Clear local realism advances Bell's ideas, demystifies QM, etc.

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Abstract: Negating the classical/quantum divide in line with Bell's hidden-variable ideas, we resolve Bell's 'action-at-a-distance' dilemma in accord with his hopes. We identify the resultant theory as clear local realism (CLR), the union of Bohr's 'measurement' insight, Einstein locality and Bell beables. Our method follows: (i) consistent with Bohr's insight, we replace EPR's elements of physical reality with Bell's beables; (ii) we let Bell's beable $\boldsymbol{\lambda}$ denote a pristine particle's total angular momentum; (iii) validating Malus' Law in our quantum-compatible equivalence relations, we deliver the hopes of Bell and Einstein for a simple constructive model of EPRB; (iv) we then derive the correct results for CHSH and Mermin's version of GHZ; (v) we thus justify EPR's belief that additional variables would bring locality and causality to QM. In short—advancing Bell's ideas in line with his expectations—we amend EPR, resolve Bell's dilemma, negate nonlocality, endorse Einstein's locally-causal Lorentz-invariant worldview, demystify the classical/quantum divide, etc. CLR: clear via Bohr's insight, local via Einstein locality, realistic via Bell beables.

Keywords: Bell's theorem, causality, CLR, completeness, EPRB, equivalence, GHZ, locality, realism

NB: This is a soliloquy. (i) Paragraphs, equations, etc, are numbered to aid discussion, improvement, correction. (ii) Key texts are freely available online; see References. (iii) We use 'particle' and 'spin' in accord with quantum conventions. (iv) We use the term *naive realism* for unbranded *realism* that is often *unreal* (as we'll show); our brand—*clear realism*—is set apart by its empirically-validated depiction of sensitive micro-realities that may be disturbed by observation. (v) Taking math to be the best logic, it may flow for several lines before we comment. (vi) Delivering results in full accord with quantum theory and experiment, we advance Bell's ideas to demystify claims like these:

> Bell (1964:199), "In a theory in which parameters are added [to QM] to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant." Aspect (2004:9), 'Bell discovered that the search for [local-realistic] models is hopeless.' Wiseman (2005:1), 'Bell (1964) strengthened Einstein's theorem (but showed the futility of his quest) by demonstrating that either reality or locality is a falsehood.' Goldstein *et al.* (2011:1), "In light of Bell's theorem, [many] experiments ... establish that our world is non-local." Maudlin (2014:25), "Non-locality is here to stay ... the world we live in is non-local." Gisin (2014:4), "For a realistic theory to predict the violation of some Bell inequalities, the theory must incorporate some form of nonlocality." Brunner *et al.* (2014:1), "Bell's 1964 theorem ... states that the predictions of quantum theory cannot be accounted for by any local theory." Norsen (2015:1), "In 1964 Bell demonstrated the need for non-locality in any theory able to reproduce the standard quantum predictions." Bricmont (2016:112), 'There are nonlocal physical effects in Nature.' *Annals of Physics* Editors (2016:67, unanimously), in the context of Bell's theorem 'it's a proven scientific fact that a violation of local realism has been demonstrated theoretically and experimentally.'

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1 Introduction

1.0. (i) '... this action at a distance business will pass. If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly, with no big new development. But anyway, I believe the questions will be resolved,' after Bell (1990:9). (ii) 'Nobody knows where the boundary between the classical and quantum domain is situated. More plausible is that we'll find that there is no boundary ...,' after Bell (2004:29-30). We agree and deliver.

1.1. Studying EPR (1935) in the context of EPRB—the EPR-based experiment in Bohm & Aharonov (1957)—Bell (1964:199) claims that EPR's program requires a grossly non-local mechanism. However, instead of amending EPR's terms (as we do at ¶1.5), Bell creates a personal dilemma—¶1.6(i)—not seeing that a theory of the type that he (and we and EPR) favored could succeed. Thus,

> after Bertlmann (2017:40): "Bell wondered, *'Where does the quantum world stop and the classical world begin?'* He wanted to get rid of that division. [Agreeing, that's what we do.] For him it was clear that hidden variable theories [HVTs], where quantum particles do *have* definite properties governed by hidden variables, would be appropriate to reformulate quantum theory: *'Everything has definite properties,'* Bell said." Now at ¶2.8, Bell (1980:7) endorses d'Espagnat's inferences to preexisting properties. So—contrary to QM orthodoxy, our theory, and Bohr's insight at ¶2.9—Bell's HVT appears to be bound by Bertlmann's (*ibid*) generalization that "HVTs in contrast postulate that the properties of individual systems—[like the spin of a particle]—do *have* preexisting values revealed by the act of measurement". But care is needed here: '*Predetermined* is Bell's original phrasing. If there is for Bell an identity between *predetermined* and *preexisting* I cannot say ad hoc, but for a realist—as Bell was—there is clearly a close connection between both phrasings,' after R. Bertlmann (pers. comm., 14 June 2017).

1.1a. NB: For us, some *revealed values* (eg, charge) may preexist, others (eg, a revealed spin-orientation) may not. So—under Bertlmann's generalization (*ibid*)—ours in not an HVT. Instead, we allow observables to be made from canonical beables whose *pristine* preexisting values *may be forever hidden* under 'measurement' interactions and transformations. We then encode the consequent incomplete information in conservation laws and probability theory to successfully advance our understanding.

1.2. Seeking unanimity, we proceed by accepting d'Espagnat's (1979:158) Bell-endorsed principles of local realism: (i) *realism* (regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers); (ii) *locality* (no influence of any kind can propagate superluminally); (iii) *induction* (legitimate conclusions can be drawn from consistent observations). So this is not a dispute about differing principles. Rather—merging our hopes with those of EPR and Bell; and given ¶1.0—we simply reject inferences that are false in quantum settings. We thus show that Bell and d'Espagnat fail under (iii): ie, ignoring consistent observations re the validity of QM and Bohr's insight—¶2.9—they draw conclusions that are false under both QM and experiment (eg, see Aspect 2004). Indeed, for us—readily accepting the commonsense in d'Espagnat's (i)-(iii) above; and succeeding with it (as we'll show)—QM seems to be better-founded than Bell imagined; eg, here's Bertlmann (2017:54; his emphasis) on Bell (with naivety, puzzlement and doubts that we do not share):

> "John was totally convinced that [naive] *realism* is the right position of a scientist. He believed that experimental results are predetermined and not just induced by the measurement process. Even more, in John's EPR analysis reality is not assumed but inferred! Otherwise (without realism), he said, *'It's a mystery if looking at one sock makes the sock pink and the other one not-pink at the same time.'* So he did hold on [to] the hidden variable program continuously, and was not discouraged by the outcome of EPR-Bell experiments but rather puzzled. For him: *'The situation was very intriguing that at the foundation of all that impressive success* [of QM] *there are these great doubts,'* as he once remarked."

1.3. Thus identifying the likely seeds of Bell's dilemma, and seeking to be clear re our position: we identify our quantum-compatible theory as *clear local realism* (CLR), the union of Bohr's insight (some physical properties change interactively; ie, Planck's quantum of action $\hbar \neq 0$), Einstein locality (no causal influence propagates superluminally; ie, $\frac{v}{c} \ngtr 1$ under relativity) and Bell beables. (Thus bypassing the Kochen-Specker theorem, CLR's physical-realism is consistent with contextuality and most interpretations of QM.) Then, taking realism to include the view that physical reality exists and has definite properties, we advance EPR's program—validating their belief; but not their famous criterion at ¶1.4—by including every relevant element of the subject reality in our analysis via ¶2.1:

> "In a complete theory there is an element corresponding to each element of reality," EPR (1935:777). "While we have thus shown that the wavefunction does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible," EPR (1935:780). [It is: as we show by introducing every relevant element of the EPRB reality at ¶2.1. Further, we show that *hidden dynamics* can be adequately described by encoding *incomplete information* in probabilistic relations; eg, via Bayes' Law: $P(XY|Z) = P(X|Z)P(Y|ZX)$. The consequent expectations can then be compared with experimental outcomes (facts).]

1.4. So (using vector-products and physical operators in 3-space; not wavefunctions, etc, in Hilbert space), we study the hidden interaction of elements of physical reality (particles) with other elements of physical reality (polarizers). That is, taking particles and polarizers to be sensitive contributors to the *veiled reality* (d'Espagnat 1983:94) of our world, we allow: (i) any interactant may be transformed under CLR; (ii) any transformation may be subject to an uncertainty induced by Planck's action-constant; (iii) every less-than-certain probability distribution represents a veiled reality. (iv) Then, identifying every relevant element of physical reality under EPRB—¶¶2.1-2.7—we provide a complete description of what can be inferred from incomplete information. (v) We also address Bell's dilemma—¶1.6(i)—by endorsing EPR's next *two* sentences: but amending—at ¶1.5—the famous EPR *criterion*.

> "The elements of physical reality ... must be found by an appeal to the results of experiments and measurements. A comprehensive definition of reality is, however, unnecessary for our purpose. We [ie, EPR; *not us*] shall be satisfied with the following criterion, which we regard as reasonable. *If, without any way disturbing a system, we can predict with certainty ... the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*," EPR (1935:777). [nb: not satisfied, we amend it at ¶1.5.]

