Lesson 9: Navier-Stokes Equations Solved Simply

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

Coincidences. The US Supreme Court consists of nine members, one of whom is the Chief Justice of the Court. So also, a one-direction Navier-Stokes equation consists of nine members, one of which is the indispensable gravity term, without which there would be no incompressible fluid flow as shown by the solutions of the N-S equations (viXra:1512.0334). Another coincidence is that numerologically, the number, 9, is equivalent to the 1800's (1 + 8 + 0 + 0 = 9) time period during which the number of the members of the Supreme Court became fixed at 9, while the formulation of the nine-term N-S equations was completed. Also, another coincidence is that the solutions of the N-S equations were completed (viXra:1512.0334) by the author in the year, 2016 (2 + 0 +1+ 6 = 9). Using a new introductory approach, this paper covers the author's previous solutions of the N-S equations (viXra:1512.0334). In particular, the N-S solutions have been compared to the equations of motion and liquid pressure of elementary physics. The N-S solutions are (except for the constants involved) very similar or identical to the equations of motion and liquid pressure of elementary physics. The results of the comparative analysis show that the N--S equations have been properly solved. It could be stated that the solutions of the N-S equations have existed since the time the equations of motion and liquid pressure of elementary physics were derived. A one-direction Navier-Stokes equation has also been derived from the equations of motion and liquid pressure of elementary physics. Insights into the solutions include how the polynomial parabolas, the radical parabolas, and the hyperbolas interact to produce turbulent flow. It is argued that the solutions and methods of solving the N-S equations are unique, and that only the approach by the author will ever produce solutions to the N-S equations. By a solution, the equation must be properly integrated and the integration results must be tested in the original equation for identity before the integration results are claimed as solutions.

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Introduction

The following examples are to convince the reader that the approach used in splitting the N-S equation and pairing the terms is valid. This approach works for partial differential equations if the single term on the right side of the equation is a constant. One can view this splitting and paring as dividing the constant term on the right side of the equation (using ratios) by the terms on the left side of the equation. It is to be understood that, to claim a solution to a partial differential equation, the integration results must be checked for identity in the original partial differential equation.
As suggested by Albert Einstein, one will think like a child at the beginning of the solution of the N-S equation. Actually, one will think like a ninth grader.
Suppose one performs the following operation:

Example 1: Addition of only two numbers

$$20 + 25 = 45 \qquad (1)$$

$$20 = 45 \times \tfrac{20}{45} = 45 \times \tfrac{4}{9} \qquad (2)$$

$$25 = 45 \times \tfrac{25}{45} = 45 \times \tfrac{5}{9} \qquad (3)$$

Equations (2), and (3), can be written as follows:

$$20 = 45a \qquad (4)$$

$$25 = 45b \qquad (5)$$

Above, a and b are called ratio terms

$$a = \tfrac{4}{9},\ b = \tfrac{5}{9} \qquad (a + b = 1\ ,\ \tfrac{4}{9} + \tfrac{5}{9} = \tfrac{9}{9} = 1)$$

Adding equations (4) and (5),

$$20 + 25 = 45(a + b)$$

One can conclude that the sum of the ratio terms is always 1.
The next example is a preparation towards the "main dish".

Example 2: There is only a single Navier-Stokes term on the left side of the equation.

If $\dfrac{dp}{dx} = \rho g_x \qquad$ (A)

find $P(x)$

Solution

$\dfrac{dp}{dx} = \rho g_x \qquad (1)$ $dp = \rho g_x dx$ $p(x) = \int \rho g_x dx$ $p(x) = \rho g_x x + C \quad (2) \qquad$ (Integrating) **Check solution for identity** in (A), From (2), $\dfrac{dp}{dx} = \rho g_x$ Substituting for the left side of (A), $\rho g_x \overset{?}{=} \rho g_x$ Yes	**Extra 1 :** $\dfrac{dp}{dy} = \rho g_y \qquad (1)$ $dp = \rho g_y dy$ $p(y) = \int \rho g_y dy$ $p(y) = \rho g_y y + C \quad (2)$ **Checking:** $\dfrac{dp}{dy} = \rho g_y$ Substituting for the left side of (1) $\rho g_y \overset{?}{=} \rho g_y$ Yes	**Extra 2** $\dfrac{dp}{dh} = \rho g \qquad (1)$ $dp = \rho g dh$ $p(h) = \rho g h + C \quad (2)$ **Note:**: In elementary physics, the liquid pressure at the bottom of a liquid of depth h units is given by $p = \rho g h$

Example 2 and its results hint at how to solve the Navier-Stokes equations.

What was done for the $\dfrac{dp}{dh}$ term to produce a correct equation for the liquid pressure must be repeated for each of the terms on the left side of the N-S equation, with the gravity term as the subject of the equation.

Example 3: There are two Navier-Stokes terms on the left side of the equation.

If $\frac{\partial p}{\partial x} + \rho \frac{\partial V_x}{\partial t} = \rho g_x$, (A)

find $P(x)$ and $V_x(t)$

Solution

Step 1: Assume that the terms on the left side of the equation are dividing the term on the right side of the equation in the ratio $d : f$. That is, the ratio terms for $\frac{\partial p}{\partial x}$ and $\rho \frac{\partial V_x}{\partial t}$ are d and f respectively.

Step 2: For $\frac{\partial p}{\partial x}$

$\frac{\partial p}{\partial x} = d\rho g_x$ (d is a ratio term)

$\frac{dp}{dx} = d\rho g_x$

(One drops the partials symbol, since a single independent variable is involved)

$dp = d\rho g_x dx$

$p(x) = \int d\rho g_x dx$

$p(x) = d\rho g_x x + C$ (integrating)

Step 3: For $\rho \frac{\partial V_x}{\partial t}$

$\rho \frac{\partial V_x}{\partial t} = f\rho g_x$ (f is a ratio term)

$\frac{\partial V_x}{\partial t} = fg_x$ (Dividing out the ρ)

$\frac{dV_x}{dt} = fg_x$

(One drops the partials symbol, since a single independent variable is involved)

$dV_x = fg_x dt$

$V_x(t) = \int fg_x dt$;

$V_x(t) = fg_x t + C$ (integrating)

Therefore, $p(x) = d\rho g_x x + C$ (B); and $V_x(t) = fg_x t + C$ (D)

To check for identity:

From (B), $\frac{dp}{dx} = d\rho g_x$ and from (D), $\frac{dV_x}{dt} = fg_x$

Substituting for $\frac{\partial p}{\partial x}$ and $\frac{\partial V_x}{\partial t}$ from (B) and (D) respectively on the left side of (A), above.

$$d\rho g_x + f\rho g_x \overset{?}{=} \rho g_x$$

$$\rho g_x (d + f) \overset{?}{=} \rho g_x$$

$$\rho g_x (1) \overset{?}{=} \rho g_x \quad \text{Yes} \quad (d + f = 1)$$

Since an identity is obtained , $p(x) = d\rho g_x x + C$, $V_x(t) = fg_x t + C$

Again, the results are similar to the equations of motion and liquid pressure of elementary physics.

Example 4: There are now three Navier-Stokes terms on the left side of the equation.

If $-\mu\dfrac{\partial^2 V_x}{\partial x^2}+\dfrac{\partial p}{\partial x}+\rho\dfrac{\partial V_x}{\partial t}=\rho g_x,$ (A)

find V_x and $P(x)$.

