International Journal of Engineering Science Invention
ISSN (Online): 2319 – 6734, ISSN (Print): 2319 – 6726
www.ijesi.org ||Volume 4 Issue 11|| November 2015 || PP.60-65

Application of Adomian Decomposition Method in Solving Second Order Nonlinear Ordinary Differential Equations

[1]E. U. Agom MSc , [2]A. M. Badmus PhD
[1]*(Department of Mathematics, University of Calabar, Calabar, Nigeria)*
[2]*(Department of Mathematics and Computer Science, Nigerian Defence Academy, Kaduna, Nigeria)*

ABSTRACT: *In this paper, we use Adomian Decomposition Method to numerically analyse second order nonlinear ordinary di_erential equations and implement the continuous algorithm in a discrete domain. This is facilitated by Maple package. And, the results from the two test problems used shows that the Adomian Decomposition Method is almost as the classical solutions.*

Key words: *Adomian Decomposition Method; Nonlinear Di_erential Equations.*

I. INTRODUCTION

Nonlinear Di_erential Equations (NDE) arise in the study of many branches of Applied Math-ematics. Like Rheology, Quantitative Biology, Physiology,Electrochemistry, scattering theory, Di_usion Transport theory, Potential theory and Elasticity. In the late 20th century, George Adomian [1] introduced a new method to solve NDE. Many of these NDE, in fact a tiny frac- tion, can be solved by analytical or closed form method. Many a times the classical method are complicated; requiring use of advanced Mathematical technique which are di_cult to un- derstand. Of all the the numerical methods available for the solution of NDE, the method of Finite Di_erence is most commonly used followed by Finite Element method. All of which are based on linearisation.
The Adomian Decomposition Method (ADM) which has been subject to much investiga- tion [1],[2], [3], [4],[6] avoids arti_cial boundary conditions, linearisation and yields an e_cient numerical solution with high degree accuracy. It enables the accurate and e_cient analytical solution of NDE without the need to resort to linearisation or perturbation approaches.

II. THE ADOMIAN DECOMPOSITION METHOD

The ADM involves separating the equation under investigation into linear and nonlinear por- tion. The linear operator representing the linear portion of the equation is inverted and the linear the linear operator is then applied to the equation. Any given conditions are taken into consideration. The nonlinear portion is decomposed into a series of what is called Adomian Polynomials. The method generates a solution in the form of a series whose terms are de- termined by a recursive relationship using the Adomian Polynomials. A brief outline of the method is a follows. Consider a general nonlinear di_erential equation as.

$$F = f \tag{2.1}$$

where F is the nonlinear di_erential operator, y and f are functions of t. In operator form equation (2.1) is

$$Ly + Ry + Ny = f \tag{2.2}$$

where L is an operator representing the linear portion of F which is easily invertible. R is a linear operator for the remainder of the linear portion, and N is a nonlinear operator representing the nonlinear term in F. Applying the inverse operator L^{-1} on both sides of equation (2.2), we obtain

$$L^{-1}Ly = L^{-1}f - L^{-1}Ry - L^{-1}Ny \tag{2.3}$$

L^{-1} is integration since F was taken to be a differential operator and L is linear. That is nth integral of y for nth order nonlinear differential equation, where n ϵ Z. Equation (3) becomes

$$y(t) = g(t) - L^{-1}Ry - L^{-1}Ny \tag{2.4}$$

where $g(t)$ represents the function generated by integrating f and using the initial/boundary conditions.

The unknown function $y(t)$ is assumed to be an infinite series given as

$$y(t) = \sum_{n=0}^{\infty} y_n(t) \tag{2.5}$$

we set

$$y_0 = g(t) \tag{2.6}$$

and the remaining terms are determined by a recursive relationship. This is found by decomposing the nonlinear term into a series of what is called **Adomian Polynomial** [1], A_n. The nonlinear term is writing as

$$Ny(t) = \sum_{n=0}^{\infty} A_n \tag{2.7}$$

In order to determine A_n, a grouping parameter, λ is introduced [ ] . The following series are established

$$y(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \tag{2.8}$$

and

$$Ny(\lambda) = \sum_{n=0}^{\infty} \lambda^n A_n \tag{2.9}$$

putting (2.5), (2.6), (2.7), (2.8) and (2.9) in (2.4), we obtain

$$\sum_{n=0}^{\infty} y_n = y_0 - L^{-1}\sum_{n=0}^{\infty} Ry_n - L^{-1}\sum_{n=0}^{\infty} A_n \tag{2.10}$$

where A_n can be determined as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} Ny(\lambda)|_{\lambda=0} \tag{2.11}$$

