Question 765: Polynomials,number pi,fractals

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abstract

This note presents some results related with fractals , polynomials and number pi.

1. Introduction

Let

$$p1(z)=z^4+2\sqrt{3}z^3+5z^2-4\sqrt{3}z-2 \tag{1}$$

$$p2(z)=z^8-2z^6+69z^4-68z^2+4 \tag{2}$$

$$p3(z)=z^{16}+1084z^{12}-1146z^8+1084z^4+1 \tag{3}$$

$$u=-\frac{\sqrt{3}}{2}+\frac{\sqrt{6}}{2}+\frac{\sqrt{6\sqrt{2}-7}}{2} \tag{4}$$

$$v=\frac{\sqrt{3}}{2}-\frac{\sqrt{6}}{2}+\frac{\sqrt{6\sqrt{2}-7}}{2} \tag{5}$$

$$s=\frac{\sqrt[4]{271+192\sqrt{2}-4\sqrt{9198+6504\sqrt{2}}}}{\sqrt{2}} \tag{6}$$

$$t=\frac{\sqrt[4]{271+192\sqrt{2}+4\sqrt{9198+6504\sqrt{2}}}}{\sqrt{2}} \tag{7}$$

We get:

Proposition 1.

$$p1(u)=p1(-v)=0 \tag{8}$$

$$p1\left(-\frac{(u-v)(17+2uv)}{3}\pm(u+v)i\right)=0 \tag{9}$$

Proposition 2.

$$p2(\pm u)=p2(\pm v)=0 \tag{10}$$

$$p2\left(\pm\frac{(u-v)(17+2uv)}{3}\pm(u+v)i\right)=0 \tag{11}$$

Proposition 3.

$$p3(\pm u\pm vi)=0 \tag{12}$$

$$p3(\pm v\pm ui)=0 \tag{13}$$

$$p3((\pm 1\pm i)s)=0 \tag{14}$$

$$p3((\pm 1\pm i)t)=0 \tag{15}$$

Proposition 4.

$$u+v=\sqrt{6\sqrt{2}-7} \tag{16}$$

$$u-v=\sqrt{6}-\sqrt{3} \tag{17}$$

$$uv=3\sqrt{2}-4 \tag{18}$$

$$u=\frac{1}{2}\sqrt{2+2\sqrt{96\sqrt{2}-135}} \tag{19}$$

$$v=\frac{1}{2}\sqrt{2-2\sqrt{96\sqrt{2}-135}} \tag{20}$$

$$u^2+v^2=1 \tag{21}$$

$$u^2-v^2=\sqrt{3}\sqrt{32\sqrt{2}-45} \tag{22}$$

$$uv(8+uv)=2 \tag{23}$$

$$\sqrt{3}(u-v)=1+uv \tag{24}$$

$$s=\frac{1}{2t} \tag{25}$$

Proposition 5.

If

$$p4(z)=z^4-z^2+34-24\sqrt{2} \tag{26}$$

then

$$p4(\pm u)=p4(\pm v)=0 \tag{27}$$

Proposition 6.

$$u^2-\left(\sqrt{6}-\sqrt{3}\right)u+4-3\sqrt{2}=0 \tag{28}$$

$$v^2+\left(\sqrt{6}+\sqrt{3}\right)v+4-3\sqrt{2}=0 \tag{29}$$

Proposition 7.

$$\pi=2\sin^{-1}u+2\sin^{-1}v \tag{30}$$

$$\pi=6\tan^{-1}u-6\tan^{-1}v \tag{31}$$

$$\pi=12\tan^{-1}\left(\frac{1-u}{1+u}\right)+12\tan^{-1}v \tag{32}$$

Proposition 8.

$$\frac{1-u}{1+u}=5-2\sqrt{3}+\left(\sqrt{3}-2\right)u^3+\left(8-5\sqrt{3}\right)u^2+\left(10\sqrt{3}-18\right)u \tag{33}$$

$$\frac{1-u}{1+u}=\frac{15-9\sqrt{3}+\sqrt{2}\left(12-6\sqrt{3}\right)+\sqrt{6\sqrt{2}-7}\left(7\sqrt{3}-13+\sqrt{2}\left(5\sqrt{3}-9\right)\right)}{2} \tag{34}$$

2. Visualization

- Roots: $p1(z)=0, p2(z)=0, p3(z)=0$.

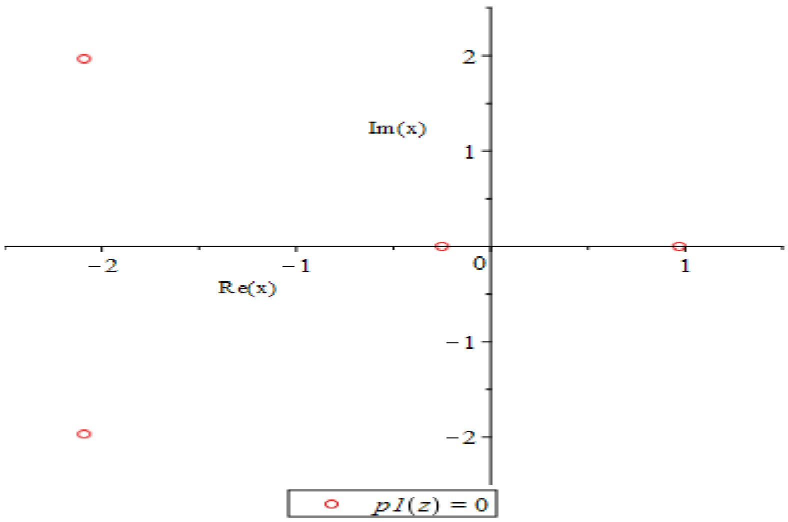
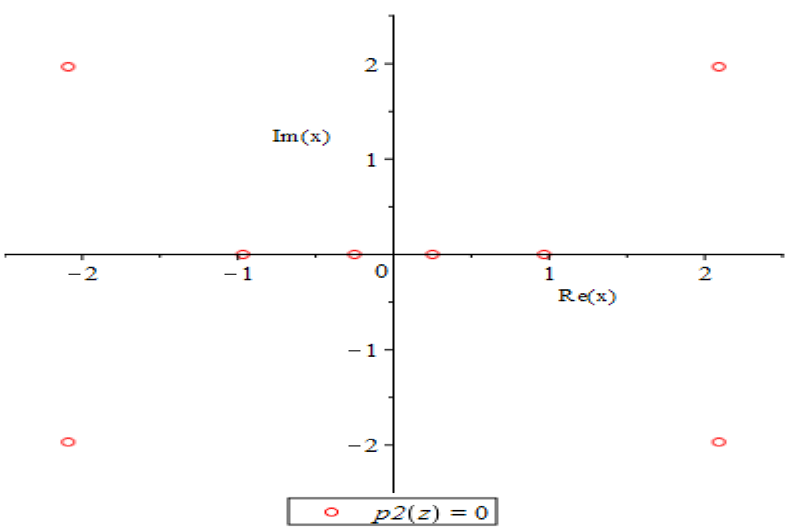


