Solving Numerically Ermakov-type Equation for Newtonian Cosmology Model with Vortex[1]

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ABSTRACT

It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as "Hubble's law". More realistic one is suggested, based on Newtonian cosmology model but here we include the vertical-rotational effect of the whole Universe. We review a Riccati-type equation obtained by Nurgaliev, and solve the equation numerically with Mathematica. It is our hope that the new proposed method can be verified with observation data.

Keywords: Riccati-type equation, computational physics, nonlinear cosmology, Newtonian cosmology, vortex.

1. Introduction

It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as "Hubble's law."

In this regard, academician Isaak Khalatnikov mentioned at the 13th Marcel Grossman Conference (http://www.icra.it/mg/mg13/) Lev Landau suggesting that something is too symmetric in the models yielding singularities, and that this problem is one of the three most important problems of modern physics. The aim of this report is to show that

[1] This paper is dedicated for 72th anniversary of Indonesia's Independence Day, 17 august 2017.

singularities are, indeed, consequences of such an overly "symmetrical approach" in building non-robust (i.e. without structural stability) toy models with singularities. Such models typically apply a synchronous system of reference and "Hubble's law", neglecting not-to-be-averaged-out quadratic terms of perturbations (specifically, differentially rotational velocities, vortexes).[1]

Only by accounting the overlooked factors instead of Einstein's ad hoc introduction of a new entity, which was later declared by him as his "biggest blunder", can we correctly interpret accelerated cosmological expansion, as well as provide possibility of static solution. The common perception of the observed accelerated expansion is that there is need either in modifying the General Relativity or discover new particles with unusual properties. Interestingly enough, both ways are possible depending on what kind of system of reference and corresponding interpretation are chosen, a decision which is usually made depending on the level of "geometrization."[1]

Local rotations (vortices) play a role in radical stabilization of the cosmological singularity in the retrospective extrapolation, making possible a static or steady-state (on the average) Universe or local region. Therefore Einstein could "permit" the galaxies to rotate instead of postulating a cosmological constant ad hoc in his general-relativistic consideration of a static Universe. Though, it does not necessarily mean that the cosmological constant is not necessary for other arguments.[2]

In this paper, more realistic one is suggested, based on Newtonian cosmology model but here we include the vortical-rotational effect of the whole Universe.

We review a Riccati-type equation obtained by Nurgaliev, and solve the equation numerically with Mathematica. It is our hope that the new proposed method can be verified with observation data.

2. Deriving Ermakov-type equation for Newtonian Cosmology model

In this section, we will derive a Riccati-type equation following Nurgaliev [1]. Then we will solve it numerically using Mathematica 11.

After he proceeds with some initial assumptions, Nurgaliev obtained a new simple local cosmological equation:[2]

$$\dot{H} + H^2 = \omega^2 + \frac{4\pi G}{3}\rho, \tag{1}$$

Where $\dot{H} = dH / dt$.

The angular momentum conservation law ωR^2 = const = K and the mass conservation law $(4\pi/3)\rho R^3$ = const = M makes equation (1) solvable:[2]

$$\dot{\mathrm{H}} + \mathrm{H}^2 = \frac{K^2}{R^4} - \frac{\mathrm{GM}}{\mathrm{R}^3}, \tag{2}$$

Or

$$\ddot{R} = \frac{K^2}{R^3} - \frac{GM}{R^2}. \tag{3}$$

Equation (3) may be written as Ermakov-type nonlinear equation as follows;

$$\ddot{R} + \frac{GM}{R^2} = \frac{K^2}{R^3}. \tag{4}$$

Nurgaliev tried to integrate equation (3), but now we will solve the above equation with Mathematica 11. First, we will rewrite this equation by replacing GM=A, K^2=B, so we get:

$$\ddot{R} + \frac{A}{R^2} = \frac{B}{R^3}. \quad (5)$$

As with what Nurgaliev did in [1][2], we also tried different sets of A and B values, as follows:

a. A and B < 0

```
A=-10;
B=-10;
ODE=x''[t]+A/x[t]^2-B/x[t]^3==0;
sol=NDSolve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]
Plot[x[t]/.sol,{t,-10,10}]
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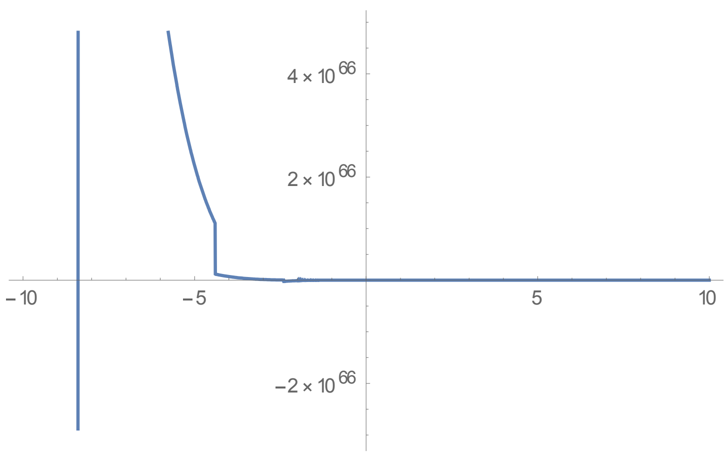
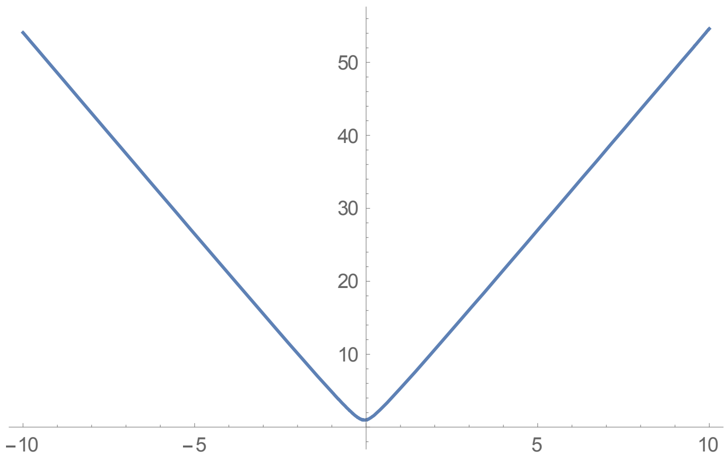


b. A < 0, B > 0

```
A=-10;
B=10;
ODE=x''[t]+A/x[t]^2-B/x[t]^3==0;
sol=NDSolve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]
Plot[x[t]/.sol,{t,-10,10}]
```



c. A > 0, B < 0

```
A=1;
B=-10;
ODE=x''[t]+A/x[t]^2-B/x[t]^3==0;
sol=NDSolve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]
Plot[x[t]/.sol,{t,-10,10}]
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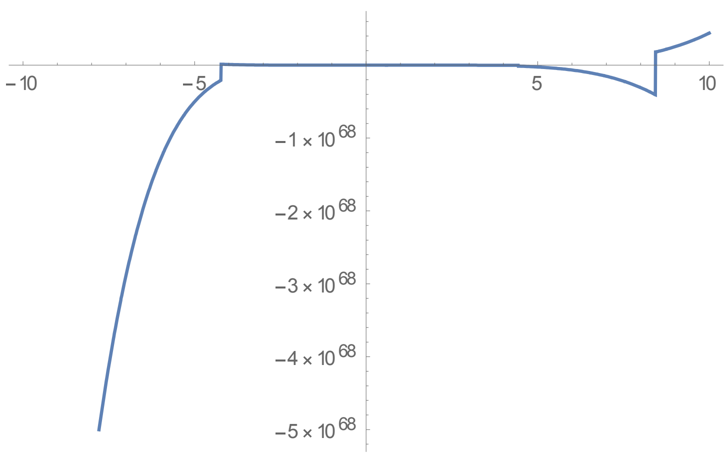
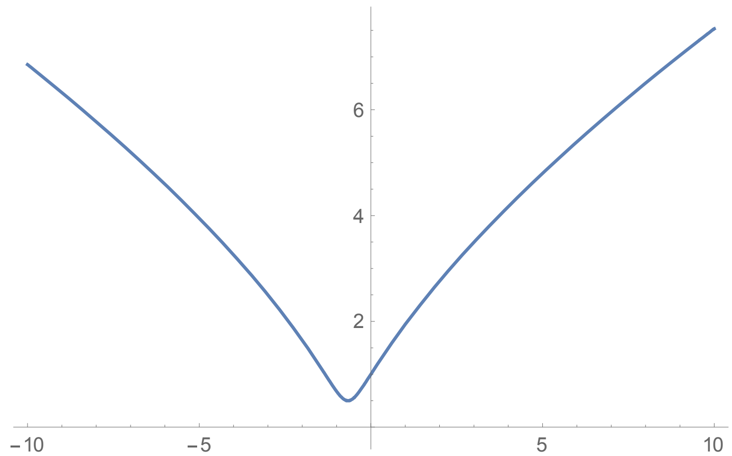


d. A >0, B > 0

```
A=1;
B=1;
ODE=x''[t]+A/x[t]^2-B/x[t]^3==0;
sol=NDSolve[{ODE,x[0]==1,x'[0]==1},x[t],{t,-10,10}]
Plot[x[t]/.sol,{t,-10,10}]
```



From the above numerical experiments, we conclude that the evolution of the Universe depends on the constants involved, especially on the rotational-vortex structure of the Universe. This needs to be investigated in more detailed for sure.

3. Concluding Remarks

It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as “Hubble’s law”. More realistic one is suggested, based on Newtonian cosmology model but here we include the vertical-rotational effect of the whole Universe. We review a Riccati-type equation obtained by Nurgaliev, and solve the equation numerically with Mathematica 11. It is our hope that the new proposed method can be verified with observation data.

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