1.5. Departing subtly from EPR, but wholly compatible with EPRB and the local-realism of ¶1.2, here's our sufficient condition for an element of physical reality—ie, for a Bellian beable—in the context of (8)-(9) below (to be clear): 'If, without in any way disturbing a system $q(\boldsymbol{\mu}_i)$, we can predict with adequate accuracy the result B_i of a test $\delta_a^{\pm}$ (which may be a disturbance) on that system $q(\boldsymbol{\mu}_i)$, then local beables (existents)—here $q(\boldsymbol{\mu}_i)$, $\delta_a^{\pm}, q(a^-)$—will mediate this result,' after Watson (1998:417). This condition delivers Bell's hope (2004:167) for 'a simple constructive model' of EPRB; see ¶2.15.

1.6. We thus arrive at: (i) Bell's unresolved dilemma re action-at-a-distance (AAD hereafter); (ii) an expansion of our shared motivation with Bell (and with EPR) from ¶1.0:

> (i) 'I cannot say that AAD is required in physics. *I can say that you cannot get away with no* AAD. You cannot separate off what happens in one place and what happens in another. *Somehow they have to be described and explained jointly.* That's the fact of the situation; Einstein's program fails ... *Maybe we have to learn to accept not so much* AAD*, but the inadequacy of no* AAD. ... **That's the dilemma**. We are led by analyzing this situation to admit that, somehow, distant things are connected, or at least not disconnected. ... I don't know any conception of locality that works with QM. So I think we're stuck with

nonlocality ... I step back from asserting that there is AAD and *I say only that you cannot get away with locality*. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is AAD,' after Bell (1990:5-13); *emphasis added*.

(ii) "Now nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary. It is hard for me to envisage intelligible discourse about a world with no classical part—no base of given events, be they only mental events in a single consciousness, to be correlated. On the other hand, it is easy to imagine that the classical domain could be extended to cover the whole. The wavefunctions—[not beables in our terms; see Bell (2004:53)]—would prove to be a provisional or incomplete description of the quantum-mechanical part, of which an objective account would become possible. It is this possibility, of a homogeneous account of the world, which is for me the chief motivation of the study of the so-called 'hidden variable' possibility," Bell (2004:29-30). Sharing this motivation, we deliver differently:

1.7. That is, we depart from Bell's analyses by using Bayes' Law safely, per (i)-(vi) below. [NB: we also prefer short-form Bayes-based expectations—like LHS (67) at ¶5.7—for they bypass a limitation in Bell's analyses at once. However, to match the style of typical Bellian essays, we defer our use of short-forms for now.] Thus, *using our notation* per ¶2.1, taking Brunner *et al.* (2014) as typical, we allow that A and B may be correlated, via our (6), under the conservation of angular momentum. The safety in our approach is then this: (i) all our probability relations are equivalent to Bell's; (ii) all our relations are experimentally validated and consistent with QM; (iii) some Bell-relations are not equivalent to ours; (iv) Bell did not review his relations in the light of experimental repudiations [see Bertlmann (2017:54) at ¶1.2 above]; (v) wrt any probability function, the inclusion of an irrelevant conditional is irrelevant; (vi) *irrelevant conditionals are best eliminated by experimental facts—not by hypotheses*—agreeing with EPR's first two sentences at ¶1.4. (vii) Here's *Bell's locality hypothesis*:

$$P(AB\,|\,ab\boldsymbol{\lambda}) = P(A\,|\,a\boldsymbol{\lambda})P(B\,|\,b\boldsymbol{\lambda}) \text{ after Brunner } et\ al. \text{ 2014:(2); though we prefer} \tag{1}$$

$$\text{(viii)}\quad P(AB\,|\,ab\boldsymbol{\lambda\mu}) = P(A\,|\,a\boldsymbol{\lambda})P(B\,|\,ab\boldsymbol{\lambda\mu}A) = \text{equation (1) amended prudently under CLR:} \tag{2}$$

because (for us) it's clearer to allow $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ to be different variables, correlated via (6); (ix) we do not confuse the consequent logical implication in our (2) with remote (nonlocal) causation; (x) we employ the principle that (2) at the macro-level may lead to a factorization like (1) at the micro-level; and vice-versa. (xi) We then use quantum-compatible (2) to deliver (1): (2) being consistent with special relativity via (6); never false if (1) is true; logically warranted by EPRB correlations; experimentally confirmed (eg, Aspect 2004); theoretically validated by the likes of LHS (67) at ¶5.7. That is, we use macro (2)—with $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ pairwise representing each twin's angular momentum—to reveal equivalence relations and micro-properties that yield a micro-version of (1). In this way we show that our world is clearly local and realistic *in CLR terms*.

1.8. So, under the principles in ¶¶1.2-1.5, eliminating false inferences and non-facts and resolving Bell's dilemma—via Analysis, Conclusions, Acknowledgment, Appendix, References—we move to defend the Abstract in line with our continuing respect for Oliver Heaviside and connected facts.

"Facts are of not much use, considered as facts. They bewilder by their number and their apparent incoherency. Let them be digested into theory, however, and brought into mutual harmony, and it is another matter. Theory is of the essence of facts. Without theory scientific knowledge would be only worthy of the madhouse," Heaviside (1893:12).

1.9. In short: CLR harmonizes many facts associated with the core principles spelt out by d'Espagnat in ¶1.2. Such facts include (i) EPR-style correlations (as in EPRB-style experiments); (ii) repeated experimental validation of QM, Bohr's insight and special relativity; (iii) the validation of (2) in theory and in practice (eg, Aspect 2004): all of which leads to ¶3.5 and CLR's realistic factoring of that famous Bellian hypothesis—(1) above—after we've established CLR's credentials beyond dispute. Here goes.

2 Analysis

2.0. Einstein "argued that the EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way," Bell (2004:86). EPR suggest that a state, 'richer in content than the quantum state, would provide a commonsense explanation of certain perfect correlations predicted by QM, which are otherwise baffling,' after GHSZ (1990:1131). Agreeing, we study EPRB logically and we deliver a clear explanation via CLR. CLR: clear via Bohr's insight, local via Einstein locality, realistic via Bell beables.

2.1. Under CLR-completeness, here's our idealization of every relevant element of the subject reality—ie, EPRB per Bell (1964)—including 3-space (since time and gravity are not essential to the analysis here). We let the beable $\boldsymbol{\lambda}$ in Bell's 1964:(1)—with the spin s implicit—denote a pristine particle's total angular momentum; and we allow that in the ith pair, $\boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0$ via the pairwise conservation of total angular momentum.

$$.A_i \equiv +1 \leftarrow \Delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \lhd\beta\rhd \ q(\boldsymbol{\mu}_i) \Rightarrow \Delta_b^{\pm} \rightarrow +1 \equiv B_i. \tag{3}$$

$$.A_i \equiv +1 = a\cdot a^{+} \leftarrow [a\cdot *] \Leftarrow q(a^{+}) \leftarrow \delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \lhd\beta\rhd \ q(\boldsymbol{\mu}_i) \Rightarrow \delta_b^{\pm} \rightarrow q(b^{+}) \Rightarrow [b\cdot *] \rightarrow b\cdot b^{+} = +1 \equiv B_i. \tag{4}$$

$$[\![\quad \text{Alice's locale} \quad]\!] \quad [\![\text{Source}]\!] \quad [\![\quad \text{Bob's locale} \quad]\!] \tag{5}$$

$$\boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0;\ i = 1, 2, ..., n.\ A_i(a, \boldsymbol{\lambda}_i) = -B_i(a, \boldsymbol{\mu}_i);\ \text{etc. } P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_j \,|\, i \neq j) << 1. \tag{6}$$

$$.A_i \equiv +1 = a\cdot a^{+} \leftarrow [a\cdot *] \Leftarrow q(a^{+}) \leftarrow \delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \lhd\beta\rhd \ q(\boldsymbol{\mu}_i) \Rightarrow \delta_a^{\pm} \rightarrow q(a^{-}) \Rightarrow [a\cdot *] \rightarrow a\cdot a^{-} = -1 \equiv B_i. \tag{7}$$

2.2. (3) shows experiment β (EPRB, with β honoring Bohm) and a test on (a decoherent interaction with) each member of the ith particle-pair; thick arrows ($\Rightarrow$) denote movement toward an interaction, thin arrows ($\rightarrow$) point to the subsequent output (typically a transformation). With pristine spin-related properties $\boldsymbol{\lambda}_i$ and $\boldsymbol{\mu}_i$ (with spin s implicit), spin-$\frac{1}{2}$ particles $q(\boldsymbol{\lambda}_i)$ and $q(\boldsymbol{\mu}_i)$ emerge from a spin-conserving decay $\lhd\,\beta\,\rhd$ such that (6) holds. Each particle interacts with a dichotomic linear-polarizer-analyzer $\Delta_x^{\pm}$—freely and independently operated by Alice (with result A) and Bob (result B)—where x denotes any relevant orientation of its principal-axis. Under EPRB: $x^{+} = +x$; $x^{-} = -x$.

2.2a. Given that A and B are discrete (± 1), and seeking generality, we employ variables like $\boldsymbol{\lambda}_i$, $\boldsymbol{\mu}_i$ (ordinary vectors with lengths in units of $\frac{\hbar}{2}$) so that our EPRB results are associated with $\frac{\hbar}{2}$. In this way linking to vector-magnitudes—eg, $\boldsymbol{\lambda}_i = |\boldsymbol{\lambda}_i|\hat{\boldsymbol{\lambda}}_i$, with $|\boldsymbol{\lambda}_i|$ in units of $\frac{\hbar}{2}$ and $\hat{\boldsymbol{\lambda}}_i$ the direction-vector—our variables may be continuous or discrete. This choice accords with the generality of our approach; and with Bell's (1964:195) indifference to whether such variables are discrete or continuous. [Then, in that we are initially searching for equivalence relations under orientations—taking Bell's (1964) a and b to be principal-axis direction-vectors—$\frac{\hbar}{2}$ is suppressed in (3)-(7). The more complete story under EPRB—$\delta_a^{\pm} q(\boldsymbol{\lambda}_i) \rightarrow q(\frac{\hbar}{2}a^{\pm})$, with allied relations under magnitudes—is developed at ¶5.3.]