Fig. 1.



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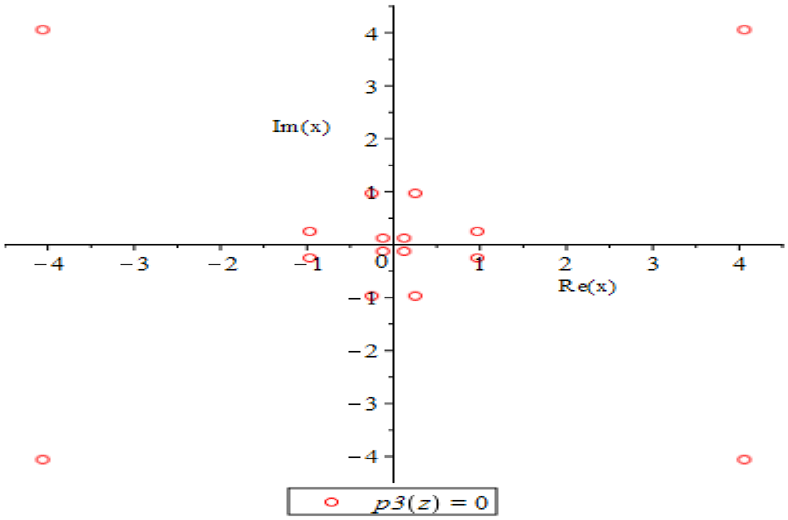


Fig. 3.

❖ $\operatorname{Re}\left(pk\left(x+yi\right)\right)=0$, $\operatorname{Im}\left(pk\left(x+yi\right)\right)=0$, $k=1,2,3$.

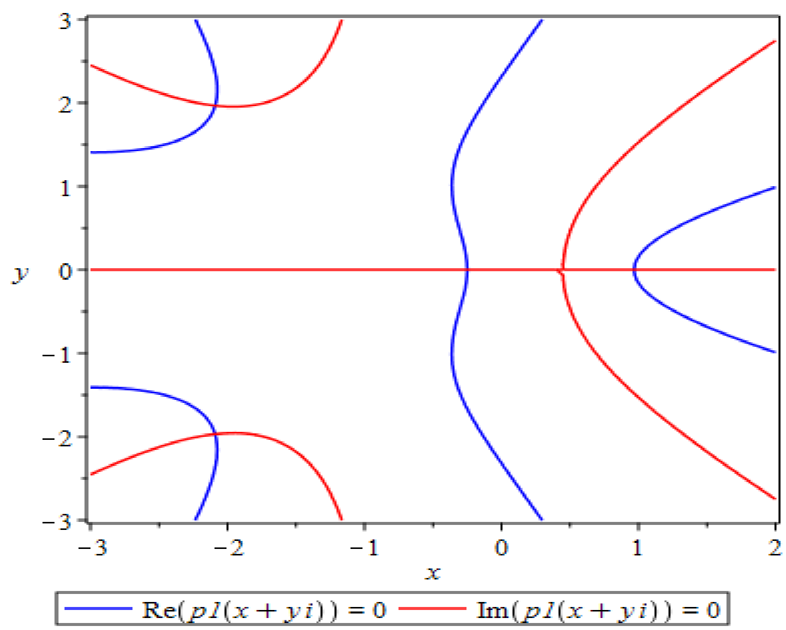
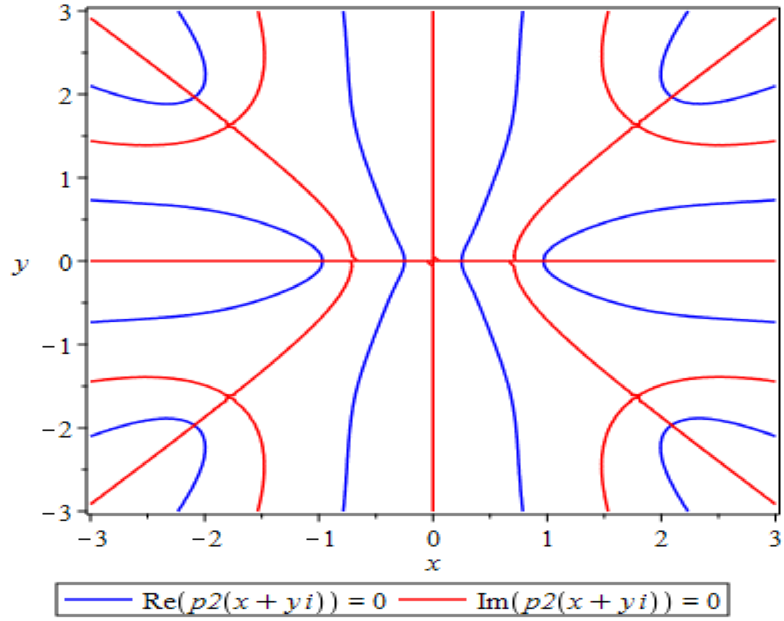
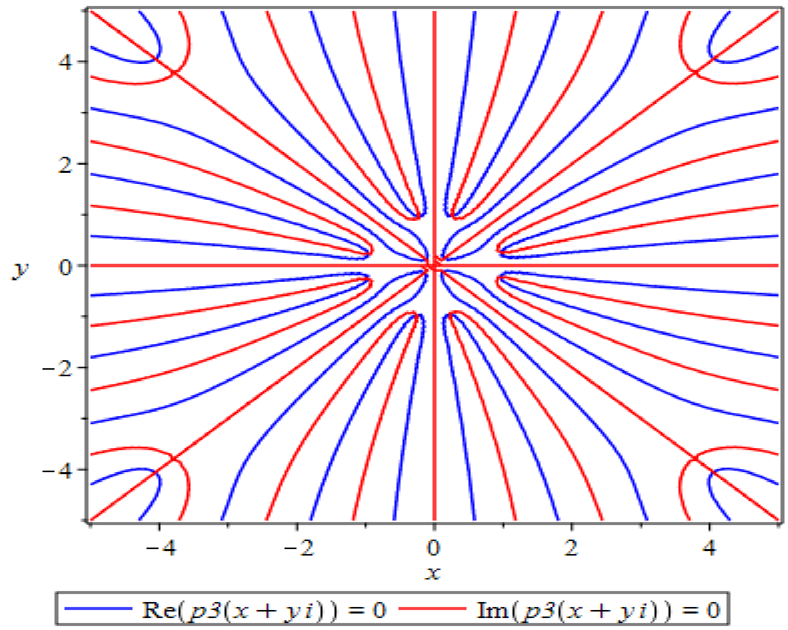


