ON THE UNIVERSAL SPEED IN RELATIVITY

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Abstract: The Michelson-Morley experiment has been considered as a verification of the validity of Einstein's postulate of the universal speed of light in vacuum. However, it has been shown that the special theory of relativity can be developed by applying only the principle of relativity without the need to postulate the universal speed of light and, as a consequence, this raises the question of what role the Michelson-Morley experiment would play in special relativity. In this work we show that the Michelson-Morley experiment not only can be used to confirm that the speed of light is not universal as postulated in Einstein's special relativity but in fact can also be used to verify that a global universal speed has the value of $\sqrt{3}c$.

The postulate of the invariance of the speed of light has been re-examined recently by many authors, and it has been shown that the special theory of relativity can be developed using only the principle of relativity, which postulates the invariance of physical laws in any inertial frame of reference [1-4]. Even though relativistic transformations can be derived from the principle of relativity alone, these formulations do not specify or determine the value of the universal invariant speed that must be accompanied the special relativity for any further development or application of the theory. It should be mentioned here that even within Einstein's theory of general relativity, whether the constancy of the speed of light has a global character is a question that has also been discussed [5-6]. In this work, we will use the relativistic transformations that are derived only from the principle of relativity to show that the Michelson-Morley experiment not only can be used to confirm that the speed of light in vacuum is not universal as postulated in Einstein's theory of special relativity but in fact can also be used to verify that a global universal speed has the value of $\sqrt{3}c$, where c is the speed of light in vacuum. For the clarity for our discussions in the following, first we recapture the necessary procedure to calculate a possible shift of the interference pattern in the Michelson-Morley experiment. In the Michelson-Morley experiment, light rays are made to travel along two optical paths l_1 and l_2 which are perpendicular to each other. In this work we assume the lengths of all optical paths to be kept constant. If the whole apparatus is moving in the direction of l_1 at speed v then by using the Galilean law of composition of velocities the times t_1 and t_2 taken for light to travel along l_1 and l_2 can be calculated as follows [7]

$$t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v} = \frac{2l_1/c}{1-v^2/c^2} \tag{1}$$

$$t_2 = \frac{2l_2/c}{(1-v^2/c^2)^{1/2}} \tag{2}$$

where v is the velocity of the earth in its orbit. From Equations (1) and (2), a time difference $\Delta_1 = t_1 - t_2$ is obtained

$$\Delta_1 = \frac{2l_1/c}{1-v^2/c^2} - \frac{2l_2/c}{(1-v^2/c^2)^{1/2}} \tag{3}$$

Now, when the whole apparatus is turned so that its direction of motion is parallel to l_2 then a new time difference $\Delta_2 = t_1 - t_2$ is obtained

$$\Delta_2 = \frac{2l_1/c}{(1-v^2/c^2)^{1/2}} - \frac{2l_2/c}{1-v^2/c^2} \tag{4}$$

From the time differences given in Equations (3) and (4), the interference pattern would shift by an amount $\delta = c(\Delta_1 - \Delta_2)/\lambda$ as follows

$$\delta = \frac{c}{\lambda}\left(\frac{2l_1/c}{1-v^2/c^2} - \frac{2l_2/c}{(1-v^2/c^2)^{1/2}} - \frac{2l_1/c}{(1-v^2/c^2)^{1/2}} + \frac{2l_2/c}{1-v^2/c^2}\right) \tag{5}$$

From Equation (5), if $v \ll c$ then using the relation $(1+x)^k \approx 1 + kx$, where k is a real number, an approximate amount of the shift of the interference pattern for the case $v \ll c$ is found as

$$\delta = \frac{c(\Delta_1 - \Delta_2)}{\lambda} \approx \frac{(l_1 + l_2)v^2}{\lambda c^2} \tag{6}$$

And when $l_1 = l_2 = l$, then

$$\delta \approx \frac{2lv^2}{\lambda c^2} \tag{7}$$

With $v \approx 30\ \mathrm{km/sec}$, $\lambda = 6 \times 10^{-7}\mathrm{m}$ and $l = 1.2\ \mathrm{m}$, the relation (7) gives $\delta \approx 0.04$ fringe. Michelson and Morley reported to observe only a small shift of the fringe pattern of at most 0.005 fringe [8]. This has been considered as a null result. The null result obtained from the Michelson-Morley experiment has been considered to be consistent with Einstein's postulate of the invariance of the speed of light in empty space, which results in the following transformation of velocities

$$u_x = \frac{u_x' + v}{1 + u_x' v/c^2}, \tag{8}$$

$$u_y = \frac{u_y' \sqrt{1 - v^2/c^2}}{1 + u_x' v/c^2} \tag{9}$$

It should be emphasized here that the shift of the interference pattern given by the relation (7) is derived from the Galilean transformation. However, the use of the Galilean transformation in the Michelson-Morley experiment is probably not appropriate to determine whether the speed of light in vacuum is universal. In fact, as will be argued in the following, when only the principle of relativity is used to formulate the special relativity then the null result obtained from the Michelson-Morley experiment not only can be used to confirm that the speed of light in vacuum is not universal as postulated in Einstein's theory of special relativity but in fact can also be used to determine a global universal speed.