The recursive relation is found to be

$$y_0 = g(t) \tag{2.12}$$

$$y_{n+1} = L^{-1}Ry_n + L^{-1}A_n \tag{2.13}$$

The ADM produces a convergent series solution which is addressed by [2] and is absolutely and uniformly convergent.

III. APPLICATION AND RESULT

In this section, we discuss the two second order NDE using the ADM.

Problem 1

Consider the second order NDE

$$y'' - y = 2, y(0) = y'(0) = 0 \tag{3.1}$$

$0 \leq x \leq 10$. Which has the exact solution

$$y(x) = e^x + e^{-x} - 2$$

We define a linear operator of

$$L = \frac{d^2}{dx^2}$$

The inverse operator is then

$$L^{-1} = \int\int_0^x (.)dxdx$$

Rewriting the problem in operator form and applying inverse operator for L we obtain that

$$A_n = 2 + y_{n-1}$$

And the recursive relation gives the first fifteen terms as

$$y_0 = 0$$
$$y_1 = y_0 + x^2$$
$$y_2 = y_1 + \frac{x^4}{12}$$
$$y_3 = y_2 + \frac{x^6}{360}$$
$$y_4 = y_3 + \frac{x^8}{20160}$$
$$y_5 = y_4 + \frac{x^{10}}{1814400}$$
$$y_6 = y_5 + \frac{x^{12}}{239500800}$$
$$y_7 = y_6 + \frac{x^{14}}{43589145600}$$

.

.

.

$$y_{15} = y_{14} + \frac{x^{30}}{132626429906095529318154240000000}$$

It must be stated here that all terms of y_n cannot be computed and the solution of (3.1) will be approximated by the series of the form

$$\chi_\eta(x) = \sum_{n=0}^{\eta-1} y_n(x) \tag{3.2}$$

With $\eta = 16$ in this case. The result of the difference between the exact and numerical solution along side the Absolute error, E_A, using a step size of 1 is as shown in Table I.

TABLE I: Exact versus ADM solution of problem 1 with step size of 1

x	Exact solution	Solution with ADM	E_A
0.5	0.2552519304	0.2552519304	0.0000000000
1.5	2.7048192304	2.7048192304	0.0000000000
2.5	10.264578959	10.264578959	0.0000000000
3.5	31.145649342	31.145649342	0.0000000000
4.5	88.028240297	88.028240297	0.0000000000
5.5	242.69601903	242.69601903	0.0000000000
6.5	663.14313648	663.14313648	0.0000000000
7.5	1806.0429675	1806.0429674	0.0000000001
8.5	4912.7690437	4912.7690392	0.0000000009
9.5	13357.726904	13357.726744	0.0000000120

With a step size of 0.1, we obtain a similar result as shown in Table II.

TABLE II: Exact versus ADM solution of problem 1 with step size of 0.1

x	Exact solution	Solution with ADM	E_A
0.1	0.0100083361	0.0100083361	0.0000000000
0.2	0.0401335112	0.0401335112	0.0000000000
0.3	0.0906770282	0.0906770282	0.0000000000
0.4	0.1621447436	0.1621447436	0.0000000000
0.5	0.2552519304	0.2552519304	0.0000000000
0.6	0.3709304364	0.3709304364	0.0000000000
0.7	0.5103380112	0.5013380113	0.0000000002
0.8	0.6748698926	0.6748698926	0.0000000000
0.9	0.8661727708	0.8661727708	0.0000000000
1.0	1.0861612696	1.0861612696	0.0000000001