Solution: Let the ratio terms for $-\mu\dfrac{\partial^2 V_x}{\partial x^2}, \dfrac{\partial p}{\partial x}, \rho\dfrac{\partial V_x}{\partial t}$ be a, d and f respectively.

(That is, the terms on the left side of the equation divide the term on the right in the ratio $a:d:f$)

Step 1: For $-\mu\dfrac{\partial^2 V_x}{\partial x^2}$

$-\mu\dfrac{\partial^2 V_x}{\partial x^2}=a\rho g_x$ (a is a ratio term)

$-\mu\dfrac{d^2 V_x}{dx^2}=a\rho g_x$

(One drops the partials symbol)

$\dfrac{d^2 V_x}{dx^2}=-\dfrac{a\rho g_x}{\mu}$

$\dfrac{dV_x}{dx}=-\dfrac{a\rho g_x x}{\mu}+C$ (integrating)

$V_x=-\dfrac{a\rho g_x x^2}{2\mu}+C_1x+C_2$

(integrating again)

Step 2: For $\dfrac{\partial p}{\partial x}$

From Step 2 of Example 3 ,

For $\dfrac{\partial p}{\partial x}$

$\dfrac{\partial p}{\partial x}=d\rho g_x$ (d is a ratio term)

$\dfrac{dp}{dx}=d\rho g_x$

(One drops the partials symbol)

$dp=d\rho g_x dx$

$p(x)=\int d\rho g_x dx$

$p(x)=d\rho g_x x+C$

(integrating.)

Step 3: For $\rho\dfrac{\partial V_x}{\partial t}$

From Step 3 of Example 3

$\rho\dfrac{\partial V_x}{\partial t}=f\rho g_x$ (f is a ratio term)

$\dfrac{\partial V_x}{\partial t}=g_x$ (Dividing out the ρ)

$\dfrac{dV_x}{dt}=fg_x$

(One drops the partials symbol since a single independent variable is involved)

$dV_x=fg_x dt$

$V_x=\int fg_x dt$

$V_x=fg_x t+C$ (integrating)

Since there are two V_x' s, one adds the results from Step 1, and Step 3.

$V_x=-\dfrac{a\rho g_x x^2}{2\mu}+C_1x+fg_x t+C_3$ (B); and $p(x)=d\rho g_x x+C$ (D)

To check for identity:

From (B) an (D) respectively, one obtains the following:

$$\boxed{\begin{array}{l}\dfrac{\partial V_x}{\partial x}=-\dfrac{a\rho g_x x}{\mu}\\ \dfrac{\partial^2 V_x}{\partial x^2}=-\dfrac{a\rho g_x}{\mu}\end{array}},\quad \boxed{\dfrac{\partial V_x}{\partial t}=fg_x},\quad \boxed{\dfrac{dp}{dx}=d\rho g_x}$$

Substituting for $\dfrac{\partial^2 V_x}{\partial x^2}$, $\dfrac{\partial V_x}{\partial t}$ and $\dfrac{\partial p}{\partial x}$ from (B) and (D) respectively on the left side of (A) above,

$$-\mu\left(-\frac{a\rho g_x}{\mu}\right)+d\rho g_x+\rho f g_x\overset{?}{=}\rho g_x$$

$$a\rho g_x+d\rho g_x+\rho f g_x\overset{?}{=}\rho g_x$$

$$\rho g_x(a+d+f)\overset{?}{=}\rho g_x$$

$$\rho g_x(1)\overset{?}{=}\rho g_x \qquad \text{Yes} \quad (a+d+f=1)$$

Since an identity is obtained,, $V_x=-\dfrac{a\rho g_x x^2}{2\mu}+C_1x+fg_x t+C_3$; $p(x)=d\rho g_x x+C$

Again, the results are similar to the equations of motion and liquid pressure of elementary physics.

Example 5: There are now five Navier-Stokes terms, including two non-linear terms on the left side of the equation.

If $\rho V_x \frac{\partial V_x}{\partial x} + \rho V_y \frac{\partial V_x}{\partial y} - \mu \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial p}{\partial x} + \rho \frac{\partial V_x}{\partial t} = \rho g_x$ (A)

find V_x and $P(x)$.

Solution

Let the ratio terms for $V_y \frac{\partial V_x}{\partial y}, -\mu \frac{\partial^2 V_x}{\partial x^2}$, $\rho \frac{\partial V_x}{\partial t}$, and $\frac{\partial p}{\partial x}$ be h, n, a, f and d respectively.
(That is, the terms on the left side of the equation divide the term on the right in the ratio $h : n : a : d : f$)

Step 1: For $\rho V_x \frac{\partial V_x}{\partial x}$

$$\rho V_x \frac{dV_x}{dx} = h\rho g_x$$

$$V_x \frac{dV_x}{dx} = hg_x \text{ (divide out } \rho)$$

$$V_x dV_x = hg_x dx$$

$$\frac{V_x^2}{2} = hg_x x$$

$$\boxed{V_x = \pm\sqrt{2hg_x x} + C_7}$$

(integrating)

Step 2: For $\rho V_y \frac{\partial V_x}{\partial y}$

$$\rho V_y \frac{\partial V_x}{\partial y} = n\rho g_x$$

$$V_y \frac{dV_x}{dy} = ng_x$$

$$V_y dV_x = ng_x \mathrm{dy}$$

$$V_y V_x = ng_x \mathrm{y} + \psi_y(V_y)$$

$$\boxed{V_x = \frac{ng_x \mathrm{y}}{V_y} + \frac{\psi_y(V_y)}{V_y}} \text{ (integrate)}$$

where $\frac{\psi_y(V_y)}{V_y}$ is an arbitrary function

Note that this is an implicit solution, since the solution contains V_y

Step 3: For $-\mu \frac{\partial^2 V_x}{\partial x^2}$

$$-\mu \frac{\partial^2 V_x}{\partial x^2} = a\rho g_x \quad (a \text{ is a ratio term})$$

$$-\mu \frac{d^2 V_x}{dx^2} = a\rho g_x$$

(One drops the partials symbol, since a single independent variable is involved)

$$-\frac{d^2 V_x}{dx^2} = -\frac{a\rho g_x}{\mu}$$

$$\frac{dV_x}{dx} = -\frac{a\rho g_x x}{\mu} + C \quad \text{(integrating)}$$

$$\boxed{V_x = -\frac{a\rho g_x x^2}{2\mu} + C_1 x + C_2}$$

(integrating again)

Step 4: For $\rho \frac{\partial V_x}{\partial t}$

From Step 3 of Example 3

$$\rho \frac{\partial V_x}{\partial t} = f\rho g_x \quad (f \text{ is a ratio term})$$

$$\frac{\partial V_x}{\partial t} = g_x \quad \text{(Dividing out the } \rho)$$

$$\frac{dV_x}{dt} = fg_x$$

((One drops the partials symbol since a single independent variable is involved)

$$dV_x = fg_x dt$$

$$V_x(t) = \int fg_x dt$$

$$\boxed{V_x(t) = fg_x t + C} \quad \text{(integrating)}$$

Step 5: For $\frac{\partial p}{\partial x}$

From Step 2 of Example 3,

$$\frac{\partial p}{\partial x} = d\rho g_x \quad (d \text{ is a ratio term})$$

$$\frac{dp}{dx} = d\rho g_x \quad \text{(One drops the partials symbol)}$$

$$dp = d\rho g_x dx$$

$$p(x) = \int d\rho g_x dx$$

$$\boxed{p(x) = d\rho g_x x + C} \quad \text{(integrating)}$$

Since there are four V_x' s, one adds the results from Steps 1, 2, 3 and 4.

$$V_x = \pm\sqrt{2hg_x x} - \frac{a\rho g_x x^2}{2\mu} + C_1 x + fg_x t + \frac{ng_x y}{V_y} + \frac{\psi_y(V_y)}{V_y} + C_3; \qquad \text{(B)}$$

and $P(x) = d\rho g_x x + C$ (D)

To check for identity

From (B) an (D) respectively,

$V_x = -\frac{a\rho g_x x^2}{2\mu} + C_1 x + fg_x t + \pm\sqrt{2hg_x x} + \frac{ng_x y}{V_y} + \frac{\psi_y(V_y)}{V_y} + C_3, P(x) = d\rho g_x x + C$, one obtains the following derivatives. However, one takes the derivative for the linear terms separately and for the non-linear terms separately.

