# Vacuum Cosmology

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## Abstract

The internal Schwarzschild solution is examined in the context of a cosmological model where the intergalactic vacuum is described by the internal metric. It is shown that the model predicts an accelerated expansion that agrees with current observations of the expansion history of our Universe, namely that the initial expansion is infinitely fast, and then the expansion slows for some time followed by an accelerated expansion. An examination of the Hubble parameter and redshift is made and it is shown that the model agrees with cosmological data in predicting the transition redshift when the expansion of the Universe changes from deceleration to acceleration. Distance modulus is plotted against redshift and compared to cosmological data. The angular portion of the metric is interpreted and it is shown that the metric can be put into the same form as the FRW metric such that when the energy density of the Universe goes to zero, the internal metric can produce an accelerated expansion without a cosmological constant.

## Expansion Along a Timelike Dimension

The current Big Bang model of the Universe says that the Universe expanded from an infinitely dense gravitational singularity at some time in the past. Current cosmological data suggests that this expansion was slowing down for some time, but is now continuing to expand at an accelerated rate. The Cosmological Principle suggests that from any reference frame in the Universe, the mass distribution is spherically symmetric and isotropic. It is proposed here that the observed expansion of the Universe is the result of a freefall in the time dimension. To analyze the spherically symmetric Universe freefalling through the time dimension, we need the Schwarzschild solution where the radial coordinate is the timelike coordinate. The interior $(r < 1)$ solution of the Schwarzschild field gives us precisely that. For $r < 1$, the signature of the Schwarzschild metric flips and the radial coordinate becomes a dimension measuring time while the $t$ coordinate becomes a dimension measuring space.

Consider a test particle in intergalactic space (or the vacuum 'bubbles' between the large-scale filaments). This vacuum will be spherically symmetric in the vicinity of test particle if it is sufficiently far from all the surrounding galaxies (and the surrounding galaxies are uniformly distributed). Since, according to Birkhoff's theorem, the Schwarzschild solution is the only spherically symmetric vacuum solution to Einstein's field equations, the vacuum occupied by the test particle must be described by the Schwarzschild metric. But it cannot be described by the internal solution since that solution has a gravitational source at the spatial center of the vacuum. It must therefore be described by the internal solution since the vacuum is surrounded by uniformly distributed masses as opposed to having a spherical mass at its center.

So let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates $(\theta, \phi)$. There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $r < 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 shows the $r < 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant $r$ represent spacelike slices of constant cosmological time and the rays of $t$ represent radial distances (each point on this plot is a 2-sphere and each hyperbola is a 3-sphere). To begin with, we will not be considering differences in angles so we only need to consider two quadrants of Figure 1. We will focus on the upper right and lower left quadrants where the quadrants represent an observer pointed in a particular direction and the positive $t$'s in those quadrants represent the coordinate distance from the observer in that particular direction.

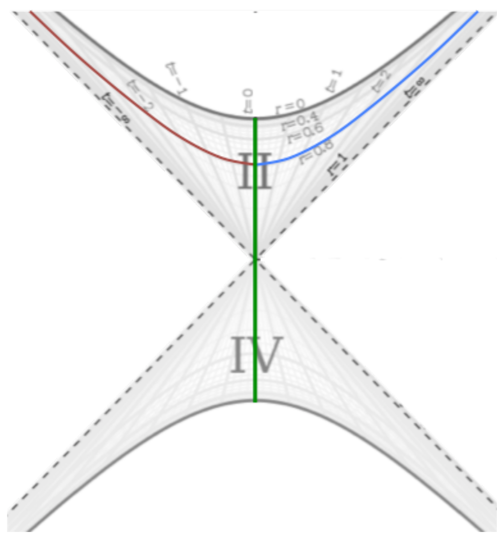


Figure 1 – Freefall Through Cosmological Time[1]

We must first determine the paths of inertial observers in the spacetime. For this we need the internal Schwarzschild metric and the geodesic equations for the internal Schwarzschild metric [1]. In these equations $u$ represents a time constant that in the external metric would be the Schwarzschild radius. In Figure 1, the value of $u$ is 1.

