Quantum equations in empty space using mutual energy and self-energy principle

Shuang-ren Zhao*

mutualenergy.org

Abstract

For photon we have obtained the results that the wave of photon obeys the mutual energy principle and self-energy principle. In this article we will extended the results for photon to other quantum. The mutual energy principle and self energy principle corresponding to the Schrödinger equation is introduced. The results are that a electron, for example, travel in the empty space from point A to point B, there are 4 different waves. The retarded wave started from point A. The advanced wave started from point B. The return waves corresponding to the above both waves. There are 5 different flow corresponding to these waves. The self-energy flow corresponding to the retarded wave, the self-energy flow corresponding to the advanced wave. The return flows corresponding the above two flows. The mutual energy flow of the retarded wave and the advanced wave. It is found the the mutual energy flow is the energy flow or the charge flow or electric current of the the electron. The electron travel in the space is a complicated process and do not only obey one Schrödinger equation. This result should be possible to further extend to to Dirac equation.

Keyword:Poynting;Maxwell;photon;retarded wave;advanced wave;time-reversal;absorber;emitter;action-at-a-distance;Schrödinger;electron

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* shrzhao@gmail.com;

I. INTRODUCTION

Maxwell equations have retarded solution and advanced solution. Wheeler and Feynman have introduced the absorber theory which involved the advanced wave [1][2]. The absorber theory is based on the action-at-a-distance [5, 16, 18]. In classical electromagnetic field theory the advanced wave is applied on mutual energy theorem, which is contribution of W.J. Welch [19], S.R. Zhao [6, 20, 21]. J. Cramer further worked on the absorber theory and introduced the transactional interpretation for quantum mechanics[3, 4].

This author combined the absorber theory and mutual energy theorem and introduced the concept that the photon energy is transferred by the mutual energy flow[9–15, 17]. And the further derived that mutual energy principle[7] and the self-energy principle[8]. The mutual energy principle says that the the electromagnetic field and the field for photon all should satisfy mutual energy principle. The solution of the mutual energy principle is retarded wave and an advanced wave. Both wave satisfies Maxwell equations. Both wave must synchronized.

The self-energy principle tells that the self-energy are returned. Hence the self-energy flow do not contributed to any energy transfer.

This article will apply the concept of the mutual energy principle and self-energy principle to other quanta for example electron. First we do not consider the spin in electron, hence assume the electron satisfy Schrödinger equation.

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II. SCHRÖDINGER EQUATION FOR RETARDED AND ADVANCED WAVE

We assume the quantum for example electron runs in the empty space from point A to B. This electron must satisfy in the the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2+V(\mathbf{r},t)\right]\Psi(\mathbf{r},t) \tag{1}$$

A. The retarded equation for point A

In empty space there is,

$$V(\mathbf{r},t)=0 \tag{2}$$

We have know that the wave $\Psi_a(\mathbf{r},t)$ is retarded wave started from point A which satisfies,

$$i\hbar\frac{\partial}{\partial t}\Psi_a(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a(\mathbf{r},t) \tag{3}$$

$\Psi_a(\mathbf{r},t)$ is retarded wave starting from point A.

B. the advanced equation for Point B

Advanced wave is obtained by a time reversal transform $\mathbf{R}$ which is defined by

$$\mathbf{R}\Psi(\mathbf{r},\mathrm{t})=\Psi(\mathbf{r},-\mathrm{t}), \tag{4}$$

Assume the Schrödinger equation is,

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2+V(\mathbf{r},t)\right]\Psi(\mathbf{r},t) \tag{5}$$

In empty space there is,

$$V(\mathbf{r},t)=0 \tag{6}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\mathbf{r},t) \tag{7}$$

The advanced wave corresponding retarded wave are,

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},-t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\mathbf{r},-t) \tag{8}$$

or

$$-i\hbar\frac{\partial}{\partial(-t)}\Psi(\mathbf{r},-t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\mathbf{r},-t) \tag{9}$$

Let $-t=\tau$

$$-i\hbar\frac{\partial}{\partial(\tau)}\Psi(\mathbf{r},\tau)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi(\mathbf{r},\tau) \tag{10}$$

This is the equation of the advanced wave. Assume in the Point B there are advanced wave which satisfy time reversal Schrödinger equation:

$$-i\hbar\frac{\partial}{\partial t}\Psi_b(\mathbf{r},t)=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b(\mathbf{r},t) \tag{11}$$

We have write τ as t. $\Psi_b(\mathbf{r},t)$ is the advanced wave starting from point B.