Test derivatives for the linear part (From Example 4))			Test derivatives for the non-linear part	
$\frac{\partial^2 V_x}{\partial x^2} = -\frac{a\rho g_x}{\mu}$	$\frac{\partial V_x}{\partial t} = fg_x$	$\frac{\partial p}{\partial x} = d\rho g_x$	$V_x^2 = 2hg_x x$ $2V_x \frac{\partial V_x}{\partial x} = 2hg_x$ $\frac{\partial V_x}{\partial x} = \frac{hg_x}{V_x}, V_x \neq 0$	$V_x = \frac{ng_x y}{V_y}$ $\frac{\partial V_x}{\partial y} = \frac{ng_x}{V_y}$ $\frac{\partial V_x}{\partial y} = \frac{ng_x}{V_y}$

Substituting for $\frac{\partial^2 V_x}{\partial x^2}$, $\frac{\partial V_x}{\partial t}$, $\frac{\partial p}{\partial x}$ $\frac{\partial V_x}{\partial x}$ $\frac{\partial V_x}{\partial y}$ from above table, respectively in the left side of (A),

$$V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} - \mu \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial p}{\partial x} + \rho \frac{\partial V_x}{\partial t} = \rho g_x$$

$$\rho V_x \frac{hg_x}{V_x} + \rho V_y \frac{ng_x}{V_y} - \mu(-\frac{a\rho g_x}{\mu}) + d\rho g_x + \rho fg_x \overset{?}{=} \rho g_x$$

$$\rho hg_x + \rho ng_x + a\rho g_x + d\rho g_x + \rho fg_x \overset{?}{=} \rho g_x$$

$$\rho g_x(h + n + a + d + f) \overset{?}{=} \rho g_x$$

$$\rho g_x(1) \overset{?}{=} \rho g_x \quad \text{Yes } (h + n + a + d + f = 1)$$

Since an identity is obtained, $V_x = -\frac{a\rho g_x x^2}{2\mu} + C_1 x + fg_x t \pm \sqrt{2hg_x x} + \frac{ng_x y}{V_y} + \frac{\psi_y(V_y)}{V_y} + C_3$;

$P(x) = d\rho g_x x + C$

Again,, except for the fifth term, the results are similar to the equations of motion and liquid pressure of elementary physics.. Note that after solving the N-S equations for the *y* and *z*-directions, the system of equations will be solved to express $(ng_x y/V_y)$ in terms of motion equations of elementary physics .

From above, if there were 100 terms on the left side of the equation, one could repeat the above procedure for each term. Therefore, using the ratio method, the N-S equations can be solved in any number of dimensions.

After having solved for a viscosity term, two convective acceleration terms, a variable acceleration term, and the pressure gradient term, one is ready to solve a complete Navier-Stokes equation.

In fact , one is ready to solve a 100-term Navier-Stokes equation.

If the reader likes the approach used so far, see epsilon-delta proofs in Calculus 1 & 2 by A. A. Frempong, published by Yellowtextbooks.com

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Solutions of 3-D Navier-Stokes Equations

By imitating the previous examples, one will now solve the x–direction Navier-Stokes equation.

$$-\mu\frac{\partial^2 V_x}{\partial x^2}-\mu\frac{\partial^2 V_x}{\partial y^2}-\mu\frac{\partial^2 V_x}{\partial z^2}+\frac{\partial p}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}=\rho g_x \qquad \text{(A)}$$

The subject of the above equation is the gravity term, because when each term of the linearized N-S equation was used as the subject of the equation, only the equation with the gravity term as the subject produced a solution (see viXra:1512.0334).

Step 1: **T**he first step here, is to split-up the equation into eight sub-equations using the ratio method. The ratio terms for the terms on the left side of the equation are a, b, c, d, f, h, n, q, respectively. $(a+b+c+d+f+h+n+q=1)$

$\boxed{1.-\mu\frac{\partial^2 V_x}{\partial x^2}=a\rho g_x;}$ $\boxed{2.-\mu\frac{\partial^2 V_x}{\partial y^2}=b\rho g_x;}$ $\boxed{3.-\mu\frac{\partial^2 V_x}{\partial z^2}=c\rho g_x;}$ $\boxed{4.\ \frac{\partial p}{\partial x}=d\rho g_x;}$

$\boxed{5.\ \frac{\partial V_x}{\partial t}=fg_x}$ $\boxed{6.\ V_x\frac{\partial V_x}{\partial x}=hg_x;}$ $\boxed{7.\ V_y\frac{\partial V_x}{\partial y}=ng_x;}$ $\boxed{8.\ V_z\frac{\partial V_x}{\partial z}=qg_x}$

Step 2: Solve the differential equations in Step 1.
Note that after splitting the equations, the equations can be solved using techniques of ordinary differential equations, since there would be a single independent variable in each equation. One can also view each of the ratio terms a, b, c, d, f, h, n, q as a fraction (a real number) of $\boxed{g_x}$ contributed by each expression on the left-hand side of equation (A) above.