[1] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

$$d\tau^2 = \frac{r}{u-r}dr^2 - \frac{u-r}{r}dt^2 - r^2 d\Omega^2 \tag{1}$$

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2}\left[\frac{u-r}{r}\left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r}\left(\frac{dr}{d\tau}\right)^2\right] \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3 with $d\Omega = 0$, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2} \tag{4}$$

Notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the Schwarzschild constant was 2*GM*, as it would be in the external solution, Equation 4 would be Newton's gravity).

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant *t* (and any such observer can choose $t = 0$ as their coordinate). The 'Big Bang' will have occurred at the center of Figure 1 at $r = 1$. We know this because the above analysis showed that space expands if $\frac{dr}{d\tau}$ is negative, so for our current cosmological time, our worldlines must be moving toward $r = 0$.

## <u>How we see the Universe</u>

Looking at Figure 1, we should note that light signals travel on 45-degree angles. So when we look out at the Universe, we can imagine that we are seeing light emitted from concentric 2-spheres from when the energy of the Universe was at the particular coordinate time corresponding to a particular 2-sphere. They are 2-spheres because each sphere represents a specific coordinate time in the past and distance from us, they are not independent. We can choose to observe the Universe at any arbitrary past time, but we cannot choose to observe the Universe at an arbitrary distance and time, the distance from us we observe depends on the present age of the Universe and the age of the 2-sphere we observe. Nonetheless, each 2-sphere will appear to us to be spatially homogeneous and isotropic and this is reflected in Equation 2 (if we fix the *r* of a 2-sphere, the space will be homogenous and isotropic).

## The Scale Factor

Expressions for the proper time interval along lines of constant *t* and Ω and the proper distance interval along hyperbolas of constant *r* and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{5}$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{6}$$

Where $a$ is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the *t* coordinate. This is good because the *t* coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time *r*.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$ when using Schwarzschild coordinates. When $r = u$, Equations 5 and 6 are both 0. At this point (the Big Bang), it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident (the observer can reach every point in space without moving through time, paths are light-like). So this why the scale factor goes to zero there and why the lines of *t* in Figure 1 converge at that point; it is an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from positive to negative (we can see that if we draw a worldline through the center point, $\frac{dr}{d\tau}$ will change signs as it passes the $r = 1$ point). In fact, for any choice of time coordinate, that point will be a stationary point in those coordinates.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed to light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite. A plot of the scale factor vs. *r* (with $u = 1$) is given in Figure 2 below:

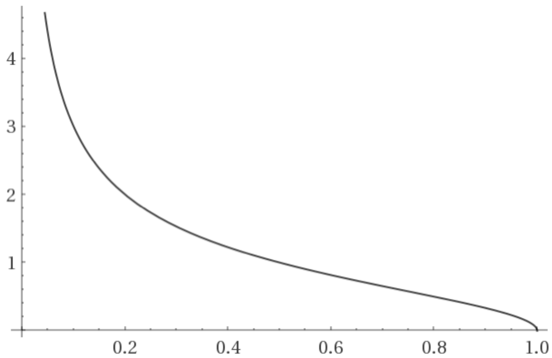


Figure 2 – Scale Factor vs. *r*

In Figure 2, there is an inflection point at $r = 0.75$. This is the point at which the expansion changes from decelerating to accelerating.