III. MUTUAL ENERGY FLOW

A. The mutual energy flow from A to B

Using Ψ_b multiply Eq(3) from right we have

$$(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b=\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_a\Psi_b \tag{12}$$

Using Ψ_a multiply Eq(11) from the left, we have

$$-i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b=\Psi_a\left[\frac{-\hbar^2}{2\mu}\nabla^2\right]\Psi_b \tag{13}$$

Subtract the Eq.(13) from Eq.(12) we obtain

$$(i\hbar\frac{\partial}{\partial t}\Psi_a)\Psi_b+i\hbar\Psi_a\frac{\partial}{\partial t}\Psi_b$$

$$=\frac{-\hbar^2}{2\mu}(\nabla^2\Psi_a\Psi_b-\Psi_a\nabla^2\Psi_b) \tag{14}$$

or

$$\frac{\partial}{\partial t}(\Psi_a\Psi_b)=-\frac{\hbar}{2\mu i}\nabla\cdot(\nabla\Psi_a\Psi_b-\Psi_a\nabla\Psi_b) \tag{15}$$

or

$$\frac{\partial}{\partial t}(\rho_{ab})=-\nabla\cdot J_{ab} \tag{16}$$

where

$$\rho_{ab}=\Psi_a\Psi_b \tag{17}$$

$$J_{ab}=\frac{\hbar}{2\mu i}(\nabla\Psi_a\Psi_b-\Psi_a\nabla\Psi_b) \tag{18}$$

The above formula are mutual energy flow principle. J_{ab} are mutual energy flow.

$$\frac{d}{dt}\iiint\limits_V\rho_{ab}dV=-\oiint\limits_\Gamma\cdot J_{ab}\hat{n}d\Gamma \tag{19}$$

This flow is not a divergence flow J_{ab}. It is a point to point converged flow. This can be proved similar to the photon as following, assume Γ is big sphere, the radius of the big sphere is infinity. Assume the wave $\Psi_a(\mathbf{r},t)$ is a short time wave. In the time $t=0$ the wave is at the place of point A. afterwards the wave begin to spread out. When the wave reached the big sphere surface Γ, it happened at a future time $t = \frac{R}{v}$, where R is the radius of the sphere. v is the speed of the wave. The advanced wave $\Psi_b(\mathbf{r}, t)$ reach the big sphere is at the past time $t = -\frac{R}{v}$. We have assume

$$d \ll R \tag{20}$$

where d is the distance from the point A to the point B.

Since the retarded wave come to the big sphere in the future, the advanced wave come to the big sphere in the past. The retarded wave and the advanced wave are not nonzero in the same time in the big sphere Γ, hence

$$\nabla\Psi_a\Psi_b - \Psi_a\nabla\Psi_b = 0 \tag{21}$$

at the sphere Γ. The J_{ab} has no any flux go out the big sphere Γ.

$$\iint\!\!\!\!\!\!\!\!\bigcirc_{\Gamma} \cdot J_{ab}\hat{n} d\Gamma dt = 0 \tag{22}$$

This means that mutual energy flow J_{ab} do not go outside our universe. Inside the volume V their is only the two sources for the charges at A and B hence the flow can only started from A to B. The flow J_{ab} is very thin in the two ends point A and B. The flow J_{ab} are very thick in the middle. The flow will has the same flux integral with time in any surface between the two point A and B. If the particle is a electron, this flow is the current. This flow is the electron itself.

IV. SELF ENERGY FLOW

A. Self-energy flow

We also know that for the retarded wave started from point A there is,

$$\frac{\partial}{\partial t}(\rho_a) = -\nabla \cdot J_a \tag{23}$$

For the advanced wave started from point B there is

$$\frac{\partial}{\partial t}(\rho_b) = -\nabla \cdot J_b \tag{24}$$

where

$$J_a = \frac{\hbar}{2\mu i}(\nabla\Psi_a\Psi_a^* - \Psi_a\nabla\Psi_a^*) \tag{25}$$

$$J_b = [\frac{\hbar}{2\mu i}(\nabla\Psi_b\Psi_b^* - \Psi_b\nabla\Psi_b^*)] \tag{26}$$

J_a is the so called probability current of retarded wave Ψ_a which is a current send energy from point A to infinite big sphere.

J_b is the so called probability current of retarded wave Ψ_b which is a current send energy from point B to infinite big sphere. Since this is advanced wave the energy current is at reversal direction. The energy flux is from infinite big sphere to the point B.

B. The self-energy flow

We know that

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} J_a \cdot \hat{n} d\Gamma dt = const \tag{27}$$

The wave started from point A is retarded wave and hence this part of energy is at a future time to reach the the big sphere Γ.

$$\int_{t=\infty}^{\infty} \oiint_{\Gamma} \cdot J_b \hat{n} d\Gamma dt = -const \tag{28}$$

The negative symbol on the left of the above formula "$-$" is because this is a advanced wave, hence the result is a negative constant. The wave started from point B is advanced wave, this is part of energy will at past time reach the big sphere. Unless our universe at the infinite big sphere is connected from future to the past, the energy send form point A can be received by the point B. Otherwise the retarded flow J_a from A will lose some energy in a future time at infinite big sphere Γ. The advanced flow J_b started from B will receive some energy in the past time at the infinite big sphere Γ. All these are not possible. This violate the energy conservation law. Our solution for this is described in the following section.

V. THE RETURN WAVES

A. the return waves

According discussion in the end of last section, we assume there are return waves for J_a and J_b. The return wave for J_a is a wave from infinite big sphere at future time to the point A. The return wave for J_b is a wave start from infinite big sphere at a past time to the point B.