Solutions of the eight sub-equations

1. $-\mu\frac{\partial^2 V_x}{\partial x^2}=a\rho g_x;$ $\frac{d^2V_x}{dx^2}=-\frac{a\rho g_x}{\mu}$ $\frac{dV_x}{dx}=-\frac{a\rho g_x}{\mu}x+C_1$ (integr.) $V_{x1}=-\frac{a\rho g_x}{2\mu}x^2+C_1x+C_2$	2. $-\mu\frac{\partial^2 V_x}{\partial y^2}=b\rho g_x$ $\frac{d^2V_x}{dy^2}=-\frac{b\rho g_x}{\mu}$ $\frac{dV_x}{dy}=-\frac{b\rho g_x}{\mu}x+C_1$ $V_{x2}=-\frac{b\rho g_x}{2\mu}y^2+C_1y+C_2$	3. $-\mu\frac{\partial^2 V_x}{\partial z^2}=c\rho g_x;$ $\frac{d^2V_x}{dz^2}=-\frac{c\rho g_x;}{\mu}$ $\frac{dV_x}{dz}=-\frac{c\rho g_x;}{\mu}z+C_5$ $V_{x3}=-\frac{c\rho g_x;}{2\mu}z^2+C_5z+C_6$	4. $\frac{\partial p}{\partial x}=d\rho g_x;$ $\frac{1}{\rho}\frac{\partial p}{\partial x}=dg_x$ $\frac{\partial p}{\partial x}=d\rho g_x$ $p=d\rho g_x x+C_7$ $\boxed{5.\frac{\partial V_x}{\partial t}=fg_x}$ $V_{x4}=fgt$
6. $V_x\frac{\partial V_x}{\partial x}=hg_x;$ $V_x\frac{dV_x}{dx}=hg_x$ $V_xdV_x=hg_xdx$ $\frac{V_x^{\ 2}}{2}=hg_xx$ $V_{x5}=\pm\sqrt{2hg_xx}+C_7$	7. $V_y\frac{\partial V_x}{\partial y}=ng_x;$ $V_y\frac{dV_x}{dy}=ng_x$ $V_ydV_x=ng_x\mathrm{dy}$ $V_yV_x=ng_x\mathrm{y}+\psi_y(V_y)$ $V_{x6}=\frac{ng_x\mathrm{y}}{V_y}+\frac{\psi_y(V_y)}{V_y}$	8. $V_z\frac{\partial V_x}{\partial z}=qg_x$ $V_z\frac{dV_x}{dz}=qg_x$ $V_zdV_x=qg_x\mathrm{dz};$ $V_zV_x=qg_x\mathrm{z}+\psi_z(V_z)$ $V_{x7}=\frac{qg_x\mathrm{z}}{V_z}+\frac{\psi_z(V_z)}{V_z}$	**Note:** $\psi_y(V_y),\psi_z(V_z)$ are arbitrary functions, (integration constants) $V_y\neq 0$ $V_z\neq 0$

Step 3: One combines the above solutions

$$
\boxed{
\begin{aligned}
&V_x(x,y,z,t) = V_{x1} + V_{x2} + V_{x3} + V_{x4} + V_{x5} + V_{x6} + V_{x7} \\
&= -\frac{ag_x}{2k}x^2 + C_1x - \frac{bg_x}{2k}y^2 + C_3y - \frac{cg_x}{2k}z^2 + C_5z + fg_xt \pm \sqrt{2hg_xx} + \frac{ng_xy}{V_y} + \frac{qg_xz}{V_z} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z} \\
&\overbrace{-\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1x + C_3y + C_5z + fg_xt}^{\text{relation for linear terms}} \pm \overbrace{\sqrt{2hg_xx} + \frac{ng_xy}{V_y} + \frac{qg_xz}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}}^{\text{relation for non-linear terms}} + C_9 \\
&P(x) = d\rho g_x x; \quad (a+b+c+d+f+h+n+q=1) \quad V_y \neq 0,\ V_z \neq 0
\end{aligned}
}
$$

Step 4: Find the test derivatives

Test derivatives for the linear part					Test derivatives for the non-linear part		
$\frac{\partial^2 V_x}{\partial x^2} = -\frac{a\rho g_x}{\mu}$	$\frac{\partial^2 V_x}{\partial y^2} = -\frac{b\rho g_x}{\mu}$	$\frac{\partial^2 V_x}{\partial z^2} = -\frac{c\rho g_x}{\mu}$	$\frac{\partial p}{\partial x} = d\rho g_x$	$\frac{\partial V_x}{\partial t} = fg_x$	$V_x^2 = 2hg_xx$ $2V_x\frac{\partial V_x}{\partial x} = 2hg_x$ $\frac{\partial V_x}{\partial x} = \frac{hg_x}{V_x}, V_x \neq 0$	$\frac{\partial V_x}{\partial y} = \frac{ng_x}{V_y}$	$\frac{\partial V_x}{\partial z} = \frac{qg_x}{V_z}$

Step 5: **S**ubstitute the derivatives from Step 4 in equation (A) for the checking.

$$-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p}{\partial x} + \rho\frac{\partial V_x}{\partial t} + \rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z} = \rho g_x \qquad \textbf{(A)}$$

$$-\mu\left(-\frac{a\rho g_x}{\mu} - \frac{b\rho g_x}{\mu} - \frac{c\rho g_x}{\mu}\right) + d\rho g_x + f\rho g_x + \rho\left(V_x\frac{hg_x}{V_x}\right) + \rho V_y\left(\frac{ng_x}{V_y}\right) + \rho V_z\left(\frac{qg_x}{V_z}\right) \stackrel{?}{=} \rho g_x$$

$$a\rho g_x + b\rho g_x + c\rho g_x + d\rho g_x + f\rho g_x + h\rho g_x + n\rho g_x + q\rho g_x \stackrel{?}{=} \rho g_x$$

$$a\rho g_x + b\rho g_x + c\rho g_x + d\rho g_x + f\rho g_x + h\rho g_x + n\rho g_x + q\rho g_x \stackrel{?}{=} \rho g_x$$

$$\rho g_x(a+b+c+d+f+h+n+q) \stackrel{?}{=} \rho g_x$$

$$\rho g_x(1) \stackrel{?}{=} \rho g_x \quad \text{Yes} \quad (a+b+c+d+f+h+n+q=1)$$

Step 6: The linear part of the relation satisfies the linear part of the equation; and the non-linear part of the relation satisfies the non-linear part of the equation.(

Analogy for the Identity Checking Method: If one goes shopping with American dollars and Japanese yens (without any currency conversion) and after shopping, if one wants to check the cost of the items purchased, one would check the cost of the items purchased with dollars against the receipts for the dollars; and one would also check the cost of the items purchased with yens against the receipts for the yens purchase. However, if one converts one currency to the other, one would only have to check the receipts for only a single currency, dollars or yens. This conversion case is similar to the linearized equations (see viXra:1512.0334), where there was no partitioning in identity checking. Note that for the Euler equations (viXra:1512.0332), there was no partitioning in taking derivatives for identity checking.

Note: After expressing $\frac{ng_xy}{V_y}$ and $\frac{q_eg_xz}{V_z}$ in terms of x, y, z, and t, there would be no partitioning in identity checking.

Summary of solutions for V_x , V_y, V_z ($P(x) = d\rho g_x x$; $P(y) = \lambda_4\rho g_y y$, $P(z) = \beta_4\rho g_z z$)

$$\boxed{\begin{array}{l} V_x = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt \pm \sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+C_9 \\ P(x) = d\rho g_x x; \quad (a+b+c+d+h+n+q=1) \quad V_y \neq 0,\ V_z \neq 0 \end{array}}$$

$$\boxed{V_y = -\frac{\rho g_y}{2\mu}(\lambda_1x^2+\lambda_2y^2+\lambda_3z^2)+C_{10}x+C_{11}y+C_{12}z+\lambda_5g_yt \pm\sqrt{2\lambda_7g_y\mathrm{y}}+\frac{\lambda_6g_yx}{V_x}+\frac{\lambda_8g_y\mathrm{z}}{V_z}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_z(V_z)}{V_z}}$$

$$\boxed{V_z = -\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt \pm\sqrt{2\beta_8g_zz}+\frac{\beta_6g_zx}{V_x}+\frac{\beta_7g_zy}{V_y}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_y(V_y)}{V_y}}$$

The above solutions are unique, because from the experience in Option 1 (viXra:1512.0334)., only the equations with the gravity terms as the subjects of the equations produced the solutions.

