Gravitational Forces Revisited

By Jack Bidnik

Abstract:

This paper explains my derivation of a number of equations to describe gravitational forces from the relativistic relative momentum of Albert Einstein's Special Relativity. One of these equations parallels Issac Newton's Gravitational Equation by replacing the Gravitational Constant, G, with a velocity dependent expression. The resulting equation is applied to the orbital parameters of the planets and a number of their moons, with very close results. The forces derived have applications in other areas of physics, including electromagnetic force, and have some surprising properties hitherto unknown in physics. I derive these results with no external forces assumed to be present, so that the only mechanical force here must be gravity.

GRAVITATIONAL FORCES RE-VISITED by Jack Bidnik June 11, 2013

If we have two masses, M and m, in space, and there is an acceleration of M with respect to m, say by a rocket motor, then, by the equivalence principle, this must be called a force between the two masses. Neglecting gravitational force, the bare mathematical fact of the acceleration can be called a force between the two masses. However, it would not be appropriate to say that mass M "exerts" a force on mass m, since the velocity of m is not changed by this at all, just the velocity of M. This would be the interpretation in the classical sense.

Now let us look at the movement of M with respect to m (which may or may not be stationary), in the relativistic sense. The relative velocity v, between them gives rise to what Einstein calls relativistic momentum,

$$\frac{(\mu_0 v)}{\sqrt{1-\frac{v^2}{c^2}}} = c\mu_0 \frac{v}{\sqrt{c^2-v^2}} \quad \text{, where } \mu_0 \text{ is the reduced mass, } \quad \mu_0 \equiv \frac{Mm}{(M+m)}$$

This suggests another force when we differentiate this momentum, namely

$$\mu_0 a_T = F_T = \frac{d}{dt}\frac{(\mu_0 v)}{\sqrt{1-\frac{v^2}{c^2}}} = \mu_0 \frac{c^3}{(\sqrt{c^2-v^2})^3}\left(\frac{dv}{dt}\right) .$$

Is this a true force, or a force in name only, like the aforementioned one? Well, for one thing, this acceleration, and force, does not necessarily require a rocket motor, as above, to exist. It exists just by virtue of the fact that there is a velocity between the two masses. It may be argued that in an inertial frame the quantity $\frac{dv}{dt}$ would be zero. Mathematically, we cannot assume this is necessarily the case, especially when there is more than one object in space.

I believe I can show here that this is a true force, and that it gives rise to Gravity as we understand it.

We begin with

$$F_T = \frac{d}{dt}\frac{(\mu_0 v)}{\sqrt{1-\frac{v^2}{c^2}}} \; = \; \mu_0 \frac{c^3}{(\sqrt{c^2-v^2})^3}(\frac{dv}{dt})$$

the total force, which has two components:

$$F_T = F_m + F_c \qquad = \; c\mu_0 \frac{1}{(\sqrt{c^2-v^2})}(\frac{dv}{dt}) \; + \; c\mu_0 \frac{v^2}{(\sqrt{c^2-v^2})^3}(\frac{dv}{dt}) \;\; \text{respectively.}$$

It is to be noted that at low values of v, $F_m \approx F_T$.

Now let's compute the energy, using the indefinite integral for now, and remembering that dr/dt=v

$$E_T = \int F_T \, dr = \int \mu_0 \frac{c^3}{(\sqrt{c^2-v^2})^3}(\frac{dv}{dt})\, dr = \mu_0 \frac{c^2}{\sqrt{1-\frac{v^2}{c^2}}} + k$$

a familiar, if incomplete, result.