2.3. Identifier i is used generically: but each particle may be tested once only in its *pristine* state, and thereafter until absorbed in an analyzer. Since the tests are locally-causal and spacelike-separated, we hold fast to Einstein's principle of local causality: the real factual situation of $q(\boldsymbol{\mu}_i)$ is independent of what is done with $q(\boldsymbol{\lambda}_i)$ which is spatially separated from $q(\boldsymbol{\mu}_i)$—and vice-versa—per Bell (1964: endnote 2; citing Einstein). Consistent with this principle, paired test outcomes are correlated via (6).

2.4. (4) expands on (3) to show that each polarizer-analyzer $\Delta_x^{\pm}$ is built from a polarizer $\delta_x^{\pm}$ and a removable analyzer $[x\cdot *]$ which responds to the polarization-vector $(*)$ of each post-polarizer particle $q(*)$ upon receipt. We assign the correct polarization $x^{\pm}$ to $q(*)$ by observing the ± 1 output of the related analyzer; or by understanding the nature of particle/polarizer interactions. So experiment β is EPRB—per Bell (1964)—with this benign finesse: to facilitate additional analysis and experimental confirmation, we can employ additional polarizers ($\delta_y^{\pm}$) to test $q(x^{\pm})$; and y may equal x.

nb: as in (3)-(4), via $q(\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm} = \delta_a^{\pm} q(\boldsymbol{\lambda}_i) \rightarrow q(a^{\pm})$, we take *interaction* and *transformation* to be concepts more fundamental than *measurement*, there being no requirement that $\boldsymbol{\lambda}_i = a^{\pm}$ prior to the interaction of $q(\boldsymbol{\lambda}_i)$ with $\delta_a^{\pm}$. "And does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts?" Bell (2004:118).

2.5. (5), with no symmetry requirements, shows the locales in (3)-(4) and (7) arbitrarily spacelike-separated from each other and from the source. Given that (3)-(7) hold over any spacelike separation, it follows that the relevant particle properties are stable between emission and interaction with a polarizer. Further, our theory is locally-causal and Lorentz-invariant because A_i and B_i are locally-caused by precedent local events $\delta_a^{\pm} q(\boldsymbol{\lambda}_i)$ and $\delta_b^{\pm} q(\boldsymbol{\mu}_i)$ respectively, which are spacelike-separated.

2.6. (6) shows $\boldsymbol{\lambda}_i$ and $\boldsymbol{\mu}_i$ pairwise correlated via the conservation of total angular momentum; our use of ordinary vectors being prompted by Dirac (1982: eqn (48), p.149) and geometric algebra as we seek a realistic replacement for Pauli's vector-of-matrices. (Note, per the Appendix: via Fröhner (1998), CLR rejects no tools of the quantum trade.) Motivated by Bell, these CLR-based variables provide a more complete specification of particle-pairs under β. Thus, for now, we allow these pristine spin-related variables to be ordinary vectors for which all magnitudes and orientations are equally probable. (New conventions begin when we integrate our approach with geometric algebra at ¶5.2-5.4.) Then, under our doctrine of cautious conservatism—and though particle responses to interactions may be similar—(6) allows it to be far less than certain that two pristine particle-pairs are physically the same.

2.7. (7) shows experiment β with Alice and Bob having the same polarizer setting a. (Per ¶2.2a, $\frac{\hbar}{2}$ is suppressed here.) Thus, as an *idealized* example—ie, by observing one result, we may predict the other (spacelike-separated) result *with certainty*—here's how Alice predicts Bob's result after observing $A_i = +1$; and vice-versa, with Bob observing $B_i = -1$ here:

$$A_i = +1 \because q(\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm} \rightarrow q(a^+) \Rightarrow [a \cdot *] \rightarrow [a \cdot a^+] = +1. \therefore - \text{ using (6) } - \tag{8}$$

$$q(\boldsymbol{\mu}_i) = q(-\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm} \rightarrow q(a^-) \Rightarrow [a \cdot *] \rightarrow [a \cdot a^-] = -1 = B_i. \; QED. \blacksquare \text{ And vice-versa.} \tag{9}$$

2.8. This is consistent with CLR's sufficient condition for a beable [¶1.5]. Without in any way disturbing $q(\boldsymbol{\mu}_i)$, Alice can predict with certainty that Bob's result will be $B_i = -1$ when he tests $q(\boldsymbol{\mu}_i)$ with $\delta_a^{\pm}$ (which may be a disturber): so beables $q(\boldsymbol{\mu}_i), \delta_a^{\pm}$ and $q(a^-)$ mediate Bob's result. Thus the beable corresponding to Bob's $B_i = -1$ result will be $q(a^-)$; a CLR outcome acceptable to EPR, but an important departure from Bell's position. For Bell endorses d'Espagnat's (1979:166) inference that the input to the polarizer equals the output from the polarizer—$q(\boldsymbol{\mu}_i) = q(a^-)$—but we do not.

[We thus come to the side-issue of Bell's likely *naive-realism* re *preexisting* beables and the possibility of clarifying ¶1.1. Now the use of *induction*—drawing legitimate conclusions from consistent observations—was foreshadowed at ¶1.2. But we will next see that d'Espagnat (and thus, seemingly, Bell) ignore a long history of consistent observations (ie, facts) that support the validity of QM and Bohr's insight. This would be OK if Bell and d'Espagnat were merely out to rebut EPR—and thus (maybe) endorse our amendment at ¶1.5—but here, as in science generally, facts and subtle distinctions matter more than differing theorems based on different assumptions. And one fact is this: d'Espagnat (1979:166) uses the phrase *'definite spin components at all times'* — which means *preexisting*.]

Here's Bell's (1980:7): "To explain this dénouement [of his theorem] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."

Here's d'Espagnat (1979:166), recast for EPRB (and our β) in our notation, *with added emphasis*: 'A physicist can infer that in every pair, one particle has the property a^+ [a positive spin-component along axis a] and the other has the property a^-. Similarly, he can conclude that in every pair one particle has the property b^+ and one b^-, and one has property c^+ and one c^-. *These conclusions require a subtle but important extension of the meaning assigned to our notation* a^+. Whereas previously a^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis a would give the definite result a^+, then that particle is said to have the property a^+. *In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times.* ... This view is contrary to the conventional interpretation of QM, but it is not contradicted by any fact that has yet been introduced.' [nb: *definite spin components at all times* = preexisting.]

2.9. However, to the contrary under CLR as we'll show: (i) d'Espagnat's inferences are false; (ii) weaker, more-general, inferences are available; (iii) there's no need to contravene known facts re QM; (iv) and no need to negate Bohr's insight: which—supported by Bell hereunder—bolsters our case against d'Espagnat's 'Bell-endorsed' inferences. See also Kochen (2015:5): in QM, physicists 'do not believe that the value of the spin component [S_z] exists' prior to the Stern-Gerlach interaction.

Here's Bell (2004: xi-xii): It's "Bohr's insight that the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems to [Bell] that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [like Bell's *impossibility* theorem, as we'll see], and most of 'quantum logic'." We agree, for in this way CLR negates the classical/quantum divide mentioned in ¶1.1(ii): under CLR's physical-realism [¶1.3]—*some physical properties change interactively*—we do not assume that all 'measured' properties already exist prior to 'measurement' interactions. Thus, via Kochen (2015:5), we come to a crucial difference between CLR and classical physics: "In classical physics, we assume that the measured properties of the system already exist prior to the measurement. ... The basic assumption is that systems have intrinsic properties and the experiment measures the value of them."

2.10. Thus, to be clear and consistent with Bohr's insight, CLR goes beyond the Bell-d'Espagnat inferences wherein the 'measured' property is *equated* to a pristine property. That is—going beyond d'Espagnat's *subtle extension* cited in ¶2.8—we instead infer to *equivalence under a 'polarizing' operator*. For *equivalence*—a weaker, more general relation than *equality*—is here compatible with QM, Bohr's view, and the consequent need to recognize the effect of 'the means of observation' under EPRB:

"... the unavoidable interaction between the objects and the measuring instruments sets an absolute limit to the possibility of speaking of a behaviour of atomic objects which is independent of the means of observation," Bohr (1958:25); see ¶1.2.