## Redshift and the Hubble Parameter

We can use the fact that $\sqrt{\frac{(u-r)}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1]:

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{(u-r)}{r}} - 1 \tag{7}$$

We can use Equation 7 to predict the redshift of the Universe at the time the expansion changed from decelerating to accelerating. First, we must find the value of *u*. For the external metric, this constant has the value of the Schwarzschild radius of a mass given by 2*GM*. For the interior metric, this constant will need to be a time; specifically, it will be the coordinate time in years from the 'Big Bang' to $r = 0$. We can use the known Hubble parameter and current age of the Universe to find this constant. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \tag{8}$$

We know that the Universe is around 13.8 billion years old, so in Equation 8 we can make the substitution $r = u - 13.8$ (because the Big Bang occurs at $r = u$). The Hubble parameter at this time has been measured to be around 67.8 (km/s)/Mpc. Converting that value to units of 1/(billion years), setting Equation 8 equal to that value and solving for *u* we get an approximate value of:

$$u \approx 28.8\ billion\ years \tag{9}$$

We can now express *r* in units of billions of years from $r = 0$ (the Big Bang occurs at $r = 28.8$). A plot of Equation 8 with the value $u = 28.8$ and the ΛCDM model [2] with $\Lambda = 0.013$ is given in Figure 3 below (our current time is shown as the dashed vertical line):

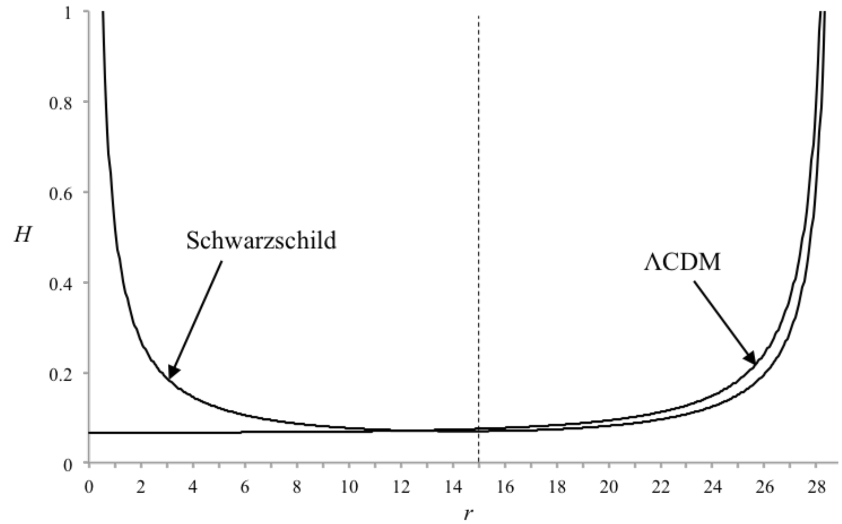


Figure 3 – Hubble Parameter vs. $r$ ($u = 28.8, \Lambda = 0.013$)

Equation 7 can be used to find the transition redshift, which is the redshift we observe at the point when the Universe transitioned from a decelerating expansion to an accelerating expansion. In Equation 7, this transition occurs at $r_{emit} = 21.6$ and our current time is $r = 0.52$. Plugging those values into Equation 8 we get an estimated transition redshift of:

$$z_t = 0.66 \quad (10)$$

This value is within the 2σ bound for the parameter [3,4], and therefore it does appear to be in agreement with cosmological measurements. A plot of redshifts measured at our current time vs. time is given in Figure 4 below:

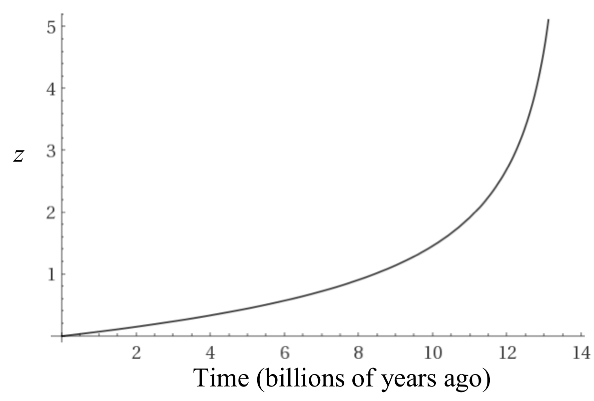


Figure 4 – Redshift vs. Time

Finally, the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 = \frac{r}{7.2} - 3 \quad (11)$$