Examination of F_T reveals that it is not dependent on distance, so that with respect to **r** it is constant. This fact is to be analyzed later, as well as the constant k, but from the integration here,we get

$$F_T \cdot r = \mu_0 \frac{c^3}{\sqrt{c^2-v^2}} + k$$

yielding

$$(F_T \cdot r - k)\sqrt{(c^2-v^2)} = \mu_0 c^3$$

and

$$\frac{\sqrt{(c^2-v^2)}}{(\mu_0 c^3)} = \frac{1}{(F_T \cdot r - k)} \quad .$$

Differentiating both sides separately, first for the left side with respect to r, we have

$$\frac{1}{(\mu_0 c^3)}\, \frac{d}{dr}\sqrt{(c^2-v^2)} = \frac{1}{\cancel{2}} \cdot \frac{1}{(\mu_0 c^3)} \cdot \frac{(-\cancel{2}v)}{\sqrt{(c^2-v^2)}}\, \frac{dv}{dr}$$

Differentiating the right side:

$$\frac{d}{dr}\frac{1}{(F_T \cdot r - k)} = -\frac{F_T}{(F_T \cdot r - k)^2}$$

and equating both:

$$\frac{1}{(\mu_0 c^3)} \cdot \frac{(-v)}{\sqrt{(c^2 - v^2)}}\frac{dv}{dr} = -\frac{F_T}{(F_T \cdot r - k)^2} \quad \text{and since} \quad \frac{dv}{dr} = \frac{dv}{dt}\frac{dt}{dr} = \frac{dv}{dt}\frac{1}{v}$$

Canceling minus signs and v's the left side reduces:

$$\frac{1}{(\mu_0 c^3)} \cdot \frac{\cancel{v}}{\sqrt{(c^2 - v^2)}}\frac{dv}{dt}\frac{1}{\cancel{v}} = \frac{1}{(\mu_0 c^3)} \cdot \frac{1}{\sqrt{(c^2 - v^2)}}\frac{dv}{dt}$$

This is equal to $\frac{1}{(\mu_0 c^3)} \times \frac{F_m}{(\mu_0 c)} = \frac{1}{(\mu_0 c^2)^2} \times F_m = K_3 F_m$, our friend F_m

from the expression for “total force”. Equating with the right hand side of the above,

$$\frac{F_T}{(F_T \cdot r - k)^2} = K_3 F_m \quad \textit{where} \quad K_3 = \frac{1}{(\mu_0 c^2)^2}$$

$$F_m = \frac{F_T}{(F_T \cdot r - k)^2} \cdot \left(\frac{1}{K_3}\right) = \frac{F_T}{(F_T^2 r^2 - 2\mathrm{F}_T rk + k^2)} \cdot \left(\frac{1}{K_3}\right)$$

Now let us extract $\frac{Mm}{r^2}$ giving $F_m = \frac{Mm}{r^2}\left[\frac{\left(\frac{1}{Mm}\right)\cdot\left(\frac{F_T}{K_3}\right)}{(F_T^2 - 2\mathrm{F}_T k/r + k^2/r^2)}\right]$

We now do the above energy integral from r=0 to r, equivalent to v=0 to v, to get the value of k:

$$\int_{r,v=0}^{r,v} F_T \cdot dr = \frac{(\mu_0 c^2)}{\sqrt{1-\frac{v^2}{c^2}}} - \mu_0 c^2$$, Einstein's equation, $so \quad k = -\mu_0 c^2$

We can now call the quantity in the square brackets above, provisionally, G, and try to determine if its value corresponds to the universally accepted value of Newton's constant.

$$G = \left[\frac{(\frac{1}{Mm})\cdot(\frac{F_T}{K_3})}{(F_T^2 - 2F_T k/r + k^2/r^2)}\right]$$ and since $$K_3 = \frac{1}{(\mu_0 c^2)^2}$$ and $$k = (-\mu_0 c^2)$$

$$G = \left[\frac{(\frac{1}{Mm})\cdot(\mu_0 \frac{a_T}{K_3})}{(\mu_0^2 a_T^2 + 2\mu_0^2 a_T c^2/r + \mu_0^2 c^4/r^2)}\right]$$ or $$G = \left[\frac{(\mu_0^2)\cdot(\frac{1}{(M+m)})\cdot(a_T c^4)}{((\mu_0^2)(a_T^2 + 2a_T c^2/r + c^4/r^2))}\right]$$

In order to evaluate G numerically, we can make a substitution that seems reasonable, somewhat analogous to the sigma substitution in the mean field approximations of Statistical Mechanics: It is reasonable to assume that the total force F_T , represented in the above equations, can be closely approximated, for a **planetary orbit**, by the centripetal force with the acceleration

$a_T = v_o^2/r$ where v_o is orbital velocity. So $$G = \left(\frac{1}{(M+m)}\right)\frac{(\frac{v_o^2}{r})c^4}{(\frac{v_o^2}{r})^2 + 2(\frac{v_o^2}{r})c^2/r + c^4/r^2}$$.