Hence for a quantum travel from A to B there 4 different waves, and 5 flows:

(1) retarded wave started from point A, which is referred as J_a

(2) advanced wave started from point B, which is referred as J_b

(3) return wave for (1), which is referred as J_{ar}

(4) return wave for (2), which is referred as J_{br}

The return wave for (1) satisfy

$$-i\hbar\frac{\partial}{\partial t}\Psi_{ar}(\mathbf{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{ar}(\mathbf{r},t) \tag{29}$$

It has the same equation with advanced wave, but it is not a advanced wave. The advanced wave is send from point A, in the $t=now$ to the time $t=past$. The returned wave Ψ_{ar} is from start from big sphere at time $t=future$ to the point A at time $t=now$.

The return wave for (2) satisfy

$$i\hbar\frac{\partial}{\partial t}\Psi_{br}(\mathbf{r},t)=\left[\frac{-\hat{\hbar^2}}{2\mu}\nabla^2\right]\Psi_{br}(\mathbf{r},t) \tag{30}$$

It has the same equation with the retarded wave, but it is not a retarded wave. The retarded wave from now to the future. $\Psi_{br}(\mathbf{r},t)$ is from big sphere at time $t=past$ to the point B at time $t=now$. The two return flow can be defined as following,

$$J_{ar}=\frac{\hbar}{2\mu i}(\nabla\Psi_{ar}\Psi^*_{ar}-\Psi_{ar}\nabla\Psi^*_{ar}) \tag{31}$$

$$J_{br}=[\frac{\hbar}{2\mu i}(\nabla\Psi_{br}\Psi^*_{br}-\Psi_{br}\nabla\Psi^*_{br})] \tag{32}$$

We have

$$J_a+J_{ar}=0 \tag{33}$$

$$J_b+J_{br}=0 \tag{34}$$

We assume that the wave Ψ_{br} and Ψ_{ar} can not interfere. If it can interfere the mutual energy flow J_{ab} will be canceled also and that is not what we hope. The above two formula tells us the J_a is canceled by J_{ar} and J_b is canceled by J_{br} hence the self-energy flow have no contribution to the energy flow from point A to the point B.

VI. SUMMARY

For a quantum for example an electron, it travel from point A to point B in the empty space, there are 4 different waves instead one Schrödinger wave. The 4 wave are retarded wave starts from A. The advanced wave starts from B, the return wave for the retarded wave and the return wave for the advanced wave. Between point A and point B there is flow J_{ab} which is transfer the energy or amount of charge from point A to point B. This flow is from point to point and do not diverge. This flow is very thin in the two ends and hence in it looks like a particle. The flow is very thick in the middle between point A and B, and hence it looks a wave. In the middle if there are double slits. This flow will go through the two slits in the same time. This explained the duality of the quantum or particle.

The self-energy flow for J_a and J_b do not transfer and energy or amount of charge. We can think they are canceled by the return flow J_{ar} and J_{br}. The above flow J_{ab}, J_a, J_b, J_{ar},J_{br} are physics flow with energy or amount of the charge and are not the probability flows.

We know the the electromagnetic field has sources which is electric current. We assume there are also some source we do not know for the wave $\Psi_a(\mathbf{r},t)$ and $\Psi_b(\mathbf{r},t)$ which is stayed at the point A and point B. The source at point A can randomly sends the retarded wave. The source at B randomly send advanced wave. Point B is the target, actually on the place close to B there are thousands point similar to point B_1, B_2, ... B_n... they all randomly send the advanced waves.

The probability come from the source of the retarded wave starts at point A and the source of the advanced wave at point B, they are synchronized concurrently, the mutual energy flow J_{ab} is produced. The retarded wave $\Psi_a(\mathbf{r},t)$ is a random events, the advanced wave $\Psi_b(\mathbf{r},t)$ is also a random events, the two random events just meet together is also a random events. This leads to the position of the particle has be received with the probability. This can be referred as the the interpretation with mutual energy principle for the quantum mechanics.

If the retarded wave flow $\Psi_a(\mathbf{r},t)$ cannot meet a advanced wave which is synchronized to the retarded wave $\Psi_a(\mathbf{r},t)$. This retarded wave flow J_a just returned through the corresponding return wave J_{ar}. If it meet the advanced wave $\Psi_b(\mathbf{r},t)$ which is synchronized with the retarded wave $\Psi_a(\mathbf{r},t)$, the mutual energy flow J_{ab} is produced. After the J_{ab}, there is the return flow J_{ar}. Hence no matter the mutual energy flow is produced or not the self-energy flow J_a always returned through J_{ar}. For the advanced wave, the similar things also happens.

VII. CONCLUSION

We have introduced mutual energy principle and self-energy principle for photon and electromagnetic fields. In this article we applied the concept of the mutual energy principle and self-energy principle to other particles for example electron. We use Schrödinger equation to study this problems, but we believe this idea are also correct for the Dirac equation.

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