A plot of the deceleration parameter is given in Figure 5 below:

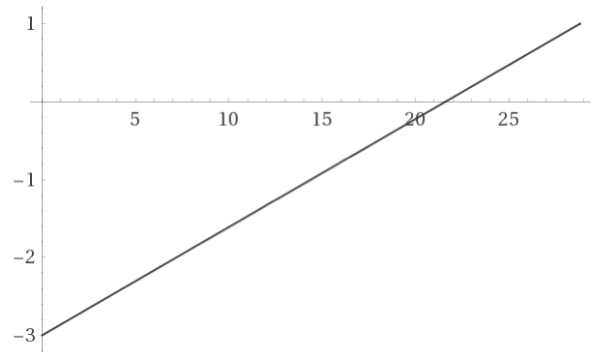

Figure 5 – Deceleration Parameter vs. *r*

**Coordinate Distance & Distance Modulus**

Figure 1 is a plot of the metric on a Kruskal-Szekeres coordinate chart where the *T*-axis is the vertical axis and the *X*-axis is the horizontal axis. The definition of *T* and *X* are given below for $u = 28.8$:

$$X = \sinh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \quad (12)$$

$$T = \cosh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \quad (13)$$

Light travels on 45-degree lines in these coordinates so if we consider our current reference frame at $t = 0$ and $r = 15$, we can find the coordinate distance *t* of some galaxy we observe along the 45-degree line at some *r* by setting $\Delta X = -\Delta T$ and solving for *t*. When we do this, we get:

$$t = 28.8\ln\left(\frac{23.23}{28.8-r}\right) - r \quad (14)$$

Where *t* is in billions of light years and $15 \leq r \leq 28.8$. Note that Equation 14 is only valid for the current cosmological time. The 23.23 constant is specific to this time so for some other time, a different constant would be required and is given by the value $C = (28.8 - r_0)e^{\frac{r_0}{28.8}}$. We can also use Equation 7 to find $r_{emit}$ as a function of *z* and substitute that into Equation 14 to get the coordinate distance as a function of redshift. If we set $r = 15$ for $u = 28.8$ in Equation 7 and solve for $r_{emit}$ we get:

$$r_{emit} = 28.8\frac{z^2+2z+1}{z^2+2z+1.92} \quad (15)$$

Substituting Equation 15 into 14 will give the coordinate distance as a function of measured redshift. A commonly used parameter in cosmology is the distance modulus, $\mu$, which is defined as:

$$\mu = 5\log_{10}\left(\frac{d}{10}\right) \quad (16)$$

Where $d$ is the distance measured in parsecs. A plot of distance modulus vs. redshift obtained by combining Equations 14, 15, and 16 (where we use $t$ measured in parsecs for $d$ in Equation 16) is shown in Figure 6 below plotted over data obtained from the Supernova Cosmology Project [6]:

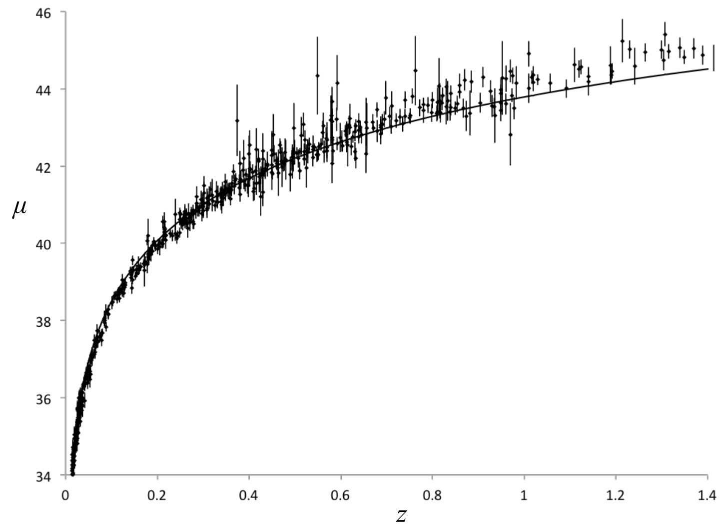


Figure 6 – Distance Modulus vs. Redshift

Note that all these predictions only required the spherical symmetry assumptions of the Schwarzschild metric and calculation of a single parameter, $u$, from cosmological data; it requires no information regarding the detailed energy distribution within the Universe. In fact, the value of $u$ only determines the units we are working in; it does not affect the form of the model. This reflects the fact that the details of the expansion are the result of the vacuum solution alone. Thus, we should expect that the model is accurate for the vacuum-dominated era of the Universe and less so for the matter and radiation dominated eras. We see this in Figure 6 where the model starts to under predict the distance modulus at high redshifts.

## **Proper Time of the Rest Observer**

Figure 7 shows the past light cone of an inertial observer at a given time during the expansion:

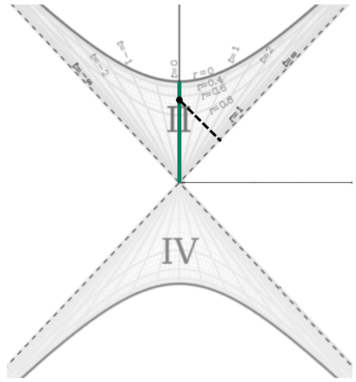


Figure 7 – Past Light Cone of Inertial Observer During the Expansion[2]

We can calculate the duration of the expansion of the Universe in the frame of an inertial observer at rest by integrating Equation 5 from 0 to *u*. The total time of expansion is therefore:

$$\tau = \frac{\pi}{2} u \tag{17}$$

Where $\tau$ is measured in billions of years. Equation 17 tells us that in the frame of an observer at rest at *t*, the time elapsed from the Big Bang to $r = 0$ measured by her clock would be around 45.2 billion years and there is only about 8.8 billion years of proper time between now and $r = 0$ for her.

Thinking of $\tau$ in Equation 17 as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2} u$ for time and space. Equation 17 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2} u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

## Metric and Geodesics in Terms of the Hubble Parameter and Scale Factor

We can re-express equations 1-4 in terms of the scale factor *a* and the Hubble parameter:

$$d\tau^2 = a^{-2} dr^2 - a^2 dt^2 - r^2 d\Omega^2 \tag{18}$$

$$\frac{d^2 t}{d\tau^2} = 2H \frac{dr}{d\tau} \frac{dt}{d\tau} \tag{19}$$

$$\frac{d^2 r}{d\tau^2} = a^2 H \left[ a^2 \left( \frac{dt}{d\tau} \right)^2 - a^{-2} \left( \frac{dr}{d\tau} \right)^2 \right] \tag{20}$$

[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

$$\frac{d^2r}{d\tau^2} = -a^2H \quad (21)$$

Equation 21 gives us a quantity analagous to the surface gravity used in the external solution. The non-zero Christoffel symbols of the model (for $d\Omega = 0$) in terms of $H$ are:

$$\Gamma^r_{rr} = -H \quad (22)$$

$$\Gamma^t_{tr} = \Gamma^t_{rt} = H \quad (23)$$

$$\Gamma^r_{tt} = a^4H \quad (24)$$

## Angular Distance and Relationship to the FRW Metric

We have to this point ignored the angular portion of the Schwarzschild metric. For the internal metric, the angular term seems initially curious because the radius associated with it is a time rather than a distance. According to Figure 1, if we look out at the Universe to a sphere of fixed *r*, we are also seeing a slice of the Universe that is a fixed *t* from our position at that *r*. Thus, in our frame, d*t* between objects on that shell is zero. But we know that some distance separates them and that distance must come from the angular part of the metric. But the radius of the angular part of the metric is independent of the distance of a shell from us. This means that the angles in the metric must have some *t* dependence built into them (because objects at that same *r* and greater *t* from us than the ones we can see must have greater angular separation from each other than the objects we actually see).