Using values obtained from http://solarsystem.nasa.gov , and at http://www.wolframalpha.com/, I have obtained values for the planets and a number of their moons, of their masses, semi-major axes, eccentricities, and orbital angular momenta. The angular momentum is used to derive the average orbital velocity in one set of cases, and the orbital velocity at equinox in another set of cases.

As can be seen from Tables 1 and 2, the results are, with some close exceptions, within .01 for most of the cases where average orbital velocity was used, and much better, to within .001 for almost all cases where the orbital velocity at equinox was used. I can only conjecture that the discrepancy for the Earth's moon is due, perhaps, to tidal effects, and Mercury's average velocity is not as good a measure as its velocity near equinox. I will explain below how I arrived at these results in more detail. The details of the data used are in Tables 2 and 3, at the end of the paper.

TABLE 1 Derived G values for planets and moons using AVERAGE velocity

The accepted value for Newton's Constant is G=0.667384e-10

For Earth	0.66717238e-10 is G	Pluto	0.63062312e-10
Mars	0.6617767e-10	Ceres	0.66285631e-10
Venus	0.6675595e-10	Io	0.6673428e-10
Mercury	0.63945953e-10	Europa	0.6671879e-10
Jupiter	0.66576582e-10	Ganymede	0.66689704e-10
Saturn	0.66546589e-10	Callisto	0.66685833e-10
Neptune	0.67184949e-10	Titan	0.66652247e-10
Uranus	0.66291788e-10	Phoebe	0.64915584e-10
Earth's Moon	0.6569555e-10		

TABLE 2 Derived G values for planets and moons at EQUINOX

The accepted value for Newton's Constant is G=0.667384e-10

For Earth	0.66735875e-10 is G	Pluto	0.6722452e-10
Mars	0.66759981e-10	Ceres	0.66703386e-10
Venus	0.66759016e-10	Io	0.66735402e-10
Mercury	0.66769372e-10	Europa	0.66724686e-10
Jupiter	0.66732819e-10	Ganymede	0.66689817e-10
Saturn	0.66740209e-10	Callisto	0.66689485e-10
Neptune	0.67189907e-10	Titan	0.66707577e-10
Uranus	0.66440166e-10	Phoebe	0.66696347e-10
Earth's Moon	0.65897801e-10		

The following BASIC program, with the appropriate values and adjustments for the two interpretations of velocity was used to obtain the derived values for G.

```
M=1.9891e30              'Mass of Sun (large mass)
m= 5.97219e24            'Mass of Earth or other planet
a=1.49598262e11          'semi-major axis (= r in this configuration)
e=0.01671123             'eccentricity
L=2.661e40               'orbital angular momentum
c=2.99792458e8           'speed of light
v=L/(m*a)                'orbital velocity in average velocity case
'v=L/(m*a*sqr(1-e^2)) use this for velocity in the equinox case
At=(v^2)/a               'centripetal acceleration
G=((1/(M+m))*At*c^4)/(At^2+(2*At*c^2)/a+c^4/(a^2))
Print G
```

I have used two different methods to compute values for G:

1. Average Orbital Velocity.
   One set of values is obtained by using the average orbital velocity, determined from the orbital angular momentum, using the length of the semi-major axis, a, for the average radius. Thus

   $$v_o = L/ma \qquad \text{and} \qquad a_T = v_o^2/a \ .$$

2. Orbital Velocity at “Equinox”.
   The second method achieves results even closer to the accepted value of Newton's constant, and is based on using the “equinox” point of the orbit. The idea is that at a point near the equinox of the orbit, the magnitude of the ***radial*** velocity reaches a point of inflection, a maximum, between those two ends of the orbit where the radial velocity is a minimum, that is zero. At that point, very near the equinox, the only component of the velocity is tangential, ie. the orbital velocity, v_o . It is similar for the moons around their planets, so the “equinox” terminology will be retained there.
   To determine v_o for these cases, we use the property of an ellipse, which says that it is the locus of points such that the sum of their distances from the foci is equal to two times the semi-major axis, ***a.***
   Thus the length of the radius to the apex of the semi-minor axis, the “equinox” point is ***r=a***. The length from that focus to the centre of the ellipse is ***ae*** where ***e*** is the eccentricity, and Θ is the angle between the radius and the major axis.

   Also $\cos\Theta = ae/a = e$, for that particular angle.

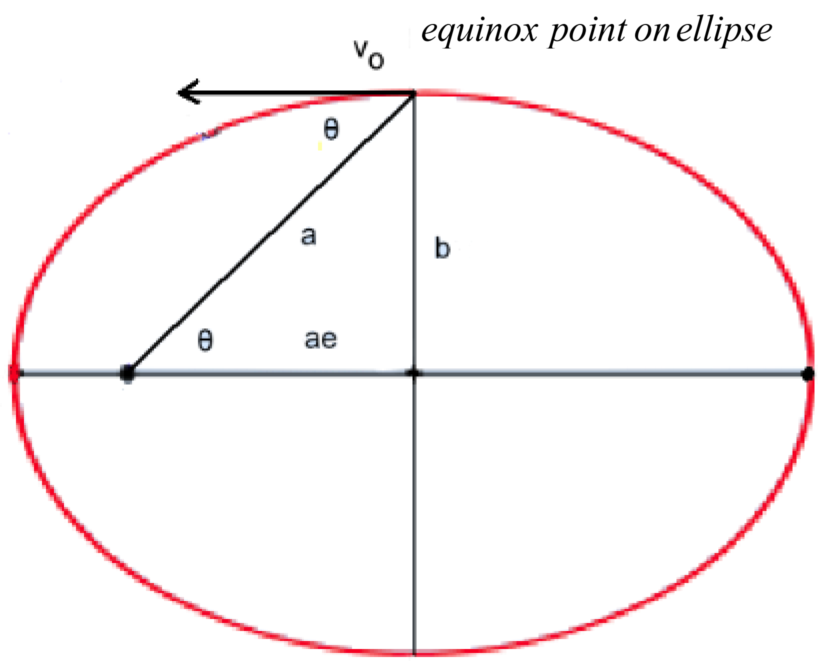
   Thus $\sin\Theta = \sqrt{1-\cos^2\Theta} = \sqrt{1-e^2}$.

   The angular momentum is $L = m v_o r \sin\Theta$, so $v_o = L/ma\sqrt{1-e^2}$.

   We use this value to compute $a_T = v_o^2/a$.

Figure 1

Near the Equinox point the magnitude of the radial component of velocity reaches a maximum, adding no radial acceleration, and only the tangential component of velocity figures in the total acceleration, or force, so that $a_T = v_o^2 / a$.



For an ellipse, the radius at the equinox is
1/2 of 2a=a

$\cos\Theta = ae/a = e \qquad \sin\Theta = \sqrt{1-\cos^2\Theta} = \sqrt{1-e^2}$

The angular momentum is $L = m v_o r \sin\Theta \qquad v_o = L/ma\sqrt{1-e^2}$

We use this value to compute $a_T = v_o^2/a$

Even though the values for G, so obtained, may technically show a variation in Newton's constant, it is not my object here to redefine the constant, or to hope that my results would duplicate the accepted value with exactitude. That my equations can come so close is indication enough that my method and theory about the forces F_T, F_m, F_c and their remarkable properties, are correct physics. I outline these properties below, and I am sure in principle, and also through my other work, that they extend to the rest of the field of physics.

Properties and implications of F_T, F_m, F_c :

$$F_T = \frac{d}{dt}\frac{(\mu_0 v)}{\sqrt{1-\frac{v^2}{c^2}}} = \mu_0 \frac{c^3}{(\sqrt{c^2-v^2})^3}(\frac{dv}{dt})$$

The maximum velocity, of separation or approach of two masses, is c.

