A SPECIAL RELATIVITY OF CIRCULAR REFERENCE FRAMES

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Abstract: Similar to Einstein's method to formulate a special relativity for inertial reference systems in classical physics, we show that a special relativity can also be formulated for systems of circular reference frames.

According to Bohr's model of the hydrogen-like atom [1], even though the electron of the atom moves in circular orbits, it does not radiate. And as shown in our previous work on the hydrogen-like atom, this result can be explained if we use a mixed Coulomb-Yukawa potential of the form $V(r) = -\alpha\, e^{-\beta r}/r + Q/r$, which results in a zero net force at $r = 1/\beta$ [2]. A curious question that can be raised, at least for the case of the hydrogen-like atom and as far as its electron is concerned, is whether these circular orbits can be considered as inertial reference frames? In the following, we will show that in fact a special relativity can be formulated for circular reference frames which rotate with respect to each other.

Consider rotating frames of reference in the form of concentric circles as shown in the figure below

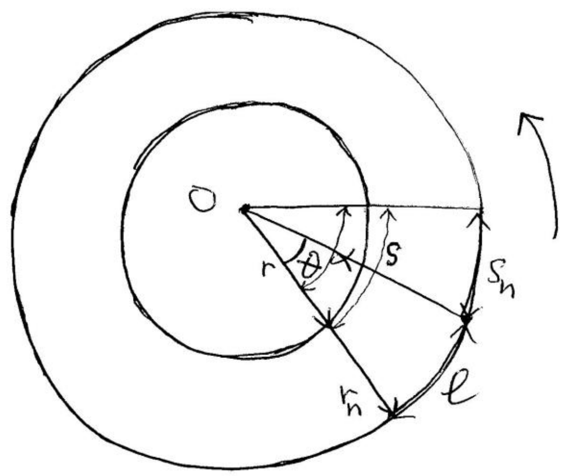

Figure 1

Let r and t be the radius and time of a circular frame which is regarded as being stationary. Let r_n and t_n be the radius and time of a circular frame which is rotating with respect to the (r, t)-frame with a constant angular speed ω about the common centre O. Denote s and s_n the arc-length positions of a particle in the (r, t)-frame and

(r_n, t_n)-frame respectively. If we assume $t_n = t$, then from Figure 1 we obtain the following relations

$$s = r\alpha \tag{1}$$

$$l = r_n\theta = r_n\omega t \tag{2}$$

$$s_n + r_n\omega t = r_n\alpha \tag{3}$$

From Equations (1) and (3), we have

$$\frac{s}{r} = \frac{s_n + r_n\omega t}{r_n} \tag{4}$$

Equation (4) is rewritten as

$$s = \frac{r}{r_n}(s_n + r_n\omega t) \tag{5}$$

Together with $t_n = t$, Equation (5) can be seen as a form of kinematical Galilean transformations of circular reference frames. In order to formulate a special relativity for circular reference frames, we assume that the relativistic transformations for rotating frames take the following forms

$$s = \frac{Rr}{r_n}(s_n + r_n\omega t_n) \tag{6}$$

$$t = \frac{Rr}{r_n}\left(t_n + \frac{r_n\omega s_n}{c^2}\right) \tag{7}$$

where R is a quantity that will be determined and c is a physical constant that plays the role of the speed of light in vacuum in Einstein's theory of special relativity [3]. The constant c can be regarded as a universal speed of a physical field that needs to be specified. It should be mentioned here that as shown in our work on the speed of light in vacuum in relativity, there may not exist a universal speed for all inertial reference frames [4]. The quantity R can be determined if we simply follow Einstein's method by assuming the following identity

$$s_n^2 - c^2t_n^2 = s^2 - c^2t^2 \tag{8}$$

With the assumed relation given in Equation (8), we obtain

$$R = \frac{r_n}{r\sqrt{1 - \frac{r_n^2\omega^2}{c^2}}} \tag{9}$$

We finally obtain the following special relativistic transformations for circular reference frames

$$s = \frac{1}{\sqrt{1 - \frac{r_n^2\omega^2}{c^2}}}(s_n + r_n\omega t_n) \tag{10}$$

$$t = \frac{1}{\sqrt{1 - \frac{r_n^2\omega^2}{c^2}}}\left(t_n + \frac{r_n\omega s_n}{c^2}\right) \tag{11}$$

References

[1] N. Bohr, *Phil. Mag*. **26** 1 (1913).
[2] Vu B Ho, On *the Stationary Orbits of a Hydrogen-like Atom* (Preprint, 2016, ResearchGate).
[3] A. Einstein, *The Principle of Relativity* (Dover, New York, 1952).
[4] Vu B Ho, *A Remark on the Universal Speed in Relativity* (Preprint, 2017, ResearchGate).