$$\boxed{\left\{\begin{array}{l} V_x = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt \pm \sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z} \\ V_y = -\frac{\rho g_y}{2\mu}(\lambda_1x^2+\lambda_2y^2+\lambda_3z^2)+C_{10}x+C_{11}y+C_{12}z+\lambda_5g_yt \pm\sqrt{2\lambda_7g_y\mathrm{y}}+\frac{\lambda_6g_yx}{V_x}+\frac{\lambda_8g_y\mathrm{z}}{V_z}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_z(V_z)}{V_z} \\ V_z = -\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt \pm\sqrt{2\beta_8g_zz}+\frac{\beta_6g_zx}{V_x}+\frac{\beta_7g_zy}{V_y}+\frac{\psi_x(V_x)}{V_x}+\frac{\psi_y(V_y)}{V_y} \end{array}\right.}$$

One will next solve the above system of solutions for V_x , V_y, V_z in order to express $\frac{ng_xy}{V_y}$ and $\frac{q_eg_xz}{V_z}$ in terms of x, y, z, and t The author used the help of the Maples software for the simultaneous algebraic solutions for V_x , V_y, V_z . The basic expressions are of the forms $-\frac{\rho g_x}{2\mu}ax^2$, $-\frac{\rho g_x}{2\mu}by^2$, $-\frac{\rho g_x}{2\mu}cz^2$, fg_xt , $\sqrt{2hg_xx}$, and $d\rho g_xx$; These expressions are similar to the terms of the equations of motion under gravity and liquid pressure of elementary physics. Note that the explicit solutions will be the results of the basic operations (addition, subtraction, multiplication, division, power finding and root extraction) on the expressions in Step 1 below.

Solving for V_x , V_y, V_z $\frac{ng_xy}{V_y}$, and $\frac{q_eg_xz}{V_z}$

Let $V_x = x$, $V_y = y$ and $V_z = z$. (x, y and z are being used for simplicity. They will be changed back to V_x , V_y, and V_z later, and they do not represent the variables x, y and z in the solutions.

Step 1 From the above system of solutions, let	**Step 2** Then the solutions to the N-S system of equations become (ignoring the arbitrary functions)
$A = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt \pm \sqrt{2hg_xx}$ $B = -\frac{\rho g_y}{2\mu}(\lambda_1x^2+\lambda_2y^2+\lambda_3z^2)+C_{10}x+C_{11}y+C_{12}z+\lambda_5g_yt \pm\sqrt{2\lambda_7g_y\mathrm{y}}$ $C = -\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt \pm\sqrt{2\beta_8g_zz}$ $D = qg_xz$; $E = ng_xy$; $F = \lambda_6g_yx$ $G = \lambda_8g_y\mathrm{z}$; $J = \beta_6g_zx$; $L = \beta_7g_zy$	$\left.\begin{array}{l} x = A+\frac{D}{z}+\frac{E}{y} \\ y = B+\frac{F}{x}+\frac{G}{z} \\ z = C+\frac{J}{x}+\frac{L}{y} \end{array}\right\} \quad \mathrm{M}$

Step 3	**Step 4**
$\left.\begin{array}{ll} xyz = Ayz + Dy + Ez & (1) \\ xyz = Bxz + Fz + Gx & (2) \\ xyz = Cxy + Jy + Lx & (3)) \end{array}\right\}$ N	$\left.\begin{array}{ll} 0 = Ayz + Dy + Ez - Bxz - Fz - Gx & (4) \\ 0 = Ayz + Dy + Ez - Cxy - Jy - Lx & (5) \\ 0 = Bxz + Fz + Gx - Cxy - Jy + -Lx & (6) \end{array}\right\}$ P

Maples software was used to solve system P to obtain

Step 5a	**Note**:
$x = \frac{L(FCD - FCJ - JLA + JCE)}{C(-BLD + BLJ + GLA - GCE)}$ $V_x = \frac{L(FCD - FCJ - JLA + JCE)}{C(-BLD + BLJ + GLA - GCE)}$ (back to V_x) $y = -\frac{L}{C}$; $V_y = -\frac{L}{C}$ (changing back to V_y as agreed to) $z = -\frac{L(D-J)}{LA - CE}$; $V_z = -\frac{L(D-J)}{LA - CE}$ (changing back to V_z as agrred to)	None of the popular academic programs could solve the system in M. Maples solved system P (step 4 above) for x, y,and z in terms of A, B, C, D. E. F, G. J. and L. Note also that x, y and z are not the same as the x, y and z in the system of equations.. They were used for convenience and simplicity .

Step 5b : Apply the following and substitute for A, B, C, D. E. F, G., J. and L in steps 6-7 below

$$A = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1x + C_3y + C_5z + fg_xt \pm \sqrt{2hg_xx}$$

$$B = -\frac{\rho g_y}{2\mu}(\lambda_1x^2 + \lambda_2y^2 + \lambda_3z^2) + C_{10}x + C_{11}y + C_{12}z + \lambda_5g_yt \pm \sqrt{2\lambda_7g_y\mathrm{y}}$$

$$C = -\frac{\rho g_z}{2\mu}(\beta_1x^2 + \beta_2y^2 + \beta_3z^2) + C_{14}x + C_{15}y + C_{16}z + \beta_5g_zt \pm \sqrt{2\beta_8g_zz}$$

$$D = qg_xz \ ; \ E = ng_xy; \ \ F = \lambda_6g_yx$$

$$G = \lambda_8g_y\mathrm{z}; \ \ J = \beta_6g_zx; \ \ L = \beta_7g_zy$$

Step 6

$$V_y = -\frac{L}{C} = -\frac{(\beta_7g_zy)}{(-\frac{\rho g_z}{2\mu}(\beta_1x^2 + \beta_2y^2 + \beta_3z^2) + C_1x + C_3y + C_5z + \beta_5g_zt \pm \sqrt{2\beta_8g_zz}}$$

$$\frac{ng_xy}{V_y} = ng_xy \div -(\frac{(\beta_7g_zy)}{(-\frac{\rho g_z}{2\mu}(\beta_1x^2 + \beta_2y^2 + \beta_3z^2) + C_1x + C_3y + C_5z + \beta_5g_zt \pm \sqrt{2\beta_8g_zz}})$$

$$\frac{ng_xy}{V_y} = -\frac{(ng_xy)[(-\frac{\rho g_z}{2\mu}(\beta_1x^2 + \beta_2y^2 + \beta_3z^2) + C_1x + C_3y + C_5z + \beta_5g_zt \pm \sqrt{2\beta_8g_zz})]}{\beta_7g_zy};$$

$$\frac{ng_xy}{V_y} = \frac{-(ng_x)(-\frac{\rho g_z}{2\mu}(\beta_1x^2 + \beta_2y^2 + \beta_3z^2) + C_1x + C_3y + C_5z + \beta_5g_zt \pm \sqrt{2\beta_8g_zz})}{\beta_7g_z}; \text{ (cancel "y")}$$

Step 7: $V_z = -\frac{L(D-J)}{LA-CE} = \frac{JL-DL)}{LA-CE}$

$$\frac{qg_xz}{V_z} = (qg_xz)\bullet\frac{(\beta_7g_zy)[-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}]-}{(\beta_7g_zy)[\beta_6g_zx-qg_xz]}$$

$$\frac{(ng_xy)[-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}]}{(\beta_7g_zy)[\beta_6g_zx-qg_xz]}$$

$$\frac{qg_xz}{V_z} = (qg_xz)\bullet\frac{(\beta_7g_z)[-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}]-}{(\beta_7g_z)[\beta_6g_zx-qg_xz]}$$

$$\frac{(ng_x)[-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}]}{(\beta_7g_z)[\beta_6g_zx-qg_xz]}$$