2.11. So now, under CLR—via the known effect of linear-polarizer $\delta_x^{\pm}$ on polarized particles $q(x^+)$—we can match interactions like $\delta_x^{\pm} q(\boldsymbol{\lambda}_i) \to q(x^+)$ with ancillary interactions like $\delta_x^{\pm} q(x^+) \to q(x^+)$. Then, since $\delta_x^{\pm}$ is a dichotomic operator that dyadically partitions its binary domain, we let $\sim$ here denote the equivalence relation *has the same output under the same operator.* Thus, in the context of EPRB:

$$\text{If } \delta_a^{\pm} q(\boldsymbol{\lambda}_i) \to q(a^+) \text{ then } q(\boldsymbol{\lambda}_i) \sim q(a^+) \because \delta_a^{\pm} q(a^+) \to q(a^+) \text{ exclusively; and } q(-\boldsymbol{\lambda}_i) = q(\boldsymbol{\mu}_i) \sim q(a^-). \quad (10)$$

$$\text{If } \delta_b^{\pm} q(\boldsymbol{\mu}_i) \to q(b^+) \text{ then } q(\boldsymbol{\mu}_i) \sim q(b^+) \because \delta_b^{\pm} q(b^+) \to q(b^+) \text{ exclusively; and } q(-\boldsymbol{\mu}_i) = q(\boldsymbol{\lambda}_i) \sim q(b^-). \quad (11)$$

2.12. That is, from (10)—consistent with Alice's frame of reference wherein Alice observes $A_i = +1$, per $q(a^+)$—we confirm $\sim$ under $\delta_a^{\pm}$ as follows: (i) polarizing-operators $\delta_a^{\pm}$ deliver $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ to the same output; (ii) it is impossible (speaking idealistically, of course) that an interaction with a $\delta_a^{\pm}$

might to deliver $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ to two different outputs; (iii) an equivalence relation $\sim$ therefore holds between $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ under $\delta_a^\pm$. (11) similarly, via Bob's frame of reference, wherein Bob observes $B_i = +1$: the equivalence relation $\sim$ now holding between $q(\boldsymbol{\mu}_i)$ and $q(b^+)$ under $\delta_b^\pm$. [NB: further, at (20)-(21) and ¶¶2.17-2.19 below, we find that particles equivalent under $\delta_a^\pm$ are also equivalent under $\delta_b^\pm$ in probability functions; an important CLR result because it licenses Malus' Law under CLR.]

2.13. Re our equivalence relations $\sim$ (using $\overset{\delta_x^\pm}{\sim}$ when clarity requires):

$$Q \equiv \{q(\boldsymbol{\lambda}_i), q(\boldsymbol{\mu}_i); q(a^\pm) \,|\, \beta, \boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0, i = 1, 2, ..., n; \delta_a^\pm\}; \tag{12}$$

$$[q(a^+)] \equiv \{q(\cdot) \in Q \mid q(\cdot) \overset{\delta_a^\pm}{\sim} q(a^+)\};\ [q(a^-)] \equiv \{q(\cdot) \in Q \mid q(\cdot) \overset{\delta_a^\pm}{\sim} q(a^-)\}; \tag{13}$$

$$Q/\!\sim\, = \{[q(a^+)],\ [q(a^-)]\}\ ; \tag{14}$$

where Q is the set of output particles $q(a^\pm)$ under $\delta_a^\pm$ and input particles $q(\cdot)$. In (13), equivalence classes $[q(a^+)]$ and $[q(a^-)]$ show Q partitioned dyadically under the mapping $\delta_a^\pm q(\cdot) \to q(a^\pm)$. So, on the elements of $\delta_a^\pm$'s domain, $\sim$ denotes: *has the same output/image under* $\delta_a^\pm$. Thus the quotient set $Q/\!\sim$ in (14)—the set of all equivalence classes under $\sim$—is a set of two diametrically-opposed extremes: a maximal antipodean discrimination; a powerful *deterministic* push-pull dynamic; see ¶¶2.16, 5.3.

2.14. We now combine (3), (4), (10), (11) into a single test on the ith particle-pair from two perspectives: (15), which Alice reads from left-to-right; (16), which Bob reads from right-to-left:

$$A_i \equiv +1 \cdots q(a^+) \leftarrow \Delta_a^\pm \Leftarrow q(\boldsymbol{\lambda}_i)\ \ \triangleleft\beta\triangleright\ \ q(\boldsymbol{\mu}_i) = q(-\boldsymbol{\lambda}_i) \sim q(a^-) \Rightarrow \delta_b^\pm \Rightarrow q(b^+) \to [b\cdot *] \to b\cdot b^+ = +1 \equiv B_i\ ; \tag{15}$$

$$A_i \equiv +1 = a\cdot a^+ \leftarrow [a\cdot *] \Leftarrow q(a^+) \leftarrow \delta_a^\pm \Leftarrow q(b^-) \sim q(-\boldsymbol{\mu}_i) = q(\boldsymbol{\lambda}_i)\ \ \triangleleft\beta\triangleright\ \ q(\boldsymbol{\mu}_i) \Rightarrow \Delta_b^\pm \to q(b^+) \cdots +1 \equiv B_i : \tag{16}$$

> Thus, in line with Bell's (1964:196) specification for his $\boldsymbol{\lambda}$: (i) seeking a physical theory of the type envisioned by Einstein/EPR, our variables have dynamical significance and laws of motion; (ii) our pristine $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$—correlated under (6)—are the initial values of such variables at some suitable instant; (iii) since different tests produce different disturbances, different relational properties may be pairwise revealed under $\sim$ without contradiction: ie, finding $q(\boldsymbol{\lambda}_i \sim a^+)$ experimentally, we learn $q(\boldsymbol{\mu}_i \sim a^-)$ relationally via (6); etc. *QED.* ■

2.15. So, from (6) and (10)-(16), with $A^\pm$ ($B^\pm$) denoting Alice's (Bob's) results (± 1), we can now provide (under β per Bell 1964): (i) the relationships missing from Bell 1964:(1); (ii) relevant *correlated* EPRB probabilities and expectations; (iii) CLR's rejection of the generality of Bell's theorem; (iv) the whole followed by explanatory comments:

$$\Delta_a^\pm q(\boldsymbol{\lambda}) \to A(a, \boldsymbol{\lambda}) = \cos(a, \boldsymbol{\lambda} \,|\, q(\boldsymbol{\lambda}) \sim q(a^\pm)) = \pm 1 \equiv A^\pm; \langle A \,|\, \beta\rangle = 0 \because P(A^+ \,|\, \beta) - P(A^- \,|\, \beta) = 0. \tag{17}$$

$$\Delta_b^\pm q(\boldsymbol{\mu}) \to B(b, \boldsymbol{\mu}) = \cos(b, \boldsymbol{\mu} \,|\, q(\boldsymbol{\mu}) \sim q(b^\pm)) = \pm 1 \equiv B^\pm; \langle B \,|\, \beta\rangle = 0 \because P(B^+ \,|\, \beta) - P(B^- \,|\, \beta) = 0. \tag{18}$$

$$P(A^+ \,|\, \beta) = P(A^- \,|\, \beta) = P(B^+ \,|\, \beta) = P(B^- \,|\, \beta) = \tfrac{1}{2} \because \boldsymbol{\lambda} \text{ and } \boldsymbol{\mu} \text{ are random variables here:} \tag{19}$$

$$P(A^+ \,|\, \beta B^+) = P(q(\boldsymbol{\lambda} \sim a^+) \,|\, \beta, q(\boldsymbol{\mu} \sim b^+)) = P(\delta_a^\pm q(b^-) \to q(a^+) \,|\, \beta) = \cos^2 s\,(a^+, b^-) = \sin^2 \tfrac{1}{2}(a, b). \tag{20}$$

$$P(B^+ \,|\, \beta A^+) = P(q(\boldsymbol{\mu} \sim b^+) \,|\, \beta, q(\boldsymbol{\lambda} \sim a^+)) = P(\delta_b^\pm q(a^-) \to q(b^+) \,|\, \beta) = \cos^2 s\,(a^-, b^+) = \sin^2 \tfrac{1}{2}(a, b). \tag{21}$$

$$\therefore P(A^+ B^+ \,|\, \beta) = P(A^+ \,|\, \beta) P(B^+ \,|\, \beta A^+) = P(B^+ \,|\, \beta) P(A^+ \,|\, \beta B^+) = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(a, b).\ QED.\ \blacksquare \tag{22}$$

$$\therefore \langle A^+ B^+ \,|\, \beta\rangle = \langle A^- B^- \,|\, \beta\rangle = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(a, b);\ \langle A^+ B^- \,|\, \beta\rangle = \langle A^- B^+ \,|\, \beta\rangle = -\tfrac{1}{2} \cos^2 \tfrac{1}{2}(a, b). \tag{23}$$

$$\therefore \langle AB \,|\, \beta\rangle \equiv \langle A^+ B^+ \,|\, \beta\rangle + \langle A^+ B^- \,|\, \beta\rangle + \langle A^- B^+ \,|\, \beta\rangle + \langle A^- B^- \,|\, \beta\rangle = -a\cdot b.\ QED.\ \blacksquare \tag{24}$$

2.16. That is. Given (15), the cosine function in (17) reads: with $q(\boldsymbol{\lambda})$ equivalent to $q(a^+)$ under $\sim$, $\cos(a, \boldsymbol{\lambda} \,|\, q(\boldsymbol{\lambda}) \sim q(a^+))$ denotes the cosine of the angle (a, a^+): ie—under the deterministic push-pull dynamic identified in ¶2.13; with $\boldsymbol{\lambda} \sim a^+$—the outcome is $+1 = A^+$ here. (18) similarly, given (16); etc. Thus, under $\sim$, CLR could embrace Bell-d'Espagnat inferences [¶2.8] to equality, but: (i) the probability that such inferences are valid is negligible; (ii) their theory does not embrace ours; (iii) under conservative CLR, we allow $P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_j \,|\, \beta, i \neq j) << 1$, per (6); etc.