Problem 2

Consider the second order NDE

$$y\prime\prime - \frac{y}{3} = 1, y(0) = y\prime(0) = 0 \tag{3.3}$$

$0 \leq x \leq 20$. Which has the exact solution

$$y(x) = \frac{3}{2}(e^{\frac{\sqrt{3}}{3}x} + e^{-\frac{\sqrt{x}}{3}x}) - 3$$

Applying equation (2.3)-(2.11), we obtain

$$A_n = 1 + \frac{y_n}{3}$$

$$y_0 = 0$$

$$y_1 = y_0 + \frac{x^2}{2}$$

$$y_2 = y_1 + \frac{x^4}{72}$$

$$y_3 = y_2 + \frac{x^6}{6480}$$

$$y_4 = y_3 + \frac{x^8}{1088640}$$

$$y_5 = y_4 + \frac{x^{10}}{293932800}$$

$$y_6 = y_5 + \frac{x^{12}}{116397388800}$$
$$y_7 = y_6 + \frac{x^{14}}{63552974284800}$$
$$\vdots$$
$$y_{12} = y_{11} + \frac{x^{30}}{214325617392584425580396544000000}$$

Similarly, we consider a finite term of the solution of equation (3.3). Applying (3.2) in this case we take η as 13. The exact solution and that of ADM with various step sizes with the E_A is as given in Table III and Table IV.

TABLE III: Exact versus ADM solution of problem 2 with step size of 2

x	Exact solution	Solution with ADM	E_A
2.0	2.2323374404	2.2323374404	0.0000000000
4.0	12.251570060	12.251570060	0.0000000000
6.0	44.968569928	44.968569928	0.0000000000
8.0	149.07359287	149.07359287	0.0000000000
10.0	479.49833254	479.49833254	0.0000000000
12.0	1527.9891340	1527.9891304	0.0000000024
14.0	4854.9361786	4854.9359042	0.0000000565
16.0	15411.585099	15411.573272	0.0000007674
18.0	48908.604319	48908.275115	0.0000067310

TABLE IV: Exact versus ADM solution of problem 2 step size of 0.1

x	Exact solution	Solution with ADM	E_A
0.1	0.0050013890	0.0050013890	0.0000000000
0.2	0.0200222321	0.0200222321	0.0000000000
0.3	0.0451126125	0.0451126125	0.0000000000
0.4	0.0803561882	0.0803561882	0.0000000000
0.5	0.1258704704	0.1258704705	0.0000000008
0.6	0.1818072154	0.1818072154	0.0000000000
0.7	0.2483529309	0.2483529309	0.0000000000
0.8	0.3257294977	0.3257294977	0.0000000000
0.9	0.4141949091	0.4141949091	0.0000000000
1.0	0.5140441318	0.5140441318	0.0000000000

The visualisation of the exact solution and that of ADM of the two test problems are given in Figures 1-4. $y = \sum_{n=0}^{16} y_n$ and $y = \sum_{n=0}^{12} y_n$ were considered for ADM result of equation (3.1) and (3.3) respectively. Although, finite series of few terms were used Figures 3 and 4 has the same resemblance to the exact results of Figures 1 and 2.