Summary for the fractional terms of the x–direction solution

$\frac{ng_xy}{V_y}$ and $\frac{qg_xz}{V_z}$ in terms of x, y, z and t

$$\frac{ng_xy}{V_y} = \frac{-(ng_x)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz})}{\beta_7g_z}\text{; (cancel "y")}$$

$$\frac{qg_xz}{V_z} = (qg_xz)\bullet\frac{(\beta_7g_z)[-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}]-}{(\beta_7g_zy)[\beta_6g_zx-qg_xz]}$$

$$\frac{(ng_x)[-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}]}{(\beta_7g_z)[\beta_6g_zx-qg_xz]}$$

$$\left(CE = -(ng_xy)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}\right)$$

$d+f+h+n+q=1;\quad \lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8=1;\quad \beta_4+\beta_5+\beta_6+\beta_7+\beta_8=1$

OR Compactly, *x*-direction solution of N-S Equation (in explicit solutions)

$\mathbf{V}_x(x,y,z,t) = A+B+F$ **Expanded** V_x

$$\mathbf{A} = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}+C_9$$

$$\mathbf{B} = \frac{-(ng_x)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz})}{\beta_7g_z}+\frac{\psi_y(V_y)}{V_y}$$

$$\mathbf{F} = (qg_xz)\bullet\frac{(\beta_7g_z)[-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt\pm\sqrt{2hg_xx}]-}{(\beta_7g_zy)[\beta_6g_zx-qg_xz]}$$

$$\frac{(ng_x)[-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt\pm\sqrt{2\beta_8g_zz}]}{(\beta_7g_z)[\beta_6g_zx-qg_xz]}+\frac{\psi_z(V_z)}{V_z}$$

$P(x) = d\rho g_xx;\quad (a+b+c+d+h+n+q=1)\quad \beta_1+\beta_2+\beta_3+\beta_5+\beta_6+\beta_7+\beta_8=1$

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Comparison of Navier-Stokes Solutions and Equations of Motion and Liquid Pressure of Elementary Physics;

Solutions of the Navier-Stokes Equations (Original) : x–direction

A $V_x = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt \pm \sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}+C_9$

$P(x) = d\rho g_x x; \quad (a+b+c+d+h+n+q=1) \quad V_y \neq 0,\ V_z \neq 0$

Summary for the fractional terms of the x–direction in terms of x, y, z and t.

$$\frac{ng_xy}{V_y} = \frac{-(ng_x)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_1x+C_3y+C_5z+\beta_5g_zt \pm\sqrt{2\beta_8g_zz})}{\beta_7g_z}$$

$$\frac{qg_xz}{V_z} = \frac{-(qg_xz)\{[(\beta_7g_zy)(-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+fg_xt \pm \sqrt{2hg_xx}]-[CE]\}}{(\beta_7g_zy)(qg_xz-\beta_6g_zx)}$$

$$(CE = -(ng_xy)(-\frac{\rho g_z}{2\mu}(\beta_1x^2+\beta_2y^2+\beta_3z^2)+C_{14}x+C_{15}y+C_{16}z+\beta_5g_zt \pm\sqrt{2\beta_8g_zz})$$

Motion equations of elementary physics:

B (B): $V_f = V_0 + gt$; (C): $V_f^2 = V_0^2 + 2gx$; (D): $V = \sqrt{2gx}$; (E): $x = V_0t + \frac{1}{2}gt^2$

The **liquid pressure**, P at the bottom of a liquid of depth h units is given by $P = \rho gh$

Similarity 1. The solutions are (except for the constants involved) very similar to the equations of motion and liquid pressure of elementary physics.

Observe the following about the Navier-Stokes Solutions Box A

1. The first three terms are parabolic in x, y, and z; the minus sign shows the usual inverted parabola when a projectile is fired upwards at an acute angle to the horizontal; also note the "gt" in $V = gt$ of (B) of the motion equations and the fg_xt in the Navier-Stokes solution.

2. The pressure, $P = \rho gh$ of the liquid pressure and the $P(x) = d\rho g_x x$ of the Navier-Stokes solution. **Note** that, only the approach in this paper could yield $P(x) = d\rho g_x x$ by integrating $dp/dx = d\rho g_x$

3. Observe the "$\sqrt{2gx}$" in $V = \sqrt{2gx}$ of (D) and the $\sqrt{2hg_xx}$ in the Navier-Stokes solution. In fact, the N-S solution term $\sqrt{2hg_xx}$ could have been obtained from $V_f^2 = V_0^2 + 2gx$,(C) , of the equation of motion by letting $V_0 = 0$ (for the convective term) ignoring the ratio term "h" of the N-S radicand. There are eight main terms (ignoring the arbitrary functions) in the N-S solution. Of these eight terms, six terms, namely, $-\frac{a\rho g_x}{2\mu}x^2$, $-\frac{b\rho g_x}{2\mu}y^2$, $-\frac{c\rho g_x}{2\mu}z^2$, fg_xt, $\sqrt{2hg_xx}$ and $d\rho g_x x$ are similar (except for the constants involved) to the terms in the equations of motion and liquid pressure. This similarity means that the approach used in solving the Navier-Stokes equation is sound. One should also note that to obtain these six terms simultaneously on integration, only the equation with the gravity term as the subject of the equation will yield these six terms. The author suggests that this form of the equation with the gravity term as the subject of the equation be called the standard form of the Navier-Stokes equation, since in this form, one can immediately split-up the equations using ratios, and integrate.

4. With regards to the variables x, y, and z, the parabolicity of the first three terms and the parabolicity of the eighth, ninth and tenth terms hint at inverse relations.. For examples, $V_x = x^2$ and $V_x = \pm\sqrt{x}$ are inverse relations of each other, $V_x = y^2$ and $V_x = \pm\sqrt{y}$ are inverse relations of each other, $V_x = z^2$ and $V_x = \pm\sqrt{z}$ are inverse relations of each other. The implications of knowing these relationships is that if one knows the steps, rules or formulas for designing for laminar flow, one can deduce the steps, rules or formulas for designing for turbulent flow by reversing the steps and using opposite operations in each step of the corresponding laminar flow design. Thus for every method, or formula for laminar flow, there is a corresponding method, formula for turbulent flow design (see also

Motion equations of elementary physics	**N-S Terms**
$V_f = V_0 + gt$	$fg_x t$,
$V_f^2 = V_0^2 + 2gx$	$\sqrt{2hg_x x}$
$x = V_0 t + \frac{1}{2}gt^2$	$-\frac{a\rho g_x}{2\mu}x^2$,
$V = \sqrt{2gx}$	$\sqrt{2hg_x x}$
$P = \rho gh$ (pressure equation)	$P(x) = d\rho g_x x$

Similarity 2. Sound approach used in pairing the terms of the equation

The approach used in splitting and pairing the terms of the equation is sound, especially by the results for the pressure $\boxed{P(x) = d\rho g_x x}$. To obtain $P(x) = d\rho g_x x$ by integrating $dp/dx = d\rho g_x$, only the equation with the gravity term as the subject of the equation will produce a solution. Therefore, if dp/dx is set to a constant multiplied by the gravity term (which is the subject of the N-S equation)) then each of the other terms must also be set equal to a constant multiplied by ρg_x

$$\boxed{-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \overbrace{\frac{\partial p}{\partial x}}^{\textbf{This}} + \rho\frac{\partial V_x}{\partial t} + \rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z} = \rho g_x}$$

From $\frac{\partial p}{\partial x} = d\rho g_x$ $P(x) = d\rho g_x x$, where d is a ratio term. Since ratio terms (constants)) were used to mathematically split-up the nine-term equation into eight sub-equations, all the terms of the equation remain "uncontaminated" and any results of the integration should differ from any other results only by constants.