The metric in this form might be thought of by considering the sun-earth-moon system where the spatial coordinates are defined as *r* being the distance from the sun and *t* being the distance from the earth. In this case, the moon will revolve around the earth with fixed *t* (assuming circular orbit) where one full orbit will correspond to an angle of $2\pi$ in the reference frame of the earth. But in the frame of the sun, the angle will be much smaller such that the orbit sweeps out a cone rather than a disk. Thus, the angle of a full orbit will be less than $2\pi$ in the frame of the sun because the moon does not revolve around the sun, but around the earth. The internal metric is similar to this, except rather than the sun being the origin of the coordinates, $r = 0$ (a time) is the center.

But what we need is the metric in a form that allows us to measure arc length in the frame of some observer at $t = 0$ and some *r*. Consider Figure 8 below:

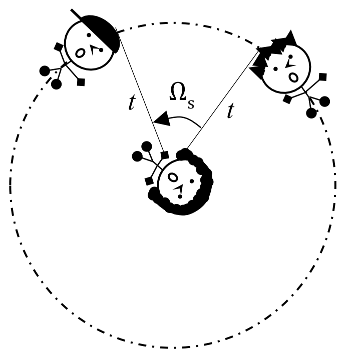

Figure 8 – Arc Length in Vacuum Cosmology

In Figure 8, we see Scout at the center of the diagram at $t = 0$ and some $r$. Dill on the left and Jem on the right are at the same $r$ as Scout (all three of them are space-like separated) and they are both the same coordinate distance $t$ from Scout. We know from Equation 6 that the proper distance between Scout and Jem/Dill is $s = at$. Multiplying $s$ by $\Omega_s$ gives us the proper arc length between Jem and Dill. But we also know that the proper arc length in the metric is given by $r\Omega$. Since these are both the proper arc lengths, we can equate them giving us:

$$\Omega = \frac{at}{r}\Omega_s \tag{25}$$

Thus, we see that if $\Omega_s = 2\pi$, $\Omega$ can be larger or smaller than that depending on the radius of the orbit and the cosmological time at which the motion is occurring. Note that this relationship will also hold true of Jem travels toward Dill along some compound angular path such that:

$$r^2 d\Omega^2 = r^2(d\theta^2 + \sin^2(\theta)d\varphi^2) = a^2t^2(d\theta_s^2 + \sin^2(\theta_s)d\varphi_s^2) \tag{26}$$

We can use Equation 26 to express the metric in a form where the angular part of the metric is centered on Scout rather than $r = 0$:

$$d\tau^2 = a^{-2}dr^2 - a^2dt^2 - a^2t^2[d\theta_s^2 + \sin^2(\theta_s)d\varphi_s^2] \tag{27}$$

Where the $t$ in the final term is the coordinate distance from Scout and $\theta$ and $\varphi$ represent the angular portions of the metric measured in Scout's frame.

Note that the metric in this form is nearly identical to the form of the FRW metric for flat space, but in this case, the stress-energy tensor is zero and the time coordinate is not the proper time of an observer at rest. Thus, this metric would correspond to an FRW cosmology with zero energy density and no cosmological constant. So as the Universe expands and the energy density drops, Equation 27 will become more and more accurate in its predictions since the Universe will become vacuum-dominated. Most notably, this metric, whose form is the same as that of the FRW metric, predicts an accelerated expansion without the need for a cosmological constant. A benefit of the internal metric's coordinates is that it captures the directionality of the time coordinate itself with the $g_{00} = a^{-2}$ function.

The geodesic equation for $r$ with these new angular coordinates included (derived from [1]) is given by:

$$\frac{d^2r}{d\tau^2} = -a^2H\left[1 + \frac{a^2t^2(3u-2r)}{u}\left(\frac{d\Omega_s}{d\tau}\right)^2\right] \tag{28}$$

Given that the speed of light has been set to 1 in Equation 28, we can see that this angular correction will only be relevant on astronomical scales for current and past times. In the future, where $a$ approaches infinity, the correction becomes very large.