Fig. 4.



3
2
1
0
−1
−2
−3
y
−3
−2
−1
0
1
2
3
x
Re(p2(x + y i)) = 0
Im(p2(x + y i)) = 0

Fig. 5.



4
2
0
−2
−4
y
−4
−2
0
2
4
x
Re(p3(x + y i)) = 0
Im(p3(x + y i)) = 0

Fig. 6.

- Fractals related with $p1(z)$.

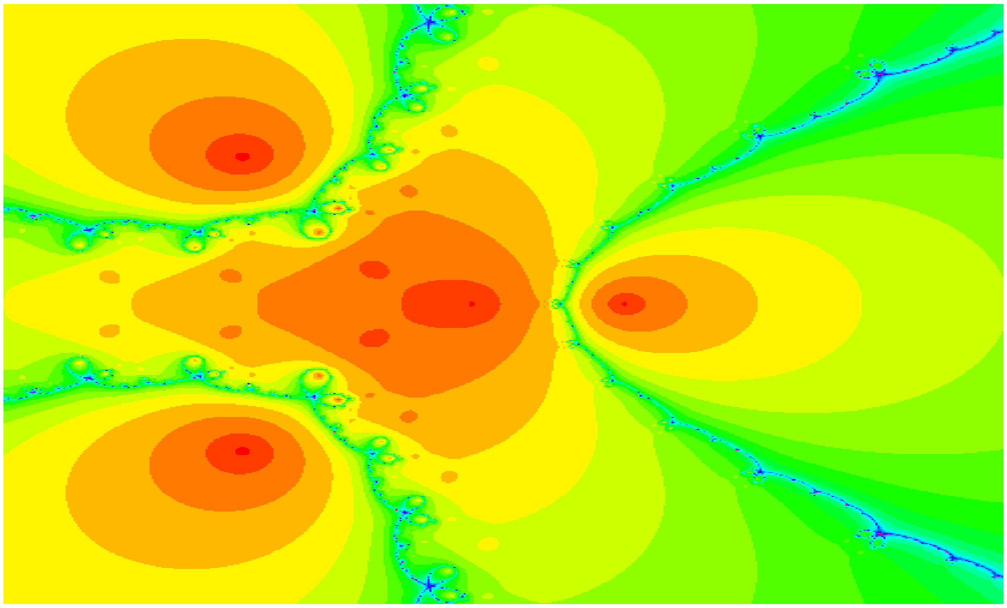


Fig. 7.

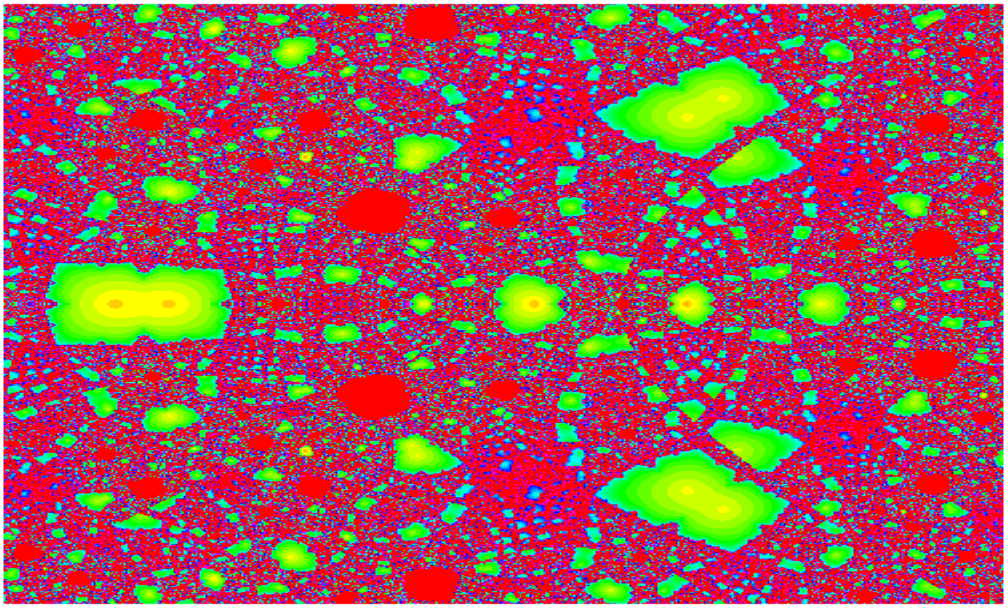


Fig. 8.

