,

$$v_R + v_L + v_C = v(t), \tag{1}$$

where $v_R, v_L, v_C$ are the voltages across $R, L, C$ respectively and $v(t)$ is the time varying voltage from the source. Substituting the corresponding physical term, in eq (1), we get the following inegral differential equation:

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\tau=t} i(\tau)d\tau = v(t). \tag{2}$$

To our goal it is sufficient to consider the more simple situation with $v = 0$. Then instead of equation (2) we write

$$L\ddot{Q} + R\dot{Q} + Q/C = 0 \tag{3}$$

with stationary solution

$$Q = Ae^{-\frac{R}{2L}t}\sin(\omega t + \alpha), \tag{4}$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \tag{5}$$

The Thomson formula for the period of oscillations is when $R = 0$

$$T = 2\pi/\omega = 2\pi\sqrt{LC}. \tag{6}$$

## 3 The relativistic transformation of $R, L, C$

The special principle of relativity states that the laws of physics should be the same in all inertial frames. It has been foundational to physical thinking since the time of Galileo, and gained renewed prominence as Einsteins first postulate of relativity theory (Einstein, 1905):

If a system of coordinates $K$ is chosen so that, in relation to it, physical laws hold good in their simplest form, the same laws hold good in relation to any other system of coordinates $K'$ moving in uniform translation relatively to $K$. This postulate we call the special principle of relativity.

The comsequences of the Lorentz transformation of special theory of relativity is dilatation of time and contraction of length and the transformation of light frequency called the relativistic Doppler effect for photons. It is well known that when photons are measured in the frame that is moving toward the photon source with velocity $v$, then the measured frequency $\omega'$ of photon is given by the formula (Rohlf, 1994):

$$\omega' = \omega\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}. \tag{7}$$

When photons are measured in the frame that is moving away from the photon source with velocity $v$, then the measured frequency $\omega'$ of photon is given by the formula (Rohlf, 1994):

$$\omega' = \omega\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}. \tag{8}$$

Let us suppose that the same formulas are valid for the Thomson frequences of the $LC$ circuit in $K'$ , or

$$\omega' = \sqrt{\frac{1}{L'C'}}. \tag{9}$$

After comparison of eq. (9) with eq. (7) we have (toward $RLC$ source)

$$L'C' = LC\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}, \tag{10}$$

from which the symmetrical solution follows:

$$L' = L\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/4} \tag{11}$$

and

$$C' = C\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/4}. \tag{12}$$

In case taht we consider the sitution when $RLC$ frequency is measured in the frame that is moving away from the $RLC$ source moving with velocity $v$, then the measured quantity $L'$ and $C'$ of the Thomson oscillator is as follows:

$$L' = L\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/4} \tag{13}$$

and

$$C' = C\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{1/4}. \tag{14}$$

The transformed resistance $R$ follows form the equation (5) for the toward photon source:

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = \sqrt{\frac{1}{L'C'} - \frac{R'^2}{4L'^2}}. \tag{15}$$

After insertion of the transformation (11) and (12) into eq. (15), we get after some elementary operations:

$$R' = 2L\sqrt{\frac{1}{LC} - \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{3/2}}. \tag{16}$$

and in the second case with $RLC$ circuit moving away from the observer we get

$$R' = 2L\sqrt{\frac{1}{LC} - \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{3/2}}. \tag{17}$$

It is not excluded that the measurement of the quantities $R, L, C$ can be the crucial problems for such famous institutions as Bell Laborratories, NASA, CERN and so on.

## 4 Two $RLC$ circuits with the mutual induction

In this case the components $R_1, L_1, C_1$ are inductive boned with $R_2, L_2, C_2$ configuration. The problem was published by Landau et al. (1989) and the frequency of the inductive system was calculated in the form (for $R_1 = R_2 = 0$).

$$\omega_{1,2}^2 = \frac{L_1C_1 + L_2L_2 \pm [(L_1C_1 - L_2C_2)^2 + 4C_1C_2L_{12}^2]^{1/2}}{2C_1C_2(L_1L_2 - L_{12}^2)}. \qquad (18)$$

After performing the transformations of all known components in the last formula, we can eliminate the term of the mutual induction $L_{12}$.

## 5 Discusion

We have applied the special theory of relativity formula for the photon Doppler effect to the problem of transformation of the $RLC$ circuit moving vith velocity $v$ relativly to the rest system. We have supposed that the relativistic transformation of the light freuency is valid also for the frequency of of the $RLC$ circuit. We have considered only the situation where components are all in series with the voltage source $(R - L - C - v)$. The generalization to the more complex situation was not considered. We have considered the symmetrical solution of the problem. The nonsymmetrical solution is also possible, however we have supposed that the nature prefers the symmetrical solution. However, the spontaneously broken symmetry is not excluded. We here do not consider this Nobelian problem. The article forms the preamble of the future investigation of relativistic electronic systems (Nillson et al. 2008) and it will be, no doubt, the integral part of such institutions as Bell Laboratories, NASA, CERN and so on.

### References

Einstein, A. (1905). Zur Elektrodynamik bewegter Körper, Annalen der Physik **17**: 891; English translation: "On the Electrodynamics of Moving Bodies" by George Barker Jeffery and Wilfrid Perrett (1923); Another English translation: "On the Electrodynamics of Moving Bodies" by Megh Nad Saha (1920).

Landau, L. D. and Lifshitz, E. M., *Electrodynamics of continuous media*, (Pergamon press, Oxford OX3 OBW, England, 1989).

Nilsson, J. W. and Riedel, S. A., *Electric circuits*, (Prentice Hall, 2008). ISBN 0-13-198925-1.

Rohlf, J. W., *Modern Physics from $\alpha$ to $Z^0$*, (John Willey & Sons, Inc., New York, 1994).