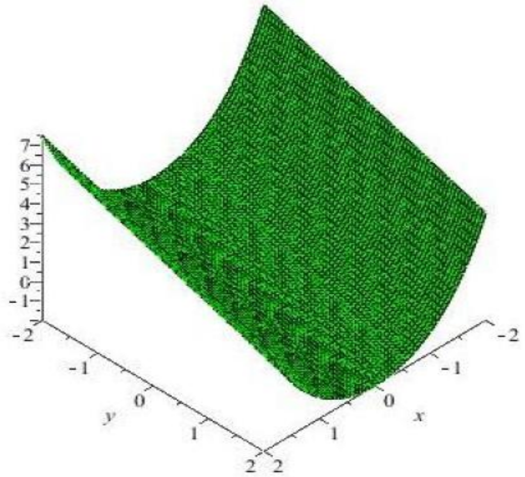

Figure 1: Exact solution of Problem 1

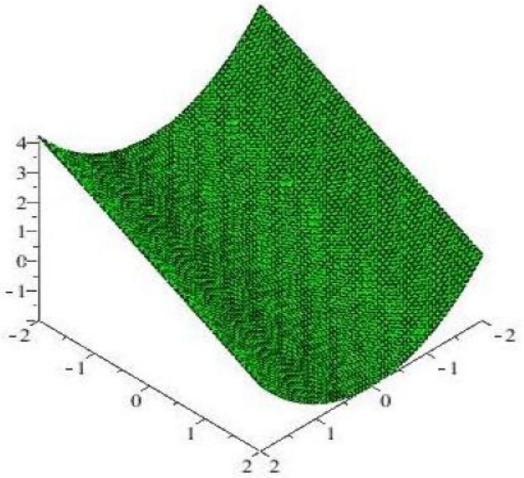

Figure 2: Exact solution of Problem 2

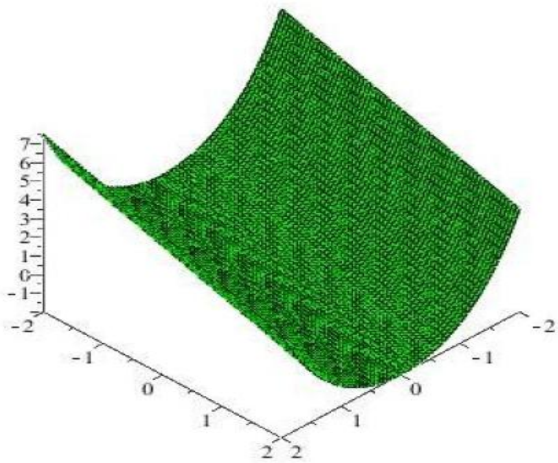

Figure 3: ADM solution of Problem 1

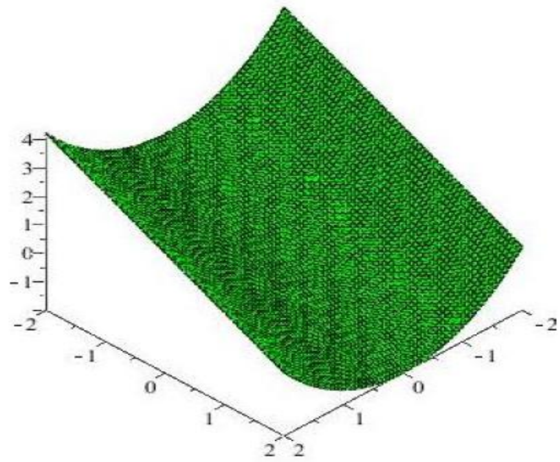

Figure 4: ADM solution of Problem 2

IV. CONCLUSION

Unlike linear Di_erential Equations, there are limited methods for obtaining classical solutions to NDE. In most cases, qualitative methods are often used in obtaining qualitative information on solution of NDE without actually solving the problem, like Phase plane method. In this paper, considering the round o_ errors inherited by taking a _nite series from an in_nite series, the result of ADM and exact solution are in strong agreement with each other. In comparison with several other methods that have been advanced for solving NDE, the result from the two test problems in this paper shows that the ADM is reliable powerful and very promising. We believe that the e_ciency of ADM gives it much wider applicability which needs to be explored further.

REFERENCE

[1] A. M. Wazwaz, A New Algorithm for Calculating Adomian Polynomials for Nonlinear Op- erator, Applied Mathematical Computation, 111(2000), 53-69. MR1745908.

[2] E. A. Ibijola and B. J. Adegboyegun, On Adomian Decomposition Method for Numerical Solution of Ordinary Di_erential Equations. Advances in Natural Applied Science, 2(2008): 165-169.

[3] G. Adomian, Solving Frontier Problem of Physics: The Decomposition Method, Boston: Kluwer Academic Publishers, (1994).

[4] G. Adomian and R. Rach, Analytic solution of Nonlinear Boundary-value Problems in Several Dimensions by Decomposition, Journal of Mathematical Analysis and Application. 174 (1993).

[5] M. J. Ablowitz and J. F. Ladik, Nonlinear Di_erential-Di_erence Equation and Fourier Analysis. Journal of Mathematics and Physics, 17(1976), 1011-1013.

[6] Q. H. Yahya and M. Z. Liu,Modi_ed Adomian Decomposition Method for Singular Initial Value Problems in Second order Ordinary Di_erential Equations, Survey in Mathematics and its Application. 3(2000), 183-193.

[7] T. Mavoungou and Y. Cherrault, Convergence of Adomian's Decomposition Method and Application to Nonlinear Di_erential Equations Kybernetes, 21(6) (1992)