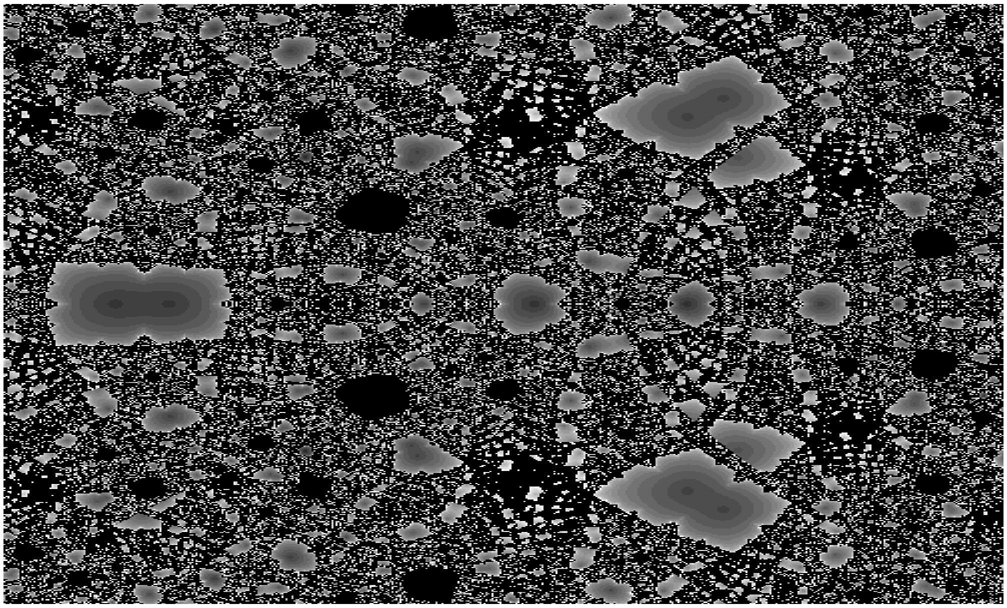
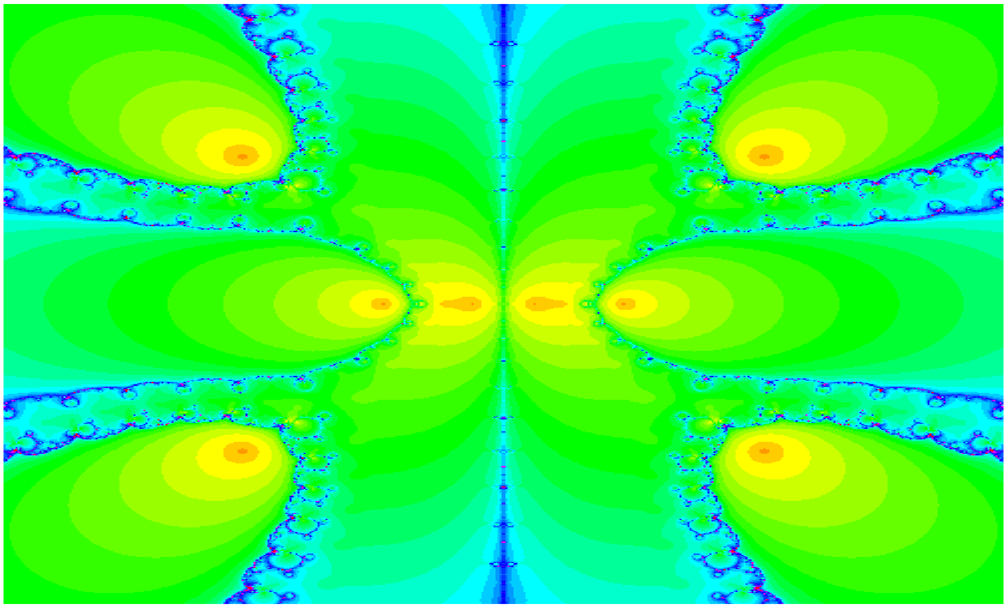


Fig. 9.

❖ Fractals related with $p2(z)$.



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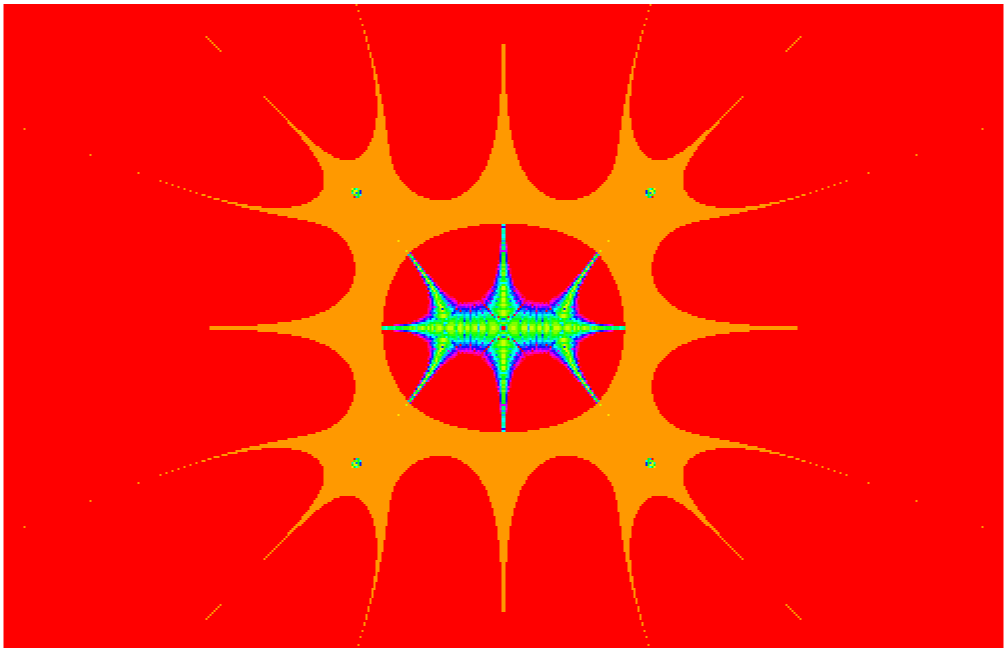
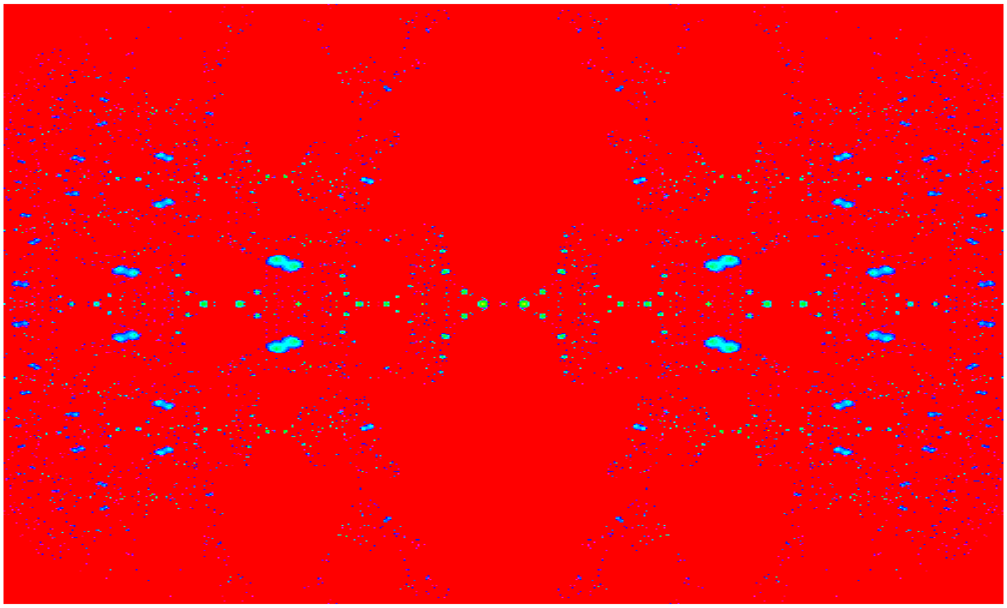