F_T does not depend on distance, as can be seen from the formula. A force is generated by relative movement of an object on earth, and another say, on alpha-centauri. The force is simultaneous with the relative motion and does not take any time at all to traverse the distance as it depends only on the velocity and acceleration between the two masses.
This is not the same as the speculative "super-luminal signal". It is precisely because c is the maximum velocity that the force is simultaneous at both ends. (It could be called, however, "'spooky' action at a distance".)

F_T is divided into two parts, F_m and F_c :

$$F_T = F_m + F_C \quad = c\mu_0 \frac{1}{(\sqrt{c^2-v^2})}(\frac{dv}{dt}) + c\mu_0 \frac{v^2}{(\sqrt{c^2-v^2})^3}(\frac{dv}{dt})$$

F_m predominates at low velocities, F_C at high velocities, with the break-even point at $v = c/\sqrt{2}$.

From the fact that we are using reduced mass we see that, since one mass may be considered fixed, then the force between them can be valued as negative when they are approaching each other, and positive when they are separating.

F_m , as shown in the above derivation of G, can be dependent on distance, ie. $1/R^2$, the remaining terms being sufficiently small due to low velocity.

The most interesting property of F_m , in general, results from the expression for the three forces when viewed thus:

$F_m = F_T - F_c$, so $F_m = F_T - F_T \cdot v^2/c^2$ upon inspection of F_T .

This last expression has the same form, when written in terms of acceleration, and if a_T is constant, as a body falling by gravity in a viscous medium, with the opposing force being proportional to the negative of the square of the velocity: $\ddot{y} = g - k\cdot(\dot{y})^2$, in the -y direction.

This means that masses that are separating in space are also attracting, while masses that are approaching each other are also repelling each other. Interesting n'est-ce pas?

Possible criticism and answer.

The analysis above has arrived at Newton's equation for gravitation from Einstein's principles without using Newton's constant as a mathematical starting point. Or has it? Perhaps one might suspect that I have smuggled Newton's constant G_N , into the math in a false bottom of the suitcase.

I had long been nervous about that possibility and I analyzed the places where G_N could possibly sneak into the data. Since I am using experimentally obtained data from NASA there seemed little to worry about, but I realized that other than the orbital velocity and the distances involved, for which I am sure that there is no reason to doubt astronomical observation, there is the matter of the mass of a planetary object. There can be no doubt that those values have always used Newton's constant in the chain of calibration of mass, as obtained experimentally from Cavendish onward.

Well, even though I have obtained these results honestly, without any reasonable mathematical expectation that the equations and figures for G would assume their final form, I recently decided to act out the worst case scenario. I have actually introduced G_N into the formula for G, outright, in the mass term, to see what would happen:

If we begin with Newton's force $F_N = G_N \frac{Mm}{r^2}$ so $M = \frac{F_N r^2}{G_N m}$, and $M + m \approx M$ for large M. Then when we invert and substitute $\frac{G_N m}{F_N r^2}$ for $\frac{1}{M+m}$ in our formula for G:

$$G = [\frac{\frac{1}{(M+m)} a_T c^4}{a_T^2 + 2 a_T c^2 / r + c^4 / r^2}]$$ we get $$G = \frac{G_N m}{F_N r^2} \frac{a_T c^4}{a_T^2 + 2 a_T c^2 / r + c^4 / r^2}$$,and since,

effectively, $F_N = m a_T$, $$G = \frac{G_N}{r^2} \frac{c^4}{(\frac{v_o^2}{r})^2 + 2(\frac{v_o^2}{r}) c^2 / r + c^4 / r^2}$$,which reduces to

$$G = G_N \frac{c^4}{(v_o^2 + c^2)^2}$$, so if v<<c then $G \approx G_N$. This also holds for the case where M=m.

