The Mother of All Field Equations

By J.A.J. van Leunen

Abstract

The first order quaternionic partial differential equation can be considered as the mother of all field equations. Second order partial differential equations describe the interaction between point-like artifacts and fields. A direct relation exists with integral balance equations.

Quaternions

Quaternions constitute nature's natural number system. Reality applies a read only repository that stores the dynamic geometric data of its inhabitants and that repository can only cope with elements of a division ring. The only suitable division rings are the real numbers, the complex numbers, and the quaternions. Quaternions form the most elaborate division ring. The repository can also store quaternionic continuums that are defined by quaternionic functions. Quaternionic differential equations can describe the dynamic behavior of quaternionic continuums. Quaternions are ideally suited for the storage of discrete dynamic geometric data. The real part of the quaternion can represent a timestamp and the imaginary part can represent a three-dimensional spatial location.

Quaternionic second order partial differential equations can describe the interaction between point-like artifacts and quaternionic continuums. In a quaternionic model of the universe these point-like artifacts constitute the objects that occur in the model.

The quaternionic first order partial differential equation appears to be the mother of all field equations. It applies the quaternionic nabla and this differential operator behaves as a quaternionic multiplier. Thus, the quaternionic multiplication rule acts as the template for the quaternionic first order partial differential equation.

$$c = c_r + \mathbf{c} = ab \equiv (a_r + \mathbf{a})\,(b_r + \mathbf{b}) = a_r b_r - \langle \mathbf{a}, \mathbf{b} \rangle + \mathbf{a}b_r + a_r\mathbf{b} \pm \mathbf{a} \times \mathbf{b}$$

Here the real part gets subscript $_r$ and the imaginary part is written in bold face.

The right side covers five different terms.
$\langle \mathbf{a}, \mathbf{b} \rangle$ is the inner product.
$\mathbf{a} \times \mathbf{b}$ is the external product.
$\pm$ indicates the choice between right and left handedness.

The quaternionic nabla

Partial quaternionic differential equations that apply the quaternionic nabla ∇ describe the interaction between a field and a point-like artifact.

$$\nabla \equiv \{\partial/\partial\tau,\ \partial/\partial x,\ \partial/\partial y,\ \partial/\partial z\}$$

$$\nabla \equiv \{\partial/\partial x, \partial/\partial y, \partial/\partial z\}$$

$$\nabla_r \equiv \partial/\partial \tau$$

τ is progression or proper time.

In the quaternionic differential calculus, differentiation with the quaternionic nabla is a quaternionic multiplication operation:

$$\Phi = \phi_r + \Phi = \nabla \psi \equiv (\nabla_r + \nabla)(\psi_r + \psi) = \nabla_r \psi_r - \langle \nabla, \psi \rangle + \nabla \psi_r + \nabla_r \psi \pm \nabla \times \psi$$

$$\phi_r = \nabla_r \psi_r - \langle \nabla, \psi \rangle$$

$$\Phi = \nabla \psi_r + \nabla_r \psi \pm \nabla \times \psi$$

$\langle \nabla, \psi \rangle$ is the divergence of ψ

$\nabla \psi_r$ is the gradient of ψ_r

$\nabla \times \psi$ is the curl of ψ

Some of the terms get new symbols

$$E = -\nabla \psi_r - \nabla_r \psi$$

$$B = \nabla \times \psi$$

Higher order differentiation

Double differentiation leads to the second order partial differential equation:

$$\rho = \nabla^* \phi = (\nabla_r - \nabla)(\nabla_r + \nabla)(\psi_r + \psi) = (\nabla_r \nabla_r + \langle \nabla, \nabla \rangle)(\psi_r + \psi) = \rho_r + J$$

This equation splits into two first order partial differential equations $\Phi = \nabla \psi$ and $\rho = \nabla^* \phi$.

$$\rho_r = \langle \nabla, E \rangle$$

$$J = \nabla \times B - \nabla_r E$$

$$\nabla_r B = -\nabla \times E$$

Two quite similar second order partial differential operators exist. The first is described above.

$$(\nabla_r \nabla_r + \langle \nabla, \nabla \rangle) \psi = \rho$$

This is still a nameless equation.

The second is the quaternionic equivalent of d'Alembert's operator $(\nabla_r \nabla_r - \langle \nabla, \nabla \rangle)$. It defines the quaternionic equivalent of the well-known wave equation.

$$(\nabla_r \nabla_r - \langle \nabla, \nabla \rangle)\, \psi = \varphi$$

Both second order partial differential operators are Hermitian differential operators.

Solutions

The homogeneous second order partial differential equations offer solutions that occur when actuators trigger them.

Waves

$$f\,(\tau, \mathbf{x}) = a \exp (i\, \omega\, (c\tau - |\mathbf{x} - \mathbf{x'}|));\ c = \pm 1$$

solves

$$\nabla_r \nabla_r\, f = \langle \nabla, \nabla \rangle\, f = -\omega^2\, f$$

Warps

$$\psi = g(\mathbf{x}\, \mathbf{i} \pm \tau)$$

Clamps

$$\psi = g(r\, \mathbf{i} \pm \tau)/r$$

Clamps and warps are shock fronts. They only occur in odd dimensions.

All solutions have advanced and retarded components.

Clamps integrate into the Green's function. They quickly fade away. For that reason they temporarily deform the carrier.

Balance equations

The first order partial differential equation is a continuity equation. It has a direct relation with integral balance equations.

With respect to a local part of a closed boundary that is oriented perpendicular to vector $\mathbf{n}$ the partial differentials relate as

$$\nabla \psi = \nabla \psi_r - \langle \nabla, \psi \rangle \pm \nabla \times \psi \Longleftrightarrow \mathbf{n}\, \psi_r - \langle \mathbf{n}, \psi \rangle \pm \mathbf{n} \times \psi$$

This is exploited in the generalized Stokes theorem

$$\iiint_V \nabla \psi = \oiint_S \mathbf{n} \psi$$

Gradient theorem

$$\iiint_V \nabla \psi_r = \oiint_S \mathbf{n} \psi_r$$

Divergence theorem (Gauss theorem)

$$\iiint_V \langle \nabla, \psi \rangle = \oiint_S \langle n, \psi \rangle$$

Curl theorem (Stokes theorem)

$$\iiint_V \nabla \times \psi = \oiint_S n \times \psi$$

References

[1] https://en.wikipedia.org/wiki/Wave_equation#General_solution

[2] https://en.wikiversity.org/wiki/Hilbert_Book_Model_Project/Quaternionic_Field_Equations

[3] https://en.wikiversity.org/wiki/Hilbert_Book_Model_Project/Extended_Stokes_Theorem

[4] Rediscovered Dark Quanta; http://vixra.org/abs/1709.0150