Fig. 11.



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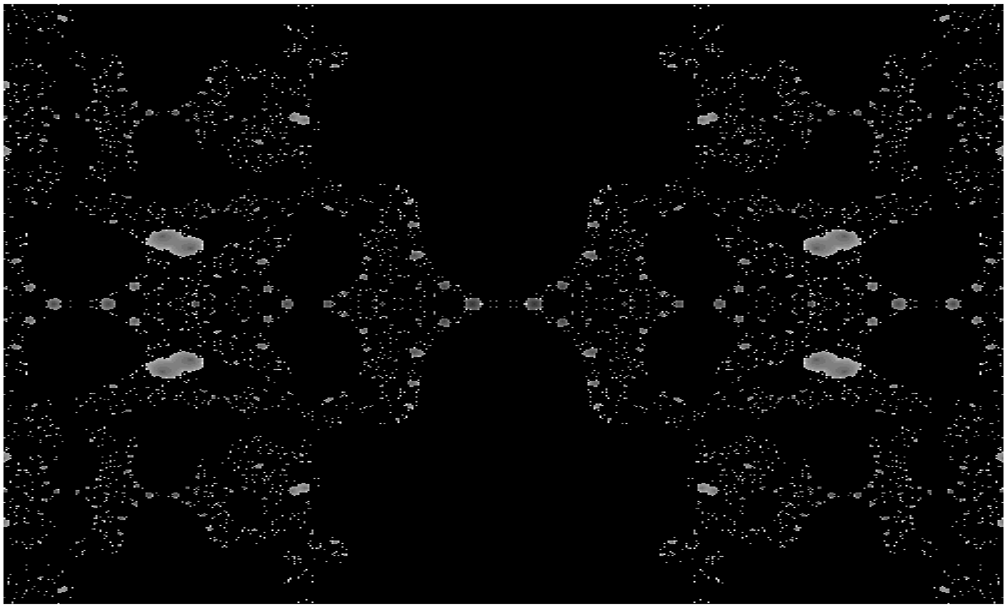


Fig. 13.