**The 'Big Bounce' and the 'Anit-Universe'**

A plot of $\tau$ vs. $r$ from the uppermost to lowermost hyperbola in Figure 1 is given in Figure 9 below. It illustrates well the relationship to typical spatial projectile motion (for $u = 1$).

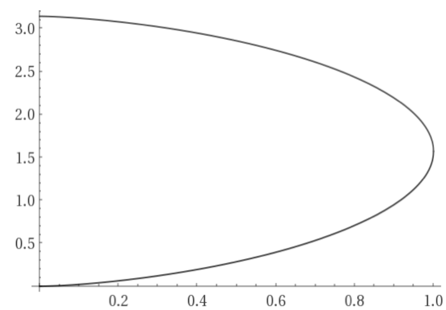


Figure 9 - $\tau$ vs. $r$

Consider a perfectly rigid and elastic ball in simple Newtonian mechanics. If we throw it straight up in the air with initial velocity $\frac{dx}{d\tau}$, the velocity will continuously decrease until at some height $\frac{dx}{d\tau} = 0$, at which point the ball will reverse direction and fall with increasingly negative $\frac{dx}{d\tau}$ until it returns to the ground. When it hits the ground (which we will assume has infinite inertia), since the ball is perfectly rigid and elastic, it will experience an infinite acceleration that will bounce it back toward its maximum height and this cycle will continue ad infinitum. So there are two turnaround points for the ball. One point is maximum height, where the ball does not experience any special acceleration; it just stops moving through space as it turns around. The second point is a hard acceleration that the ball can really feel a (infinite) force changing its direction.

Likewise, we can see that the Schwarzschild cosmology is a similar situation except that the Universe is the ball and the acceleration is through time rather than space. The Big Bang corresponds to maximum height, where the Universe's velocity through time changes sign. The $r = 0$ hyperbolas are, perhaps, the 'bounce'. When the ball bounced, it experienced an infinite acceleration. In the cosmological case, when $r = 0$ the curvature of the spacetime is infinite [1]. This infinite curvature may be a point in time where the worldlines of the Universe turn back on themselves as if the spacetime is folded there and the worldlines go up one side and down the other (the infinite curvature is at the fold).

But what would the Universe bounce off of? The author can only speculate about this, but here is one possibility. We have until now only discussed the positive *t* axes in describing our Universe, but there is also the collection of negative *t*'s that are present in Figure 1. Perhaps Figure 1 is showing 2 Universes, one corresponding to the positive *t*'s and one corresponding to the negative *t*'s. We might think of these as a Universe and an anti-Universe. So in Figure 9, the anti-Scout's real axes in Figure 9 would correspond to the negative *t* axes in Figure 1.

We said that the Universe corresponds to positive *t*'s on Figure 1 whereas the negative *t*'s correspond to the 'anti-Universe'. At $r = 0$, the proper distances expand out to infinity, so perhaps as the worldlines enter $r = 0$ they expand out to positive infinity and then come back in from negative infinity, where the Universe begins its recollapse, essentially becoming the aforementioned 'anti-Universe'. This process would be the 'bounce' resulting from the infinite curvature at $r = 0$.

Of course, there is currently no evidence directly supporting these hypotheses at this time.

## Relationship to the External Solution

Let us consider a meter stick at rest at the center of a collapsing spherically symmetric collapsing shell. The meter stick inside the shell stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat space). An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [5]:

$$\frac{dr}{d\tau} = -\sqrt{\frac{2GM}{r}} \tag{29}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 29. Since the meter stick is also at rest relative to observers at rest at any *r*, Equation 29 will also give the relative velocity between the freefall observer and the meter stick when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1 - V^2} = dt\sqrt{1 - \frac{2GM}{r}} \tag{30}$$

Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1 - \frac{2GM}{r}} \tag{31}$$

We see from Equation 31 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. So observers in freefall will see the space beyond $r = 2GM$ fully contracted as they approach $r = 2GM$.