In other words, although the equations for F_T, F_m, F_c , do not yield the exact value for G_N which would be astonishing and impossible, they do yield a formula which can be vanishingly close to it, depending only on orbital velocity. Is this not just as good as getting G_N outright, perhaps even better, since this expands the idea beyond the constant? Einstein's insights and his equations also expand on G_N by adding factors to it, but he assumes G_N .

In any case, it is totally counter-intuitive to expect that merely by extracting a factor Mm/r^2 from such a complex expression, the remaining factor would evaluate to almost G_N .

I hope this is a convincing argument for this different approach to relativity. I have found that it can also extend into the quantum sphere, and I believe it can be extended to electromagnetic forces too.

Post-Script Page

There are many implications in all areas of physics, from the existence and nature of these forces, not the least of which is in Quantum Mechanics. I have already made considerable progress in the application of these ideas to waves and particles on the quantum level, which I will present at the appropriate time.
I am also engaged in quantifying applications in electromagnetic theory.
There is no doubt that these ideas present immediate theoretical bases in particle physics, whose equations are already not too dissimilar to the ones presented here.
Applications in general astrophysics and cosmological theory are an obvious extension, for those so inclined.
The existence of a true force F_T , which is independent of distance, may account for behaviours of spins of entangled particles at great distances.

I have also formulated a possible explanation for the double-slit problem, based on these forces, but it may require an experiment to help solidify the theory.
Please note that I am not asserting "the aether" here, nor am I necessarily alleging determinism, although it may so come to appear, but we must leave that to future philosophical discussions.

I do not believe that my equations cast any doubt on the validity of Albert Einstein's theories, nor on their continued usefulness in analyzing physical phenomena, but I think they provide another avenue to view some things which have so far proven elusive.

TABLE 3 Data and Results for Average Orbital Velocities

The accepted value for Newton's Constant is G=0.667384e-10

M=1.9891e30 'Mass of the Sun 'a=average radial travel =major axis 'For Earth 0.66717235e-10 is G m= 5.97219e24 a=1.49598262e11 e=0.01671123 L=2.661e40 'Mars 0.66177668e-10 m=.641693e24 a= 2.27943824e11 e= 0.0933941 L= 3.515e39 'Venus 0.66755947e-10 m=4.867320e24 a= 1.08209475e11 e= 0.00677672 L= 1.845e40 'Mercury 0.63945947e-10 m=.330104e24 a= 5.7909227e10 e= 0.20563593 L= 8.959e38 'Jupiter 0.66576581e-10 m=1898.130e24 a= 7.78340821e11 e=0.04838624 L=1.928e43 'Saturn 0.66546589e-10 m= 568.319e24 a= 1.426666422e12 e= 0.05386179 L= 7.811e42 'Neptune 0.67184949e-10 m=102.41e24 a= 4.498396441e12 e= 0.00859048 L=2.511e42	'Uranus 0.66291788e-10 m=86.8103e24 a= 2.870658186e12 e= 0.04725744 L=1.689e42 ' 'Pluto 0.63062312e-10 m= 13.09e21 a= 5.906440628e12 e=0.2488273 L= 3.563e38 'Moon 0.6569555e-10 M= 5.97219e24 m= .073477e24 a=.3844e9 e= 0.0554 L= 2.871e34 'Ceres 0.66285631e-10 M=1.9891e30 'mass of Sun m=.947e21 a=4.1369025e11 e=0.079138251 L=6.994e36 'Io 0.66734279e-10 M=1898.130e24 'Jupiter m=.0893193797311089e24 a=4.218e8 e=0.0041 L=6.529e35 'Europa 0.6671879e-10 M= 1898.130e24 'Jupiter m=.0479984383874927e24 a= 6.711e8 e= 0.0094 L= 4.425e35 'Ganymede 0.66689704e-10 M= 1898.130e24 'Jupiter m= .148185846875052e24 a= 1.0704e9 e= 0.0013 L= 1.725e36	'Callisto 0.66685833e-10 M= 1898.130e24 'Jupiter m= .107593737963819e24 a= 1.8827e9 e= 0.0074 L= 1.661e36 'Titan 0.66652247e-10 M= 568.319e24 ' mass of Saturn m= .134552523083241e24 a= 1.221865e9 e= 0.0288 L= 9.155e35 'Phoebe 0.64915584e-10 M= 568.319e24 'Saturn m= 8.29063377189367e18 a= 12.947913e9 e= 0.1634 L= 1.812e32 Formulae and Basic Program Replace hi-lite with above formulas M=1.9891e30 'Phoebe 0.64915584e-10 is G M= 568.319e24 'Saturn m= 8.29063377189367e18 a= 12.947913e9 e= 0.1634 L= 1.812e32 Mu=M*m/(M+m) c=2.99792458e8 v=L/(m*a) At=(v^2)/a G=((1/(M+m))*At*c^4)/ (At^2+(2*At*c^2)/a+c^4/(a^2)) Print G