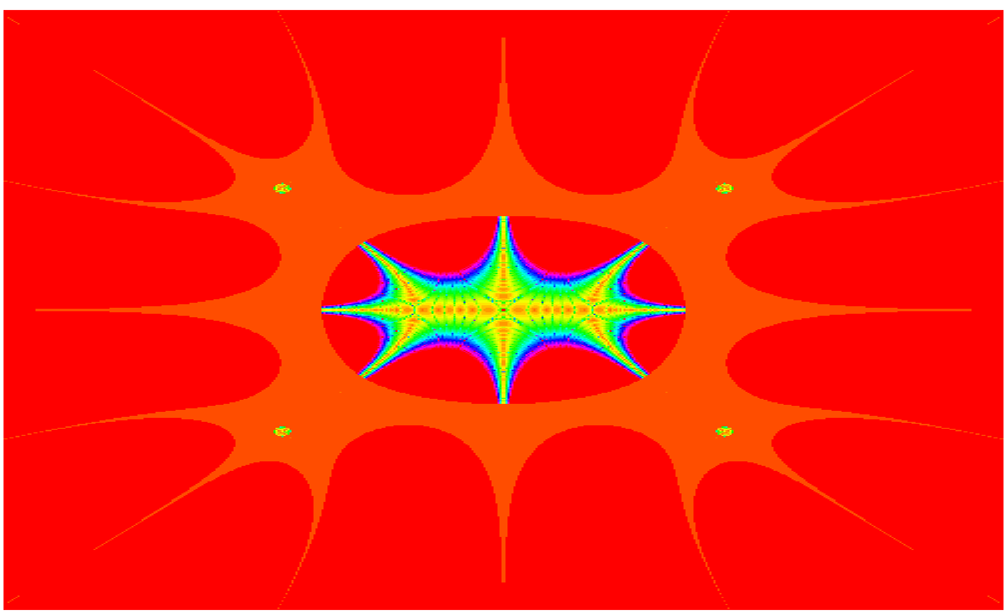
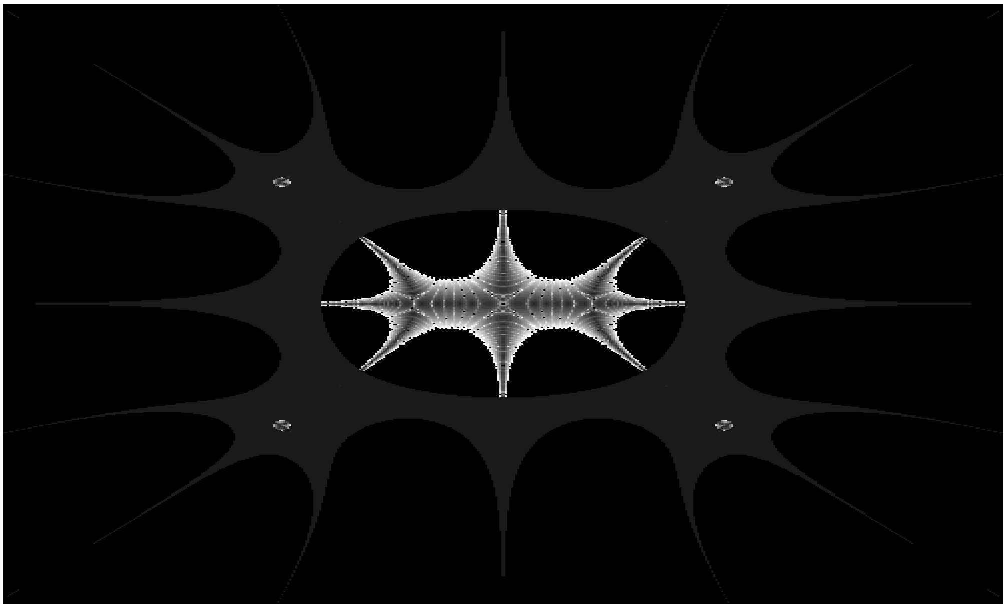


Fig. 14.



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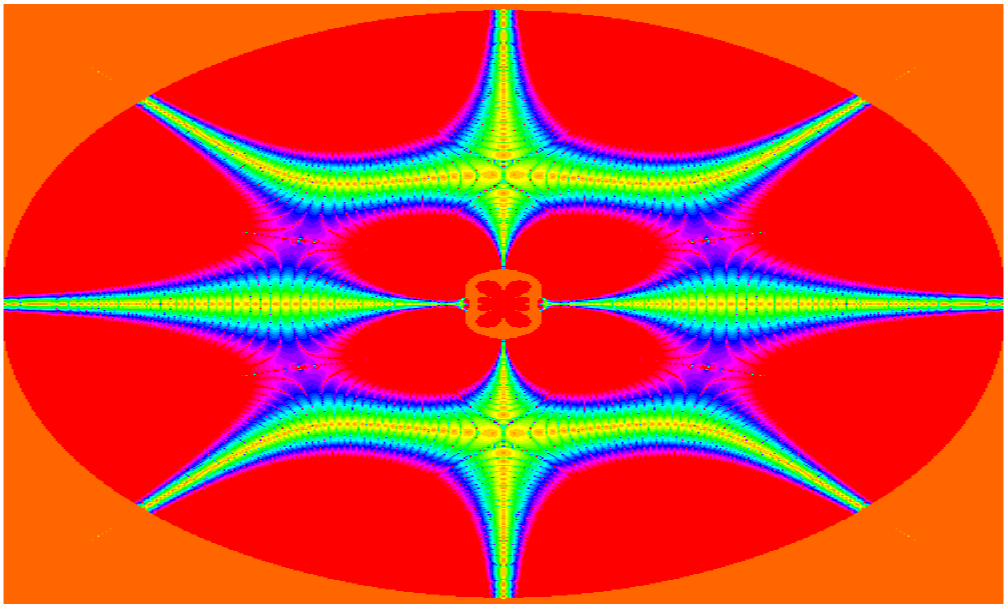

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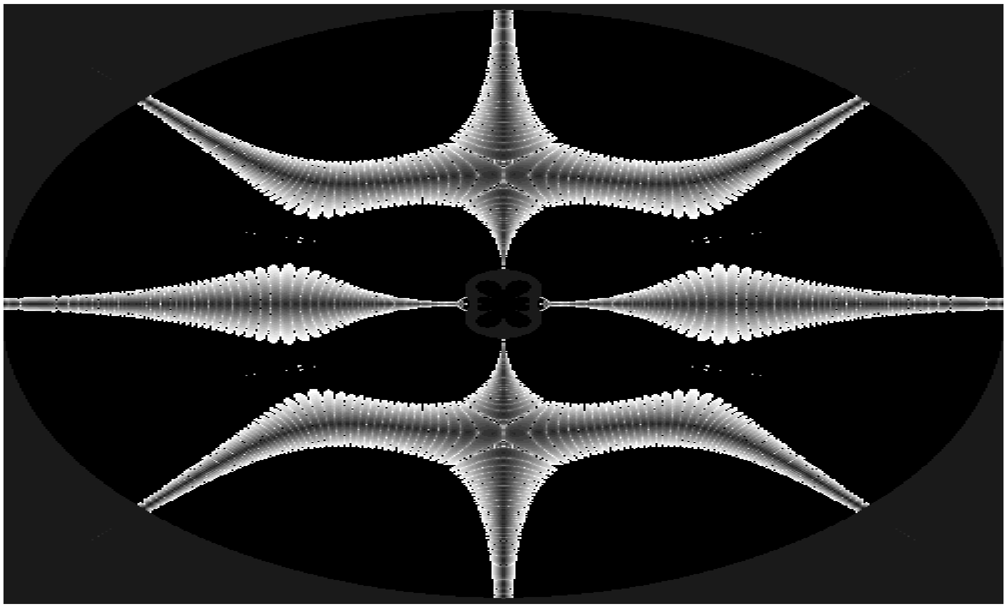
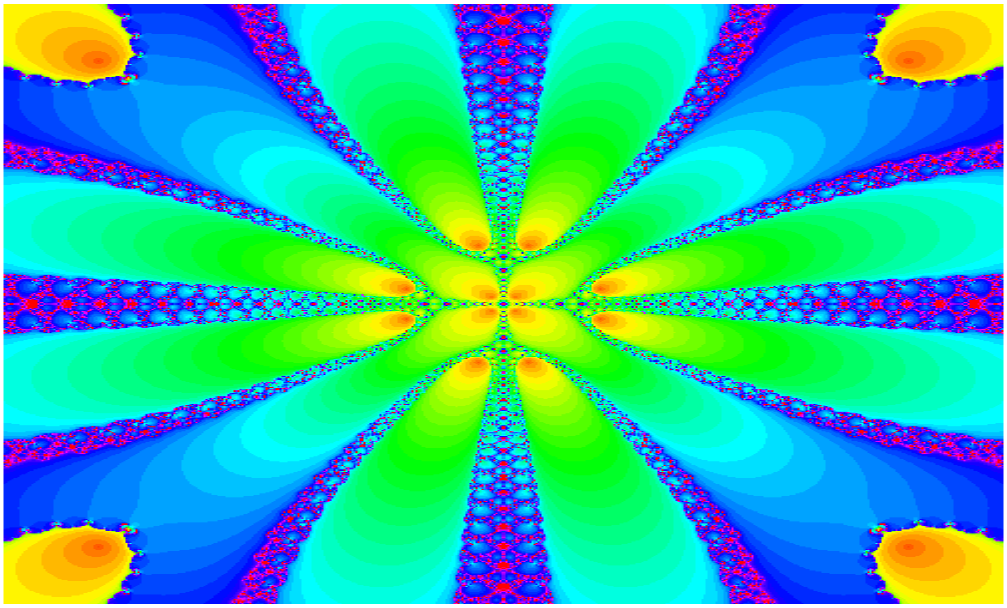