When each term of the linearized Navier-Stokes equation was made subject of the N-S equation, only the equation with the gravity term as the subject of the equation produced a solution. (viXra:1512.0334).

Similarity 3: Insight of the N-S solutions and equations of motion and fluid pressure of elementary physics

$$V_x = -\frac{\rho g_x}{2\mu}(ax^2 + by^2 + cz^2) + C_1 x + C_3 y + C_5 z + fg_x t \pm \sqrt{2hg_x x} + \frac{ng_x y}{V_y} + \frac{qg_x z}{V_z} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z} + C_9$$

$$P(x) = d\rho g_x x; \quad (a + b + c + d + h + n + q = 1) \quad V_y \neq 0,\ V_z \neq 0$$

One observes above that the most important insight of the above solution is the **indispensability** of the gravity term in incompressible fluid flow. Observe that if gravity, g_x, were zero, the first three terms, the seventh, the eighth, the ninth, the tenth terms of the velocity solution and $P(x)$ would all be zero. For equations of motion and liquid pressure of elementary physics, each equation contains an indispensable gravity term.See above table.

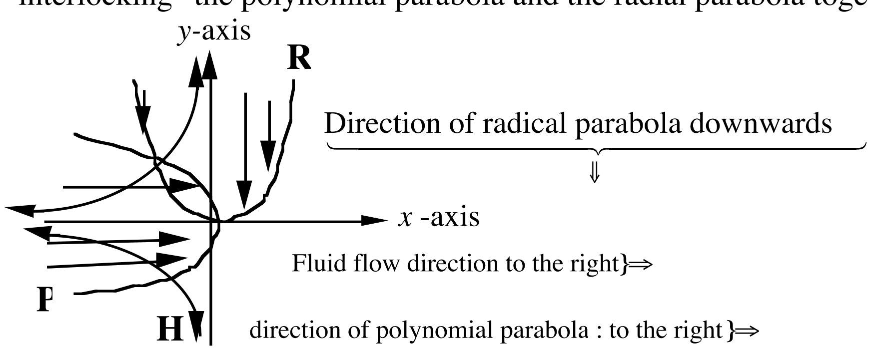
Velocity profile; Laminar flow and Turbulent Flow

For communication purposes, each of the terms containing the even powers x^2, y^2 and z^2 will be called a polynomial parabola, and each of the terms containing the square roots $\pm\sqrt{x}$, $\pm\sqrt{y}$ and $\pm\sqrt{z}$ will be called a radical parabola. Also, each of the terms containing variables in the denominator will be called a hyperbola. The terms, polynomial parabola, radical parabola and hyperbola will be used interchangeably with what produces these profiles.

The fluid flow in the Navier-Stokes solution may be characterized as follows. The x–direction solution consists of linear, parabolic, and hyperbolic terms. The first three terms characterize polynomial parabolas. The characteristic curve for the integral of the x–nonlinear term is a radical parabola. The integral of the y–nonlinear term is similar parabolically to that of the x–nonlinear term. The integral of the z–nonlinear term is a combination of two radical parabolas and a hyperbola.

Turbulence occurrence

In the N-S solution, during fluid flow, the polynomial parabolas, the radical parabolas, and the hyperbolas are present at any speed. The polynomial parabolas are prominent and dominate flow while the radical parabolas are dormant at low speeds, and consequently, the flow is laminar. At a low speed, a radical parabola (or a polynomial parabola susceptible to radicalization) is not active, since the radicand of the parabola is small and consequently, the square root is small. When the speed becomes large, the "x" in $\sqrt{2hg_x x}$ becomes large and therefore the radical parabola becomes active. Note that the radical parabola will be moving at right angles to the direction of fluid flow, the direction of which is also that of the axis of symmetry of the dominating polynomial parabola. In the figure below, assume that flow is in the positive x–direction. Then while the axis of symmetry of the polynomial parabola (**P**) is in the x-direction, the axis of symmetry of the radical parabola (**R**) would be in the direction of the negative y-axis (that is, at right angles to fluid flow direction). Also, note the branches of the hyperbola (**H**) in the second and third quadrants. For each branch of the hyperbola, one end becomes asymptotic to the axis of symmetry of the polynomial parabola, while the other end becomes asymptotic to the axis of symmetry of the radical parabola, and thereby "interlocking" the polynomial parabola and the radial parabola together.



P--Polynomial parabola; R--Radical parabola; H--Hyperbola;

Therefore, the polynomial parabola, the radical parabola and the branches of the hyperbola become connected together to form a system such that any changes in one of them affect the behavior of the others. Any action that increases the velocity and consequently the "x" in $\sqrt{2hg_x x}$ increases the effect of the radical parabola which is in direction at right angles to the fluid flow direction. The radical parabolas would be moving, from various positions, to the left or to the right, at right angles to the direction of fluid flow, noting that the direction of fluid flow is the direction of the polynomial parabola, and at the same time, the hyperbolas will be moving asymptotically to fluid flow direction and asymptotically to direction of the radical parabola as in the figure.

Thus, while the dominating polynomial parabolas are moving in the positive x–direction, and the radical parabolas are moving at right angles to direction of flow, the hyperbolas would be moving asymptotically to the axes of symmetry of the polynomial and radical parabolas, resulting in deviation from laminar flow and producing flows such as vortex flow, swirling flow, and turbulent flow. Imagine the polynomial parabolas pulling to the right, while the radical parabolas are pushing downwards with the hyperbola halves pressing against the axes of the parabolas and the resulting deviation from laminar flow to turbulence and chaos.

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Derivation of the Navier-Stokes equations from equations of motion under gravity and liquid pressure of elementary physics

Motion equations of elementary physics	N-S Terms
$V_f = V_0 + gt$	$fg_x t,$
$V_f^2 = V_0^2 + 2gx$	$\sqrt{2hg_x x}$
$x = V_0 t + \frac{1}{2}gt^2$	$-\frac{a\rho g_x}{2\mu}x^2,$
$V = \sqrt{2gx}$	$\sqrt{2hg_x x}$
$P = \rho gh$ (liquid pressure equation)	$P(x) = d\rho g_x x$

1-2. From $P = \rho gh$, one obtains the N–S terms $\frac{dp}{dx}$ and ρg_x

3. From $V = gt$, one obtains the N–S term $\frac{dV}{dt}$ (note : $\frac{dV}{dt} = g$)

4. From $V = \sqrt{2gx}$, one obtains the N–S term $Vx\frac{dVx}{dx}$

$$(Vx^2 = 2gx; \leftrightarrow 2Vx\frac{dVx}{dx} = 2g \leftrightarrow Vx\frac{dVx}{dx} = g)$$

5. *Adding* the derivative with respect to y, one obtains the N–S term $Vy\frac{dV_x}{dy}$

6. *Adding* the derivative with respect to z, one obtains the N–S term $Vz\frac{dV_x}{dz}$

From 1 to 6, one obtains the terms

$$\frac{dp}{dx}, \rho g_x, \frac{dV}{dt}, Vx\frac{dV}{dx}, Vy\frac{dV_x}{dy}, Vz\frac{dV_x}{dz},$$