2.17. Next, re (20); and (21) similarly. Via standard probability theory and Bayes' Law [¶1.3]: (i) the correlation of $A^{\pm}$ and $B^{\pm}$ via (6) induces the probability relation at LHS (20); (ii) A^+ and B^+ *are* correlated via (6); (iii) such correlation is recognized by Bell (in our favor) under EPRB (as follows):

> Recasting Bell (2004:208) in line with EPRB: "There are no 'messages' in one system from the other. The inexplicable [sic] correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system [eg, from Alice's A^+] to events in the other [eg, to Bob's B^+] are possible."

2.18. Further. In (20) under $\sim$, the LHS probability relation is—from the middle term in (20)—equivalent to a classical test on spin-$\frac{1}{2}$ particles of known polarization. So we derive RHS (20) by extending Malus' *classical* $\cos^2 s\,(a^+, b^-)$ Law (c.1808)—re the relative intensity of beams of polarized photons ($s = 1$)—to spin-$\frac{1}{2}$ particles ($s = \frac{1}{2}$). (21) similarly. Then, since our equivalence relations hold under probability functions P, P is well-defined under $\sim$ and is that same law—Malus' Law—now quantum-compatible by extension; doubly testable and validated, as formatted in (20)-(21); eg:

> re Aspect's (2004:5-7) 'concerned' discussion of Malus' Law, our trigonometric arguments represent clear law-based dynamical processes under (10)-(11) and ¶¶2.3, 2.13: eg, $(q(\boldsymbol{\lambda}_i) \sim q(a^+)) \equiv (\delta_a^{\pm} q(\boldsymbol{\lambda}_i) \to q(a^+))$; the a in $\delta_a^{\pm}$ denoting the orientation of a non-uniform field with which $q(\boldsymbol{\lambda}_i)$ interacts. A wire-grid microwave-polarizer provides a macroscopic analogy. With its conducting wires represented by a direction-vector in 3-space, an impinging unpolarized beam of microwaves drives electrons within the wires, thereby generating an alternating current (Hecht 1975:104). So the wires become polarizing-operators (in our terms), the transmitted beam being strongly linearly polarized orthogonal to the wires. (Polaroid®-sheet is a molecular equivalent for photons.) This suggests (see ¶5.3), that the micro-dynamics of particle/polarizer interactions may be represented by a suitable vector-product with two boundary-conditions: (i) the remnant angular momentum finally aligned ($\pm$) with the field is *always* the spin $s\hbar$; (ii) *each pairwise correlation* arises from the pairwise-dynamics associated with the conservation of total angular momentum in (6).

2.19. Thus, from (20), $P(A^+ \mid \beta B^+)$ under $\sim$ is Malus' Law generalized to entangled particles: so we can let Malus' *quantum* Law be the new QM-compatible law that links the first and last terms in (20) directly. And (21) similarly. Then, seeking one unified law under CLR, we might say that Malus' Law applies classically to the relational properties of the classical beams that Malus employed: and quantum-mechanically to the relational properties of particles. [To maintain this law-based unity in physics—bypassing much debate in philosophy; and backing a reconstruction of QM per Kochen (2015)—we would need to define absolute (intrinsic) and relational (extrinsic) properties accordingly.] However, using (6) and (10) with the generalization $\boldsymbol{\lambda} + \boldsymbol{\mu} = 0$, the expanded version of (20) is:

$$P(A^+ \mid \beta B^+) \equiv P(\delta_a^{\pm} q(\boldsymbol{\lambda}) \to q(a^+) \mid \beta, \delta_b^{\pm} q(\boldsymbol{\mu}) \to q(b^+)) = P(\delta_a^{\pm} q(\boldsymbol{\lambda}) \to q(a^+) \mid \beta, q(\boldsymbol{\mu}) \sim q(b^+)) \quad (25)$$

$$= P(\delta_a^{\pm} q(\boldsymbol{\lambda}) \to q(a^+) \mid \beta, q(-\boldsymbol{\lambda}) \sim q(b^+)) = P(\delta_a^{\pm} q(b^-) \to q(a^+) \mid \beta) = \cos^2 s\,(a^+, b^-) = \sin^2 \tfrac{1}{2}(a, b). \quad (26)$$

2.19a. Given (25)-(26), we're in the following good company: "Nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary," Bell (2004:29-30). QM 'can be understood as a powerful extension of ordinary probability theory,' Fröhner (1998:652). 'The major transformation from classical to quantum physics,' in Kochen's (2015:26) approach, 'lies not in modifying the basic classical concepts such as state, observable, symmetry, dynamics, combining systems, or the notion of probability, but rather in the shift from intrinsic to extrinsic properties.' Via our CLR approach to QM, we take these ideas further in the Appendix.

2.20. However—in passing—allowing that every extrinsic property in QM can be converted to a relational property under an equivalence relation: our unified Malus' Law (henceforth, Malus' Law without qualification) would apply to relational properties generally. To put it another way, in Malus' 19th-century context—and in our soon-to-be CLR (post-EPR) world—consider two photons (with $s = 1$, as we now know): (i) $q(\boldsymbol{\lambda}_j \overset{\delta_x^\pm}{\sim} x^+) \equiv (\delta_x^\pm q(\boldsymbol{\lambda}_j) \to q(x^+))$ in our terms; (ii) $q(\boldsymbol{\lambda}_k = x^+)$ is our notation for an x^+-polarized photon in an x^+-polarized classical beam that Malus worked with. We then say: for each $q(x^+)$ here, (x^+) is a relational property under $\sim$. For, as relational properties—with P well-defined under $\sim$ from ¶2.17—they yield identical (and valid) results (with $s = 1$ here):

$$P(\delta_a^\pm q(\boldsymbol{\lambda}_j \overset{\delta_x^\pm}{\sim} x^+) \to q(a^+)) \;\sim\; P(\delta_a^\pm q(\boldsymbol{\lambda}_k = x^+) \to q(a^+)) = \cos^2 s(a^+, x^+) = \cos^2(a, x). \tag{27}$$

> "It is not easy [maybe] to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision [as is our aim here] might be possible by concentration on the *be*ables, which can be described 'in classical terms', because they are there [like our $q(\boldsymbol{\lambda}_j)$, with $q(\boldsymbol{\lambda}_j \sim x^+)$ under $\delta_x^\pm$; and $q(\boldsymbol{\lambda}_k = x^+)$]. ... 'Observables' [like A_j and A_k in our notation] must be *made*, somehow, out of beables [as ours are; eg, in (27)]. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables [as CLR does]," Bell (2004:52).

2.21. Returning to the logic of (17)-(24): (22) follows from (19)-(21) via Bayes' Law; which is applicable here—and thus applicable to EPR studies generally—since $A^\pm$ and $B^\pm$ are correlated via (6). The expectations in (23) follow from (22) via the definition of an expectation. Then, with (24) from (23) via the definition of the overall expectation, we have the expectation $\langle AB\,|\,\beta\rangle$. *Thus*—with Bell claiming in the line below his 1964:(3) that (24) is impossible—*the generality of Bell's theorem is constrained by the limited generality of his inferences*. With ▲ denoting absurdity, the source of Bell's 'impossibility theorem'—ie, the mathematical consequence of Bell's false inference [¶¶2.8-2.9]—follows:

2.22. Under '**Contradiction**: The main result will now be proved', Bell (1964:197) takes us via his 1964:(14), direction-vector c, and three unnumbered equations—say, (14a)-(14c)—to his 1964:(15); ie:

$$|\,\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle\,| \leq 1 + \langle BC\,|\,\beta\rangle\,;\text{ ie, using our (24): } |\,(a\cdot c) - (a\cdot b)\,| \leq 1 - (b\cdot c);\ \blacktriangle \tag{28}$$

$$\text{ie, Bell 1964:(15) is absurd under CLR, mathematics and QM } \because |\,(a\cdot c) - (a\cdot b)\,| \leq \tfrac{3}{2} - (b\cdot c). \tag{29}$$

2.23. To pinpoint the source of this absurdity (and avoid any defective intermediaries), we now link LHS Bell 1964:(14a) directly to LHS Bell 1964:(15). Using illustrative angles, Bell's 1964:(15) allows:

$$0 \leq \langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle \leq 1 + \langle BC\,|\,\beta\rangle\,; \tag{30}$$

$$\text{ie, using our (24), } 0 \leq (a\cdot c) - (a\cdot b) \leq 1 - (b\cdot c); \tag{31}$$

$$\text{so, if } (a, b) = \tfrac{\pi}{4} \text{ and } (a, c) = (c, b) = \tfrac{\pi}{8}, \text{ then } 0 \leq 0.217 \leq 0.076 \text{ (conservatively);}\ \blacktriangle \tag{32}$$

$$\text{ie, Bell 1964:(14a)} \neq \text{Bell 1964:(14b)} = \text{Bell 1964:(14c)} = \text{Bell 1964:(15)}.\ QED.\ \blacksquare \tag{33}$$

2.24. Thus, under EPRB and CLR: Bell's theorem (and related inequalities) stem from the $\neq$ in (33); ie, they begin with Bell's move from his *valid* (14a) to his *invalid* (14b). Now via Bell's note at 1964:(14b), we find that Bell moves from (14a) to (14b) via the generalization $(A(b, \boldsymbol{\lambda}))^2 = 1$. But if $i \neq j$, $A(b, \boldsymbol{\lambda}_i)A(b, \boldsymbol{\lambda}_j) = \pm 1$; ie, the product of uncorrelated scalars—each of which may take the value ± 1—is ± 1. So, as we'll show, Bell's generalization is invalid under EPRB, with the following consequences: (i) absurdities—like (28) and (32)—flow from Bell's limiting generalization under EPRB; (ii) Bell's theorem is limited to systems for which this limited generalization holds; (iii) EPRB-based settings are not such systems; (iv) Bell's generalization has nothing to do with local causality; (v) based on such a constrained 'realism'—naive realism—Bell's ambit claims are misleading. Let's see:

2.25. Using our (3)-(7) and a particle-by-particle analysis of β, let $3n$ random particle-pairs be equally distributed over three randomized polarizer-pairings $(a, b), (b, c), (c, a)$. Allowing each particle-pair to be unique—and here, uniquely indexed—let n be such that (for convenience in presentation and to an adequate accuracy hereafter):

$$\text{Bell 1964:(14a)} = \langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i) - A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i})] \tag{34}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}) - 1] \tag{35}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)[A(b,\boldsymbol{\lambda}_i)A(c,\boldsymbol{\lambda}_i) - 1]\ \textbf{(after using } \boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i}\ \textbf{[sic])} = \text{Bell 1964:(14b)}: \blacktriangle \tag{36}$$

for, under EPRB, and CLR per (6): $P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i}\,|\,\beta) << 1$. So (36) joins (24) & (28) under ▲. (37)

2.26. Thus, under his generalization $(A(b, \boldsymbol{\lambda}))^2 = 1$ at ¶2.24, Bell has a quantum-incompatible generalization akin to an ordered sample of n objects subject to repetitive non-destructive testing, with $\boldsymbol{\lambda}_i \equiv \boldsymbol{\lambda}_{n+i}$ per (36). Allowing that quantum-compatibility will eliminate such absurdities, we derive the valid consequences: since the average of $|\,A(a, \boldsymbol{\lambda}_i)A(b, \boldsymbol{\lambda}_i)\,|$ is ≤ 1, *valid* (35) reduces to *valid* (38).

$$\text{Bell 1964:(14a)} = |\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle| \leq 1 - \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}). \tag{38}$$

2.27. Now: (i) from (6), the independent and uncorrelated random variables $\boldsymbol{\lambda}_i$ and $\boldsymbol{\lambda}_{n+i}$ generate independent and uncorrelated random variables (ie, the binary outputs ± 1); (ii) the expectation over the product of two independent and uncorrelated random variables is the product of their individual expectations; (iii) so (38) reduces to:

$$\text{Bell 1964:(14a)} = |\,\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle\,| \leq\ 1 - \langle AB\,|\,\beta\rangle\langle AC\,|\,\beta\rangle \neq \text{Bell 1964:(14b)}; \tag{39}$$

ie, $|\,(a \cdot b) - (a \cdot c)\,| \leq 1 - (a \cdot b)(a \cdot c) \neq$ RHS Bell 1964:(15) unless $a = b \vee c$, which is absurd. ▲ (40)

2.28. In short: Since LHS (40) is a fact, Bell's 1964:(15) under EPRB is absurd and false. In passing, the CHSH (1969) inequality—eg, Peres (1995:164)—falls to similar analysis under straight mathematics:

$$|\,(a{\cdot}b) + (b{\cdot}c) + (c{\cdot}d) - (d{\cdot}a)\,| \leq 2\sqrt{2}.\ \therefore\ |\,(a{\cdot}b) + (b{\cdot}c) + (c{\cdot}d) - (d{\cdot}a)\,| \leq 2 \text{ is absurd.} \blacktriangle \tag{41}$$

2.29. Finally, to complete our analysis, we consider experiment γ, Mermin's (1990) 3-particle variant of GHZ (1989); widely-regarded as the best variant of Bell's theorem. *Respectively, hereafter*: three spin-$\frac{1}{2}$ particles with spin-related properties $\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}$ emerge from a spin-conserving decay such that:

$$\boldsymbol{\lambda} + \boldsymbol{\mu} + \boldsymbol{\nu} = \pi.\ \therefore \boldsymbol{\nu} = \pi - \boldsymbol{\lambda} - \boldsymbol{\mu} \text{ (for convenience; the choice matters not).} \tag{42}$$

2.30. The particles separate in the y-z plane and interact with spin-$\frac{1}{2}$ polarizers that are orthogonal to the related line of flight. Let a, b, c here [nb: elsewhere, they are direction-vectors] be the angle of each polarizer's principal-axis relative to the positive x-axis; and let the equivalence relations for $\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}$ be expressed in similar terms. Finally, let the test results be A, B, C. Then, based on LHS (17)-(18) in short-form—ie, $A^+ = \cos(a, \boldsymbol{\lambda}\,|\,q(\boldsymbol{\lambda}) \sim q(a^+)) = \cos(a - \boldsymbol{\lambda}\,|\,\boldsymbol{\lambda} \sim a^+) = 1$; etc—let

$$A^+ = \cos(a - \boldsymbol{\lambda}\,|\,\boldsymbol{\lambda} \sim a) = 1.\ B^+ = \cos(b - \boldsymbol{\mu}\,|\,\boldsymbol{\mu} \sim b) = 1.\ C^+ = \cos(c - \boldsymbol{\nu}\,|\,\boldsymbol{\nu} \sim c) = 1. \tag{43}$$

2.31. Via the principles in (3)-(24)—and nothing more—we now derive $\langle ABC\,|\,\gamma\rangle$, the expectation for the Mermin/GHZ experiment γ. (Explanatory notes follow.)

$$\langle A^+B^+C^+ \,|\, \gamma\rangle \equiv$$

$$P(\boldsymbol{\lambda}\sim a\,|\,\gamma)\cos(a-\boldsymbol{\lambda}\,|\,\boldsymbol{\lambda}\sim a)\cdot P(\boldsymbol{\mu}\sim b\,|\,\gamma)\cos(b-\boldsymbol{\mu}\,|\,\boldsymbol{\mu}\sim b)\cdot P(\boldsymbol{\nu}\sim c\,|\,\gamma,\boldsymbol{\lambda}\sim a,\boldsymbol{\mu}\sim b)\cos(c-\boldsymbol{\nu}\,|\,\boldsymbol{\nu}\sim c) \quad (44)$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{2}\cdot P(\boldsymbol{\nu}\sim c\,|\,\gamma,\boldsymbol{\lambda}\sim a,\boldsymbol{\mu}\sim b) = \tfrac{1}{4}P((\pi-\boldsymbol{\lambda}-\boldsymbol{\mu})\sim c\,|\,\gamma,\boldsymbol{\lambda}\sim a,\boldsymbol{\mu}\sim b) \quad (45)$$

$$= \tfrac{1}{4}P((\pi-a-b)\sim c\,|\,\gamma) = \tfrac{1}{4}\cos^2\tfrac{1}{2}(\pi-a-b-c) = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a+b+c). \quad (46)$$

$$\text{Similarly: } \langle A^+B^-C^-\,|\,\gamma\rangle = \langle A^-B^+C^-\,|\,\gamma\rangle = \langle A^-B^-C^+\,|\,\gamma\rangle = \tfrac{1}{4}\sin^2\tfrac{1}{2}(a+b+c), \text{ and} \quad (47)$$

$$\langle A^+B^+C^-\,|\,\gamma\rangle = \langle A^+B^-C^+\,|\,\gamma\rangle = \langle A^-B^+C^+\,|\,\gamma\rangle = \langle A^-B^-C^-\,|\,\gamma\rangle = -\tfrac{1}{4}\cos^2\tfrac{1}{2}(a+b+c). \quad (48)$$

$$\therefore \langle ABC\,|\,\gamma\rangle \equiv \Sigma\,\langle A^\pm B^\pm C^\pm\,|\,\gamma\rangle = \sin^2\tfrac{1}{2}(a+b+c) - \cos^2\tfrac{1}{2}(a+b+c) = -\cos(a+b+c).\ QED.\blacksquare \quad (49)$$

2.32. (44) defines the required expectation. (45) follows (44) by reduction using (17)-(19). (46) follows from (45) by allocating the equivalence relations in the conditioning space to the related variables. Thus, in words, LHS (46) is one-quarter the probability that $\boldsymbol{\nu}$ — ie, $\boldsymbol{\nu} \sim (\pi - a^+ - b^+)$ — will be equivalent to c^+ under $\delta_c^\pm$. In other words: LHS (46) $= \frac{1}{4}P(\delta_c^\pm q(\boldsymbol{\nu} \sim \pi - a^+ - b^+) \to q(c^+)\,|\,\gamma) =$ RHS (46) via Malus' Law. So (46) is the three-particle variant of (23) in the two-particle EPRB experiment sketched in (3)-(6). (47)-(49) then follow naturally.

2.33. Thus, delivering Mermin's (1990:11) *crucial minus sign*, (49) is the correct result for γ; ie, when $(a+b+c) = 0$, $\langle ABC\,|\,\gamma\rangle = -1$. So, consistent with QM—using CLR and its rules for physical operators and EPRB-based interactions in 3-space—we again deliver classically-intelligible EPR correlations. (This ends our use of a, b, c as the angle of a polarizer's principal-axis relative to the positive x-axis.)

2.34. Via CLR's valid results for EPRB at (24), CHSH at (41), Mermin/GHZ at (49), Aspect (2004) at (68)—and such results so clearly in conflict with Bellian conclusions—we rest our case: CLR is a valid general theory. So, with CLR's credentials established after ¶1.9, we move to Conclusions: which includes a correct factoring of (1)—Bell's locality hypothesis—at ¶3.5.