Fig. 17.

❖ Fractals related with $p3(z)$.



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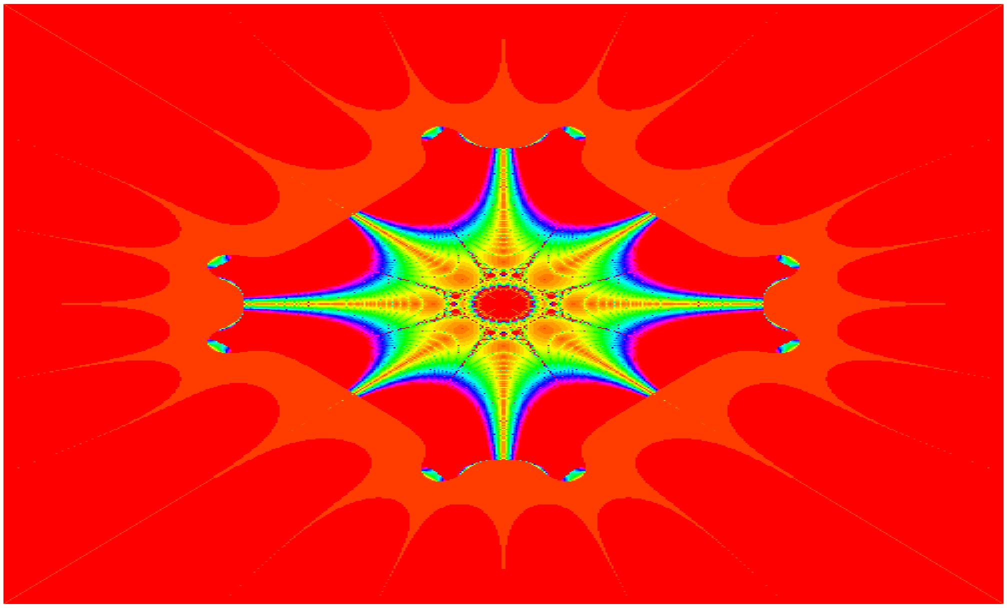

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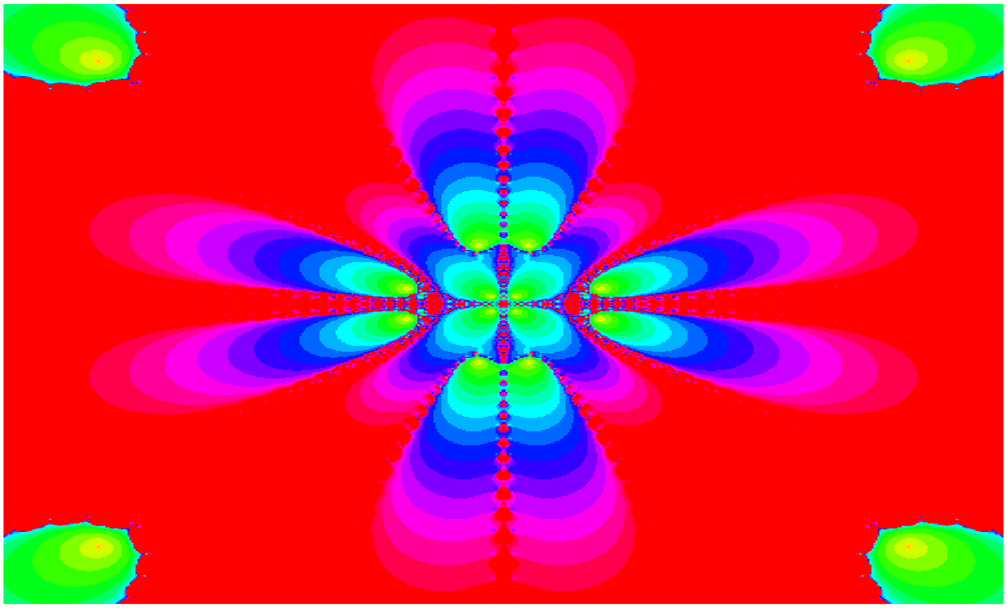
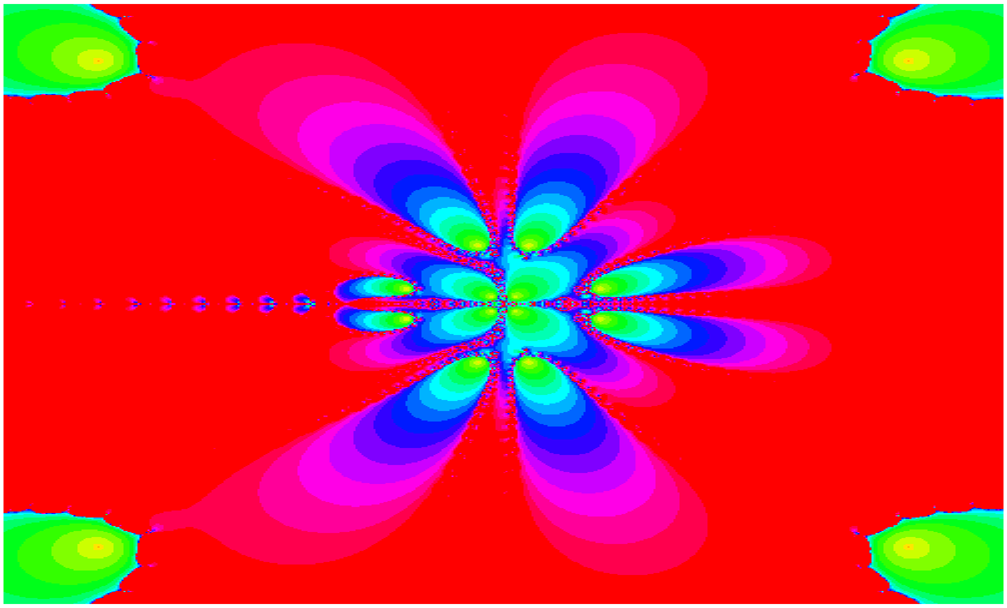