Adding the viscous terms $-\mu\frac{\partial^2 V_x}{\partial x^2}$, $-\mu\frac{\partial^2 V_x}{\partial y^2}$, $-\mu\frac{\partial^2 V_x}{\partial z^2}$ one obtains

$$\frac{dp}{dx}, \rho g_x, \frac{dV}{dt}, Vx\frac{dV}{dx}, Vy\frac{dV_x}{dy}, Vz\frac{dV_x}{dz}, -\mu\frac{\partial^2 V_x}{\partial x^2}, -\mu\frac{\partial^2 V_x}{\partial y^2}, -\mu\frac{\partial^2 V_x}{\partial z^2}$$

Introducing ρ and putting the terms into an equation, one obtains the x-direction Navier-Stokes Equation

$$-\mu\frac{\partial^2 V_x}{\partial x^2} - \mu\frac{\partial^2 V_x}{\partial y^2} - \mu\frac{\partial^2 V_x}{\partial z^2} + \frac{\partial p}{\partial x} + \rho\frac{\partial V_x}{\partial t} + \rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z} = \rho g_x$$

The above equation is the nine-member supreme equation of incompressible fluid flow.

Overall Conclusion

The Navier-Stokes (N-S) equations in 3-D have been solved analytically. It was also shown that without gravity forces on earth, there would be no incompressible fluid flow on earth as is known. Note that so far as the general solutions of the N-S equations are concerned, one needs not find the specific values of the ratio terms involved. The N-S solutions were compared to the equations of motion under gravity and liquid pressure of elementary physics, and it was found out that, except for the constants involved, the N-S solutions are very similar or identical to the equations of motion and liquid pressure of elementary physics. Such agreement shows that the N--S equations were properly solved. It could be stated that the solutions of the N-S equations have existed since the time the equations of motion and liquid pressure of elementary physics were derived. Insights into the solutions include how the polynomial parabolas, the radical parabolas, and the hyperbolas interact to produce turbulent flow. Also, the *x*-direction N-S equation was derived from the equations of motion under gravity and liquid pressure of elementary physics. Finally, for any fluid flow design, one should always maximize the role of gravity for cost-effectiveness, durability, and dependability. Perhaps, Newton's law for fluid flow should read "Sum of everything else equals ρg" ; and this would imply that the other terms of the N-S equation divide the gravity term in a definite ratio, and each term utilizes gravity to function.

Uniqueness of the solution of the Navier-Stokes equation on
When each term of the linearized Navier-Stokes equation (see viXra:1512.0334) was made subject of the N-S equation, only the equation with the gravity term as the subject of the equation produced a solution. Similarly, the solution of the Navier-Stokes equation solution is unique.

About the solutions of the N-S Equations
1. In the CMI requirements paper, it is suggested that one can assume that gravity is zero. From the author's solutions of the N-S papers (See viXra:1512.0334), gravity cannot be zero, otherwise, there would be no fluid flow.. If one assumes that gravity, g, is zero, then one should also assume that $\frac{\partial p}{\partial x} = 0$ in the N-S equation, since in elementary physics $P = \rho gh$**.** If one is designing oil or water pipelines or water channels, one cannot assume zero gravity, since there would be no fluid flow without gravity.

2. A number of papers sometimes mention periodic solutions of N-S. equations. There are no periodic solutions, but periodic relations, because the integration results do not satisfy the N-S equations completely. Note that on integrating the N-S equations, and obtaining sines and cosines in the integration results, one should not mention "periodic" solutions until one has successfully checked the results for identity in the original equation. In any case, N-S equations have **no periodic** but perhaps, quasiperiodic solutions. . See viXra:1512.0334

After comparison with the equations of motion and liquid pressure of elementary physics, the author believes that tthe Navier-Stokes equations have finally been solved analytically.

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Spin-off: CMI Millennium Prize Problem Requirements Proof

For the Navier-Stokes equations (Original Equations)

Proof of the existence of solutions of the Navier-Stokes equations

From page 10, if $y=0,\ z=0$ in

$$V_{x(x,y,z,t)} = \overbrace{-\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2)+C_1x+C_3y+C_5z+\underbrace{fg_xt}_{\text{continued}}}^{\textbf{Solution to Linear part}} \underbrace{\pm\sqrt{2hg_xx}+\frac{ng_xy}{V_y}+\frac{qg_xz}{V_z}+\frac{\psi_y(V_y)}{V_y}+\frac{\psi_z(V_z)}{V_z}}_{\textbf{solution of Euler equation}}$$

$$P(x)=d\rho g_x x$$

one obtains

$$\boxed{V_x(x,t)=-\frac{\rho g_x}{2\mu}ax^2+C_1x+fg_xt\pm\sqrt{2hg_xx}+C_9\,;\quad P(x)=d\rho g_x x;}$$

$$V_x(x,0)=V_x^0(x)=-\frac{\rho g_x}{2\mu}ax^2+C_1x\pm\sqrt{2hg_xx}+C_9\,;\quad P(x)=d\rho g_x x;$$

$V_x(x,t)=-\frac{\rho g_x}{2\mu}ax^2+C_1x+fg_xt\pm\sqrt{2hg_xx}+C_9\,;\ P(x)=d\rho g_x x$; are solutions of

$-\mu\frac{\partial^2V_x}{\partial x^2}+\frac{\partial p}{\partial x}+\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}=\rho g_x$ (deleting the $y-$ and $z-$terms of (A)), p.8,

Therefore, smooth solutions to the above differential equation exist, and the proof is complete.

Finding $P(x,t)$:

1. $V_x(x,t)=-\frac{\rho g_x}{2\mu}ax^2+C_1x+fg_xt\pm\sqrt{2hg_xx}+C_9;\ P(x)=d\rho g_x x;$ **2.** $\frac{\partial p}{\partial x}=d\rho g;$

$$\frac{dp}{dt}=\frac{dp}{dx}\frac{dx}{dt}$$

$$\frac{dp}{dt}=\frac{dp}{dx}V_x \qquad (\frac{dx}{dt}=V_x)$$

$$\frac{dp}{dt}=d\rho g_x\left(-\frac{\rho g_x}{2\mu}(ax^2)+C_1x\pm\sqrt{2hg_xx}+fg_xt+C_9\right) \qquad (\frac{dp}{dx}=d\rho g_x)$$

$$P(x,t)=\int d\rho g_x\left(-\frac{\rho g_x}{2\mu}(ax^2)+C_1x\pm\sqrt{2hg_xx}+fg_xt+C_9\right)dt$$

$$P(x,t)=d\rho g_x\left(-\frac{a\rho g_x}{2\mu}x^2t+C_1xt\pm t\sqrt{2hg_xx}+\frac{fg_xt^2}{2}+C_9t\right)+C_{10}$$

References: For paper edition of the above paper, see Appendix 9 ((p,245) of the book entitled "Power of Ratios" , Second Edition, by A. A. Frempong, published by Yellowtextbooks.com. Without using ratios or proportion, the author would never be able to split-up the Navier-Stokes equations into sub-equations which were readily integrable. The impediment to solving the Navier-Stokes equations for over 150 years (whether linearized or non-linearized) has been due to finding a way to split-up the equations. Since ratios were the key to splitting the Navier-Stokes equations, and solving them, the solutions have also been published in the " Power of Ratios" book which covers definition of ratio and applications of ratio in mathematics, science, engineering, economics and business fields.

Adonten

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