As shown in the above-mentioned references [2-4], without postulating the universal speed of light in vacuum, the principle of relativity alone can be used to derive the relativistic addition law for parallel velocities as follows

$$u_x = \frac{u_x' + v}{1 + K u_x' v} \tag{10}$$

where K is a universal constant. If the optical path l_1 is along the direction of u_x and if $u_x' = c$ then the time t_1 for light to travel along l_1 is given by

$$t_1 = \frac{l_1}{\frac{c - v}{1 - Kcv}} + \frac{l_1}{\frac{c + v}{1 + Kcv}} = \frac{2l_1}{c}\left(\frac{1 - Kv^2}{1 - v^2/c^2}\right) \tag{11}$$

To calculate the time t_2 for light to travel along l_2 in the direction perpendicular to the direction of v, we note that in order to be consistent with the transformation of the perpendicular component in Einstein's theory of special relativity given in Equation (9), the perpendicular component of velocity in the special relativity that is derived only from the principle of relativity should be transformed as

$$u_y = \frac{u_y' \sqrt{1 - Kv^2}}{1 + K u_x' v} \tag{12}$$

In the case when $u_x' = 0$ and $u_y' = c\sqrt{1 - v^2/c^2}$, we then obtain

$$u_y = c\sqrt{1 - v^2/c^2}\,\sqrt{1 - Kv^2} \tag{13}$$

The time $t_2 = 2l_2/u_y$ for light to travel along l_2 is calculated as

$$t_2 = \frac{2l_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}\,\sqrt{1 - Kv^2}} \tag{14}$$

Similar to the case of Galilean transformation, using the relativistic transformations given in Equations (11) and (14), the interference pattern would shift by a calculated amount $\delta = c(\Delta_1 - \Delta_2)/\lambda$ given as

$$\delta = \frac{c}{\lambda}\left(\frac{2l_1}{c}\left(\frac{1 - Kv^2}{1 - v^2/c^2}\right) - \frac{2l_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}\,\sqrt{1 - Kv^2}} - \frac{2l_1}{c}\frac{1}{\sqrt{1 - v^2/c^2}\,\sqrt{1 - Kv^2}} + \frac{2l_2}{c}\left(\frac{1 - Kv^2}{1 - v^2/c^2}\right)\right) \tag{15}$$

For the case when $l_1 = l_2 = l$, Equation (15) is reduced to

$$\delta = \frac{4l}{\lambda}\left(\left(\frac{1 - Kv^2}{1 - v^2/c^2}\right) - \frac{1}{\sqrt{1 - v^2/c^2}\,\sqrt{1 - Kv^2}}\right) \tag{16}$$

Using the relation $(1 + x)^k \approx 1 + kx$ for each term in Equation (16), we obtain an approximate amount of the shift of the interference pattern for the case $v \ll c$ as

$$\delta \approx \frac{4l}{\lambda}\left((1 + v^2/c^2)(1 - Kv^2) - \left(1 + \tfrac{1}{2}v^2/c^2\right)\left(1 + \tfrac{1}{2}Kv^2\right)\right) = \frac{2lv^2}{\lambda}\left(\frac{1}{c^2} - 3K\right) \tag{17}$$

The result given in Equation (17) reduces to that obtained from Galilean transformation given in Equation (7) which corresponds to the infinite value of the universal speed. However, if an absolute null result is obtained from the Michelson-Morley experiment, $\delta \equiv 0$, then $K = 1/\left(\sqrt{3}c\right)^2$. In this case the value of the universal speed c_U is given as $c_U = 1/\sqrt{K} = \sqrt{3}c$. Therefore, the null result from the Michelson-Morley experiment shows that the speed of light in vacuum is not universal and can be used to verify that photons are in fact massive particles [9]. As a further remark, we would like to mention here that in addition to Minkowski pseudo-Euclidean relativity it is possible to formulate a special Euclidean relativity in which there is no upper limit for the speed of transmission and the assumed universal speed is not the speed of any physical object or physical field but rather the common speed of expansion of the spatial space of all inertial frames [10].

References

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