Fig. 20



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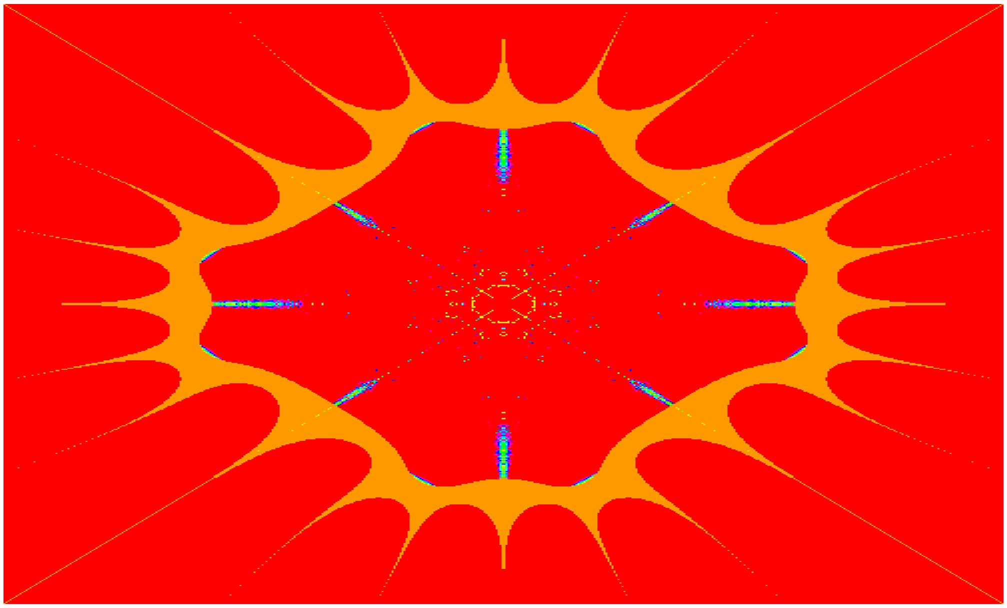

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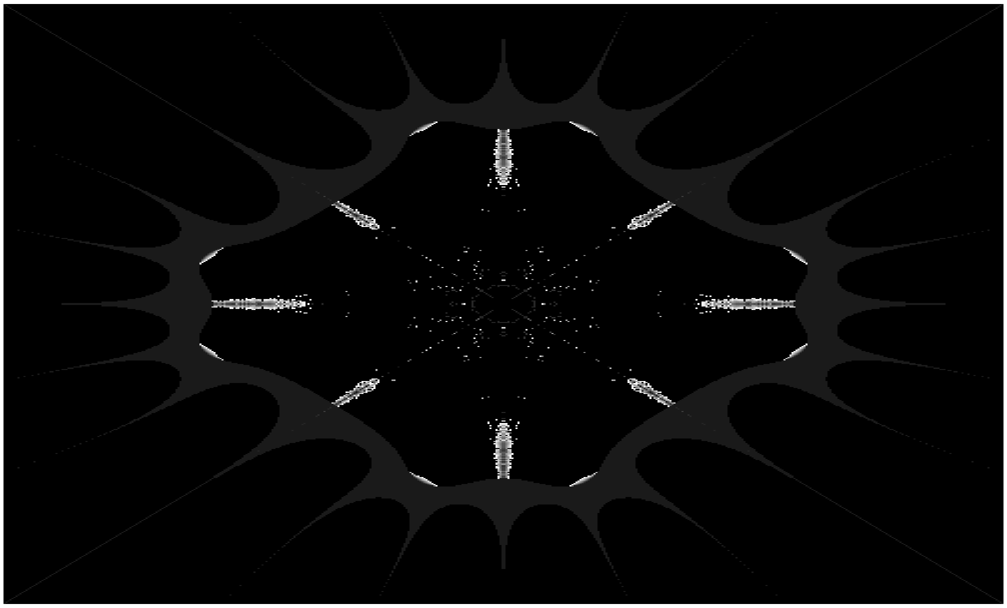

Fig. 23.

References

1. M. Abramowitz and I.A. Stegun. Handbook of Mathematical Functions. Dover Publications, New York , 1970.