3 Conclusions

3.0. We conclude: CLR resolves Bell's dilemma re AAD and fulfills this hope: 'Let us hope that these analyses [local-causality impossibility proofs] also may one day be illuminated, perhaps harshly, by a simple constructive model. However long that may be, long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination,' Bell (2004:167). For Bellian difficulties arise from incorrectly imagining the nature of micro-reality; not from locality: our p.1, NB:(vi), notwithstanding!

3.1. To be clear: accepting the Bell-endorsed principles at ¶1.2—then deriving the correct results for EPRB at (24), CHSH at (41), Mermin (1990) at (49), Aspect (2004) at (68)—CLR resolves Bell's AAD/locality dilemma in line with his hope for a simple constructive model of EPRB. And though we amend EPR's realism at ¶1.5, CLR justifies EPR's belief that additional variables would bring locality and causality to QM. We accordingly conclude that CLR validly rejects all claims like those at our p.1, NB:(vi)—which are mostly based on naive-realism—for our world is Einstein-local.

3.2. Further, under realism: against false Bell/d'Espagnat inferences to equality—¶¶2.8-2.10—our weaker more-general equivalence relations ($\sim$) in (10)-(11) correctly relate quantum things $q(\boldsymbol{\lambda})$ to more familiar things $q(a^\pm)$; etc. So Bellian absurdities arise under equality relations—most typically associated with naive realism—while (as in CLR), science is hardly possible without equivalence relations under operators. Nevertheless, in and from Bellian studies—and honoring Bohr—we conclude that Bohr's insight into realism should henceforth rank equally with Einstein's insight into locality.

3.3. Thus, while most Bellians (typically wedded to what we term naive realism) agree with Bell's Einstein-based locality hypothesis (1)—but do not progress it—we use Bohr's insight: (i) to amend EPR's sufficient condition for a beable; (ii) to correct Bell/d'Espagnat inferences; (ii) to breach the classical/quantum divide; (iv) to be a differentiating factor in our approach to realism via *clear realism*. We conclude that Bohr's insight reveals itself to be a surprising Bellian oversight wrt realism.

3.4. This leads to Bell's (1980:13-15) claim—under *local explicability*; his (11), our (1)—that the macro-factoring [our term, since Bell's $\boldsymbol{\lambda}$ is an undifferentiated set of unknown (and possibly unknowable) hidden-variables] of joint EPRB probabilities should accord with logical independence. In that case, given the outputs A and B in (1), we should find $\langle AB\,|\,\beta\rangle = \langle A\,|\,\beta\rangle\langle B\,|\,\beta\rangle = 0$. However,

$$\text{from (17)-(18)},\ \langle A\,|\,\beta\rangle = \langle B\,|\,\beta\rangle = 0; \text{but from (24)}, \langle AB\,|\,\beta\rangle \neq 0: \tag{50}$$

so, with A and B independent *but* correlated, we conclude that this difference makes all the difference.

3.5. For—(i) replacing Bell's (2004:240) "*full specification* of local [EPRB] beables in a given space-time region" (our emphasis) with CLR's *adequate specification* of local beables; (ii) given Bell (2014:243) referring to logically independent correlations which permit symmetric factorizations as *locally explicable*; (iii) taking such factorizations to be a consequence of local causality and not a *formulation* thereof; (iv) and using (6) and (19)—we conclude that CLR's adequacy goes beyond Bell to deliver:

$$P(A^+B^+\,|\,\beta, a, q(\boldsymbol{\lambda}\sim a^+), b, q(\boldsymbol{\mu}\sim b^+)) = P(A^+\,|\,\beta, a, q(\boldsymbol{\lambda}\sim a^+))P(B^+\,|\,\beta, b, q(\boldsymbol{\mu}\sim b^+)) = 1. \tag{51}$$

3.6. We therefore conclude that Bell's focus on (an improbable) *full* specification—in typical HVT fashion—prevents him from deriving the result that follows via CLR's (perhaps less improbable) *adequate* specification. For, more prudently—with $\oplus$ denoting xor; using (19)-(22) at the end—CLR allows us to factor Bell's locality hypothesis (1) like this:

$$P(A^+B^+\,|\,\beta, a, q(\boldsymbol{\lambda}), b, q(\boldsymbol{\mu})) = P(A^+\,|\,\beta, a, q(\boldsymbol{\lambda}))P(B^+\,|\,\beta, b, q(\boldsymbol{\mu})) \tag{52}$$

$$= P(q(\boldsymbol{\lambda}) \overset{\delta_a^\pm}{\sim} q(a^+)\,|\,\beta)P(q(-\boldsymbol{\lambda}) \overset{\delta_b^\pm}{\sim} q(b^+)\,|\,\beta) = \tfrac{1}{2}P(q(a^-) \overset{\delta_b^\pm}{\sim} q(b^+)\,|\,\beta) \oplus \tfrac{1}{2}P(q(b^-) \overset{\delta_a^\pm}{\sim} q(a^+)\,|\,\beta) \tag{53}$$

$$= \tfrac{1}{2}\sin^2\tfrac{1}{2}(a,b) = P(A^+|\beta)P(B^+|\beta A^+) \oplus P(B^+|\beta)P(A^+|\beta B^+) = P(A^+B^+\,|\,\beta).\ QED. \blacksquare \tag{54}$$

3.7. (51)-(54) shows that logical independence at the micro-level—in (51), with 1x1 = 1; or in (52)-(53)—may lead to Bayes' Law at the macro-level, per (54); and vice-versa. Moreover, against Aspect (2004:9 and that *hopeless* search) and Bell generally, our alternative (CLR) factorizations under Bayes' Law are licensed by the experimentally-verified generality of Malus' Law: note the link between (53) and (54) under our equivalence relations. (Moreover, contra Bell and his dilemma at ¶1.6(i), CLR can of course explain things by events in their neighbourhood.) In passing, the symmetry associated with $\oplus$ in (53)-(54) shows that Alice's factoring is—of course—the same as Bob's. Importantly, wrt Bayes' Law at ¶1.3: valid *equivalence relations* allow us the encode *better information* about *random beables* and their *hidden dynamics* in our probability relations; thus (52) leads to (54), and vice-versa.

3.8. Many agree with du Sautoy (2016:170), "Bell's theorem is as mathematically robust as they come." However, Bell's use of $[A(b,\boldsymbol{\lambda})]^2 = 1$ (see ¶2.24), renders Bell's theorem unphysical under EPRB, mathematically false at (24), absurd at (28) and (32). For, per ¶2.25, Bell's use of $[A(b,\boldsymbol{\lambda})]^2 = 1$ is invalid under EPRB due to multiple pairing/matching problems; ie, under $i \neq j$, the product of uncorrelated scalars is: $A(b,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_j) = \pm 1$. We conclude: Bayes' Law for the probability of joint outcomes is never false here (neither mathematically nor experimentally), confirming Bell's (2004:239) *utmost suspicion*: he did throw the baby—baby Bayes—out with the bathwater. For, since A and B are independent but correlated per ¶3.2, Bayes' Law is central to our understanding of macro-EPRB.

3.9. Re ¶¶2.19-20, we conclude that opportunities for a wholesale reinterpretation of QM remain: 'collapse' as the Bayesian updating of an equivalence class via prior correlations; 'states' as states of information about multivectors; 'measurements' as the outcomes of interactions involving physical operators; 'wave-particle duality' as an equivalence relation; more physically-significant CLR-style approaches, like that at (56) re Pauli's vector-of-matrices. For: (i) our Lorentz-invariant analysis resolves Bell's AAD/locality dilemma; (ii) we've dispensed with AAD; (iii) we've validated Einstein's program; (iv) we do get away with locality; (v) we've thus justified Bell's motivation and validated our common enterprise, based on ¶¶1.0, 1.6(ii).

3.10. Finally, re our opening position at ¶1.2—*taking QM to be better-founded than Bell imagined; based on doubts, puzzlements and mysteries that we do not share*—we have justified our concern (under CLR) re each sentence in Bertlmann's (2017:54) remarks there.

3.11. In sum; consistent with Einstein's locally-causal Lorentz-invariant worldview: (i) Bell's theorem is bypassed; (ii) its naive realism—via (36)—leads to its consequent lack of generality; (iii) Bell's dilemma at ¶1.6(i) is resolved; (v) Bell's chief motivation via ¶1.6(ii) is justified; (vi) his locality hypothesis at ¶1.7 is developed; (vii) his questions answered via (22), (24), (33), (49), (51)-(54), etc. We thus conclude that—at peace with QM and relativity—a clearly realistic account of the world beckons: CLR—clear local realism—per (51)-(54); etc.

CLR: clear via Bohr's insight, local via Einstein locality, realistic via Bell beables.

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5 Appendix

5.1. The CLR analysis above—via equivalence relations under orientations; and consistent with EPRB, QM and experiment—is decisive in resolving the Bellian dilemma defined at ¶1.6(i). So, per ¶2.2a and ¶2.6, we now show CLR's accord with QM via direct relations under magnitudes. To this end: (i) from Bell 1964:(1) and ¶2.1, we let the beable $\boldsymbol{\lambda}$ denote a pristine particle's total angular momentum; (ii) from ¶2.15 we get the relationships missing from Bell 1964:(1); (iii) from ¶2.4, such relationships are experimentally-validated; (iv) new relationships may be validated similarly.