Let us make a radial coordinate change for the freefalling observer. We choose $R$ such that $\frac{dR}{dr} = \frac{r}{r-2GM}$. This coordinate varies identically to the $r$ coordinate for large $r$ and then diverges from it at the horizon. Note that $R \to \infty$ as $r \to \infty$ and $R \to -\infty$ as $r \to 2GM$. The coordinate velocity of the freefalling observer with this coordinate is given by:

$$\frac{dR}{dt} = -\sqrt{\frac{2GM}{r}} = -\sqrt{\frac{2GM}{W(e^{R-1})+1}} \tag{32}$$

Where $W$ is the product-log function. This coordinate choice is also useful because the speed of light in these coordinates is 1 independent of $R$ and $t$. The external Schwarzschild metric with the new coordinate becomes:

$$d\tau^2 = \frac{r-2GM}{r}\left[dt^2 - dR^2\right] \tag{33}$$

A plot of the integral of Equation 32 is given in Figure 10 below:

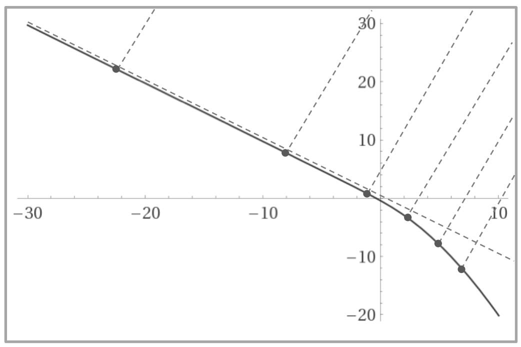

Figure 10 – Light Signals on *t*-*R* Chart

Figure 10 is a *t*-*R* chart that shows a single infalling signal representing the signal to which the freefall worldline is asymptotic. The freefalling observer will never receive this signal or any subsequent signal before the Universe reaches $r = 0$. The dots in Figure 10 represent intervals of equal proper time along the worldline and we can see that rest observers will receive signals from the freefalling observer at longer and longer intervals.

In the frame of the freefalling observer, rest observers will be moving away from her at the speed given in Equation 32. Therefore, she will see the external Universe accelerating away from her at an even faster rate than observers at infinity see other observers at infinity accelerating away from them, their signals increasingly redshifted as time passes. Nonetheless, the freefalling observer will never fall into a 'black hole'. It would take an infinite amount of time in the frame of an observer at infinity for the freefalling observer to reach the event horizon. But the intergalactic bubbles in the Universe will expand and recollapse in a finite amount of time in the frame of the infinite observer and therefore the freefalling observer will only reach the $r = 2GM$ location when the Universe itself has recollapsed. We know this because the proper time of an observer at rest in the internal solution is the coordinate time of the external solution:

$$dt_{external} = adr_{internal} \quad (34)$$

Since it takes a freefalling observer an infinite amount of coordinate time to reach the horizon in the external solution, but there is only a finite amount of proper time to $r = 0$ and then back to $r = u$ in the internal solution, the freefaller can never reach the horizon during the expansion or collapse of the Universe. When she reaches $r = 2GM$, the entire Universe will be fully contracted (it will have reached the $a = 0$ state described in the previous sections) as though everything in the Universe has collapsed to the same $r = 2GM$, and the observer as well as the entire Universe will have reached the next 'Big Bang' state at which point it will presumably begin its expansion once more. This is how the internal and external Schwarzschild solutions relate to one another, they both correspond to the 'Big Bang' state of the Universe.

## Conclusion

It has been shown that the internal Schwarzschild metric will give observations that very closely resemble cosmological observations in our Universe. So either the internal solution is in fact a cosmological solution, or observers inside a Black Hole will see a spacetime that evolves in a strikingly similar way to the evolution of large-scale Universe.

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