TABLE 4 Data and Results for Orbital Velocities at Equinox

The accepted value for Newton's Constant is G=0.667384e-10

M=1.9891e30 'Mass of the Sun
'a=major axis = identically radial
'distance at equinox

'For Earth 0.66735872e-10
m= 5.97219e24
a=1.49598262e11
e=0.01671123
L=2.661e40

'Mars 0.66759981e-10
m=.641693e24
a= 2.27943824e11
e= 0.0933941
L= 3.515e39

'Venus 0.66759013e-10
m=4.867320e24
a= 1.08209475e11
e= 0.00677672
L= 1.845e40

'Mercury 0.66769365e-10
m=.330104e24
a= 5.7909227e10
e= 0.20563593
L= 8.959e38

'Jupiter 0.66732818e-10
m=1898.130e24
a= 7.78340821e11
e=0.04838624
L=1.928e43

'Saturn 0.66740208e-10
m= 568.319e24
a= 1.426666422e12
e= 0.05386179
L= 7.811e42

'Neptune 0.67189907e-10
m=102.41e24
a= 4.498396441e12
e= 0.00859048
L=2.511e42

'Uranus 0.66440166e-10
m=86.8103e24
a= 2.870658186e12
e= 0.04725744
L=1.689e42

'Pluto 0.6722452e-10
m= 13.09e21
a= 5.906440628e12
e=0.2488273
L= 3.563e38

'Moon 0.65897801e-10
M= 5.97219e24
m= .073477e24
a=.3844e9
e= 0.0554
L= 2.871e34

'Ceres 0.66703385e-10
M=1.9891e30 'mass of Sun
m=.947e21
a=4.1369025e11
e=0.079138251
L=6.994e36

'Io 0.66735401e-10
M=1898.130e24 'Jupiter
m=.0893193797311089e24
a=4.218e8
e=0.0041
L=6.529e35

'Europa 0.66724685e-10
M= 1898.130e24 'Jupiter
m=.0479984383874927e24
a= 6.711e8
e= 0.0094
L= 4.425e35

'Ganymede 0.66689816e-10
M= 1898.130e24 'Jupiter
m= .148185846875052e24
a= 1.0704e9
e= 0.0013
L= 1.725e36

'Callisto 0.66689485e-10
M= 1898.130e24 'Jupiter
m= .107593737963819e24
a= 1.8827e9
e= 0.0074
L= 1.661e36

'Titan 0.66707577e-10
M= 568.319e24 ' mass of Saturn
m= .134552523083241e24
a= 1.221865e9
e= 0.0288
L= 9.155e35

'Phoebe 0.66696347e-10
M= 568.319e24 'Saturn
m= 8.29063377189367e18
a= 12.947913e9
e= 0.1634
L= 1.812e32

Formulae and Basic Program:

M=1.9891e30 ' the Sun
'Insert planet data from above 'here, eg.

'For Earth 0.66735872e-10
m= 5.97219e24 'planet mass
a=1.49598262e11 'semi-major
e=0.01671123 'eccentricity
L=2.661e40 'angular mom.

c=2.99792458e8

v=L/(m*a*SQR(1-e^2)) ' at equi.
At=v^2/a

G=((1/(M+m))*At*c^4)/_
(At^2+(2*At*c^2)/a+c^4/(a^2))

Print G