5.2. The link between CLR and geometric algebra follows: (i) let a_1, a_2, a_3 be a right-handed set of orthonormal basis vectors; (ii) let our $a \equiv a_3$; (iii) let a be our preferred term. As the original identifier of the principal axis of Alice's polarizer (from ¶2.1), a is the unit-vector denoting the key variable of polarizing-operator $\delta_a^\pm$ with respect to spin-$\frac{1}{2}$ particles $q(\boldsymbol{\lambda})$ under EPRB. Then, in conventional short-form notation under geometric algebra:

$$a_i a_j = a_i \cdot a_j + a_i \wedge a_j = \delta_{ij} + \imath\epsilon_{ijk} a_k; \imath \equiv a_i a_j a_k; \imath^2 = (a_i a_j a_k)^2 = -1; a_1 a_2 = a_1 \wedge a_2 = \imath a_3. \quad (55)$$

So our real vectors satisfy the defining relation of the Pauli matrices: $\sigma_i\sigma_j = \delta_{ij} + \imath\epsilon_{ijk}\sigma_{k.}$ (56)

The equiprobable spin-bivectors under the interaction $\delta_a^\pm q(\boldsymbol{\lambda})$ are then: $\pm|\,\mathbf{s}\,|\,\mathrm{a}_1\mathrm{a}_2 = \pm|\,\mathbf{s}\,|\,\imath\mathrm{a}_3$; (57)

where the spin-vector is: $\mathbf{s} = \pm\frac{\hbar}{2}a_3 = \pm\frac{\hbar}{2}a$; $+$ denoting spin-up wrt a; etc. (58)

5.3. Based on ¶2.18, we now represent particle/polarizer interactions by a new vector-product. Symmetrically, under the deterministic push-pull dynamics of ¶2.13, let a_- be appropriately orthogonal to a^+ as determined by the relevant spin; see (59). Then—with $\approx$ denoting equiprobability; $\oplus \equiv$ xor; a^- antiparallel to a^+; $a^\perp$ perpendicular to a^+—we define the spin-product $a\{s\hbar\}\boldsymbol{\lambda}$, a fair-coin:

$$a\{s\hbar\}\boldsymbol{\lambda} \approx s\hbar a_-^+\text{: if } s = \tfrac{1}{2}, a_-^+ \approx a^+ \oplus a^-\text{; if } s = 1, a_-^+ \approx a^+ \oplus a^\perp. \tag{59}$$

5.4. For digital outputs, eg Bell 1964:(1), here's the reduced spin-product $a\{s\}\boldsymbol{\lambda}$, another fair-coin:

$$a\{s\}\boldsymbol{\lambda} \approx \cos 2s(a, a_-^+) = \pm 1 = A^\pm;\ \text{with } a_-^+ \text{ defined in (59).} \tag{60}$$

5.5. Thus, using (4) to create two examples, we have for Alice under β (where $s = \frac{1}{2}$):

$$(\Delta_a^\pm q(\boldsymbol{\lambda}_i) \to A(a, \boldsymbol{\lambda}_i) \,|\, \beta) = +1 \equiv A^+ = \cos 2s(a, \boldsymbol{\lambda}_i \,|\, q(\boldsymbol{\lambda}_i) \sim q(a^+)) \equiv a\{s\}\boldsymbol{\lambda}_i = a \cdot a^+ = +1; \tag{61}$$

$$(\Delta_a^\pm q(\boldsymbol{\lambda}_j) \to A(a, \boldsymbol{\lambda}_j) \,|\, \beta) = -1 \equiv A^- = \cos 2s(a, \boldsymbol{\lambda}_j \,|\, q(\boldsymbol{\lambda}_j) \sim q(a^-)) \equiv a\{s\}\boldsymbol{\lambda}_j = a \cdot a^- = -1. \tag{62}$$

5.6. Then, given (60), Bob's corresponding results $B^\pm$ are correlated with Alice's $A^\pm$ via (6). So, using the most basic (ie, a probability-based) definition of an expectation—eg, Whittle (1976:20)—we take the expectation $\langle X \,|\, \beta \rangle$ to be the conventional arithmetic mean of X under the conditional β:

$$\therefore\ \langle X \,|\, \beta \rangle \equiv \sum_{i=1}^{n} P_i\, x_i\ :\ \text{given } P_i \equiv P(X = x_i \,|\, \beta);\ \sum_{i=1}^{n} P_i = 1. \tag{63}$$

$$\therefore\ \langle AB \,|\, \beta \rangle = P(AB = +1 \,|\, \beta) - P(AB = -1 \,|\, \beta) = 2P(AB = 1 \,|\, \beta) - 1 = 4P(A^+B^+ \,|\, \beta) - 1 \tag{64}$$

$$= 2P(B^+ \,|\, \beta, A^+) - 1 = 2P(b\{s\}\boldsymbol{\mu} = 1 \,|\, \beta, a\{s\}\boldsymbol{\lambda} = 1) - 1, \text{ using Bayes' Law and (60),} \tag{65}$$

$$= 2\sin^2 \tfrac{1}{2}(a, b) - 1 = -a \cdot b, \text{ using Malus' Law as in (20). QED.} \blacksquare \tag{66}$$

5.7. Note that the short-form representation of the expectation on LHS (65) is our preferred format. [Earlier (per ¶1.7), to be more in line with typical Bell essays, we refrained from using it.] By way of experimental confirmation, using α; the experiment in Aspect (2004) with photons ($s = 1$):

$$\langle AB \,|\, \alpha \rangle = 2P(B^+ \,|\, \alpha, A^+) - 1 = 2P(b\{s\}\boldsymbol{\mu} = 1 \,|\, \alpha, a\{s\}\boldsymbol{\lambda} = 1) - 1, \text{ using (65),} \tag{67}$$

$$= 2\cos^2(a, b) - 1 = \cos 2(a, b), \text{ using Malus' Law as in (27). QED.} \blacksquare \tag{68}$$

5.8. Thus, with Bayes' Law and Malus' Law to the fore here in our short-form expressions, and in the light of CLR, we now analyze Fröhner 1998:(75). There we see the inner products of the polarizer direction-vectors a and b with 'the spin $\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2$ taken to be an ordinary vector for which all orientations are equally probable'. Fröhner is thus able to 'equal the QM result' (in his terms):

$$\text{Fröhner 1998:(75): } \langle (a \cdot \boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_2 \cdot b) \rangle = -\langle (a \cdot \boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_1 \cdot b) \rangle = -\tfrac{\langle \sigma_1^2 \rangle}{3}(a \cdot b). \tag{69}$$

5.9. Thus, in our terms, and to match Bell 1964:(1), (69) needs to be solved for:

$$(a \cdot \boldsymbol{\sigma}_1) = \pm 1; (\boldsymbol{\sigma}_2 \cdot b) = \pm 1; \tfrac{\langle \sigma_1^2 \rangle}{3} = 1. \tag{70}$$

5.10. Fröhner (1998:647) does this—see his (70)-(73)—by describing the spin-coordinates via Pauli matrices and in the terms of EPR's criterion at ¶1.4 [that we amend at ¶1.5]. In that our method is coordinate-free, we now show our CLR resolution of (69)-(70). Under CLR—using the statistical terms variance (var), covariance (cov) and statistical-correlation (cor); with $\langle A \,|\, \beta \rangle = \langle B \,|\, \beta \rangle = 0$ from (17)-(18)—we have:

$$\text{cov}\,(A, B \,|\, \beta) \equiv \langle (A - \langle A \rangle)(B - \langle B \rangle) \,|\, \beta \rangle = (AB \,|\, \beta) = -a \cdot b\text{: from (24) or (66);} \tag{71}$$

$$\operatorname{var}(A\,|\,\beta) \equiv \left\langle (A - \langle A\rangle)^2\,|\,\beta\right\rangle = \left\langle A^2\,|\,\beta\right\rangle = 1; \tag{72}$$

$$\operatorname{var}(B\,|\,\beta) \equiv \left\langle (B - \langle B\rangle)^2\,|\,\beta\right\rangle = \left\langle B^2\,|\,\beta\right\rangle = 1. \tag{73}$$

$$\therefore \operatorname{cor}(A, B\,|\,\beta) \equiv \frac{\operatorname{cov}(A, B\,|\,\beta)}{\sqrt{\operatorname{var}(A\,|\,\beta)}\sqrt{\operatorname{var}(B\,|\,\beta)}} = \langle AB\,|\,\beta\rangle = -a{\cdot}b.\, QED. \blacksquare \tag{74}$$

5.11. Thus, independent of (71)-(74): our spin-products in (59)-(60), with their fair-coin outputs, deliver the correct (ie, QM-compatible) results.

5.12. In relation to EPRB and Bell (1964)—and more particularly to EPR-completeness at ¶1.3 and our EPR-amendment at ¶1.5—CLR leads us to conclude that $\boldsymbol{\lambda}$ represents the total angular momentum of a particle in units of $s\hbar$; ie, in units of spin (the intrinsic angular momentum). It follows that our spin-product [¶5.3] represents the reduction of $\boldsymbol{\lambda}$ and the collateral rotation of the remnant angular momentum—ie, per ¶2.18, the rotation of the irreducible spin $s\hbar$—onto a relevant axis via each particle/polarizer interaction. With $\boldsymbol{\mu}$ similarly, under its pairwise correlation with $\boldsymbol{\lambda}$ at (6): via the EPRB centrality—and the Aspect (2004) validated generality—of Malus' Law.

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