Sigma J Eng & Nat Sci 8 (1), 2017, 53-64

Publications Prepared for The 4th International
Fuzzy Systems Symposium 2015
4. Uluslararası Bulanık Sistemler Sempozyumu 2015
için Hazırlanan Yayınlar

Research Article / Araştırma Makalesi

A DECISION MAKING METHOD BY COMBINING TOPSIS AND GREY RELATION METHOD UNDER FUZZY SOFT SETS

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Received/Geliş: 07.09.2016 Accepted/Kabul: 24.11.2016

ABSTRACT

In this study, we first introduce the fuzzy sets, soft sets, fuzzy soft sets and their related properties. We then present the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) that is one of classical Multiple Attribute Decision Making (MADM) methods. We also present the Grey Relation Method. In the main part of this study, we extend the TOPSIS method on the fuzzy soft set theory to construct a decision making method to deal with problems that contain uncertainties. To make it we combine the TOPSIS and The Grey Relational Analysis (GRA) under fuzzy soft sets. We finally give an illustrative application for drug selection.
Keywords: Soft sets, fuzzy sets, fuzzy soft sets, TOPSIS, multi-criteria decision making, grey relational analysis, drug selection.

1. INTRODUCTION

There are a lot of uncertainties in many areas such as business, service, management, military, economy, engineering, etc. To deal with this uncertainty, there some mathematical models. One of them is fuzzy set theory is suggested by Zadeh [34] in 1965. The other one is soft set theory is proposed by Molodtsov [24] in 1999. The Molodtsov's theory was a new approach different from the Zadeh's theory for modeling uncertainty. Some operations of soft sets and their properties were defined by Maji et al. [22]. Then some modifications of operations of soft sets and their properties were given by some researchers such as Ali et al. [1], Çağman and Enginoğlu [7], Zhu and Wen [36], Çağman [8]. In 2001, Maji et al. [21] defined the concept of fuzzy soft set and fuzzy soft set operations that are a generalization of Molodtsov's soft set. Then Roy and Maji [29] applied the fuzzy soft sets to a decision making problem. Majumdar and Samanta [23] defined the generalized fuzzy soft sets and investigated their properties. Zhou et al. [35] suggested the generalized interval valued fuzzy soft sets and investigated their properties.

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TOPSIS being one of classical MADM methods such as ELECTRE [28], VIKOR [27], PROMETHEE [2], developed by Hwang and Yoon [17]. Chen et al. [4] extended the TOPSIS method for solving Multi Criteria Decision Making (MCDM) problems in fuzzy environment. Boran et al. [3] developed TOPSIS method for MCDM problems based on intuitionistic fuzzy sets. Chi and Liu [6] extended TOPSIS to interval neutrosophic sets, and with respect to the MADM problems in which the attribute weights were unknown and the attribute values take the form of interval neutrosophic sets. Eraslan [13] gave a decision making method by using TOPSIS on soft set theory.

The Grey theory, proposed by Deng [9] in 1982, similar to fuzzy set theory, is an effective mathematical means to deal with systems analysis characterized by incomplete information. Grey theory is widely applied in fields such as systems analysis, data processing, modeling and prediction, as well as control and decision-making [10], [11], [12], [31]. The GRA is part of grey system theory, which is suitable for solving problems with complicated interrelationships between multiple factors and variables [25]. The GRA has been successfully applied in solving a variety of MADM problems, such as the hiring decision [26], the restoration planning for power distribution systems [5], the inspection of integrated-circuit marking process [18], the modeling of quality function deployment [37], the detection of silicon wafer slicing defects [20], etc.

The GRA has been widely used to solve the uncertainty problems under the discrete data and incomplete information [11], [12]. In addition, the GRA method is one of the very popular methods to analyze various relationships among the discrete data sets and make decisions in multiple attribute situations. The major advantages of the GRA method are that the results are based on the original data, the calculations are simple and straightforward and it is one of the best methods to make decisions under business environment [16].

The GRA and TOPSIS [17], [19], [33], both use the idea of minimizing a distance function. Feng and Wang [15], applied grey relation analysis to select representative criteria among a large set of available choices, and then used TOPSIS for outranking. The aim of this paper is to extended the concept of the grey relation based on the concepts of TOPSIS to solve the multiple criteria decision making problem for selection of drugs problem and it appears to be more appropriate.

The rest of the paper is organized as follows: in the Section 2, we present briefly some definitions and properties required in the other sections of study. In Section 3, we give an group decision making method combining TOPSIS and GRA method under fuzzy soft environment. In Section 4, we present an application of suggested method to a real life problem containing drug selection. In Section 5, conclusions are presented.

This work is presented in The 4th International Fuzzy Systems Symposium (FUZZYSS15), 5-6 November 2015.

2. PRELIMINARY

In this section, we summarize the preliminary TOPSIS method and definitions which are fuzzy set [34], soft set [24], [7], fuzzy soft set and their results.

Definition II.1. *[34] Let U be an initial universe set. A fuzzy set X over U is a set defined by a function μ_X representing a mapping*

$$\mu_X : U \to [0, 1]$$

Here, μ_X called membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set X. Thus, a fuzzy set X over U can be represented as follows,

$$X : \{(\mu_X(u)/u) : u \in U;\ \mu_X(u) \in [0,1]\}$$

Note that the set of all the fuzzy sets over U will be denoted by F(U).

Definition II.2. *[24] Let U and X be two non empty set and P(U) is the power set of U. Then, a soft set f over U is a function defined by*
$f : X \to P(U)$
where U refer to an initial universe and X is a set of parameters. In other words, the soft set is a parametrized family of subsets of the set U. A soft set over U can be represented by the set of ordered pairs
$f = \{(x; f(x)) : x \in X\}$

Note that the set of all soft sets defined from X to $P(U)$ will be denoted by S_X^U.

Definition II.3. *[7] Let* $f, g \in S_X^U$. *Then,*

- *f is called an empty soft set, denoted by* Φ_X, *if* $f(x) = \emptyset$, *for all* $x \in X$.
- *f is called a universal soft set, denoted by* $f_{\tilde{X}}$, *if* $f(x) = U$, *for all* $x \in X$.
- *The set* $Im(f) = \{f(x) : x \in X\}$ *is called image of f*.
- *f is a soft subset of g, denoted by* $f \,\tilde{\subseteq}\, g$, *if* $f(x) \subseteq g(x)$, *for all* $x \in X$.
- *f and g are soft equal, denoted by* $f = g$, *if and only if* $f(x) = g(x)$ *for all* $x \in X$.
- *the set* $(f \,\tilde{\cup}\, g)(x) = f(x) \cup g(x)$ *for all* $x \in X$ *is called union of f and g.*
- *the set* $(f \,\tilde{\cap}\, g)(x) = f(x) \cap g(x)$ *for all* $x \in X$ *is called intersection of f and g.*
- *the set* $f^c(x) = U \setminus f(x)$ *for all* $x \in X$ *is called complement of f.*

Definition II.4. *[21] Let U be an initial universe set, X be a set of all parameters, μ be a fuzzy set over U for every* $x \in X$ *and F(U) denote the set of all fuzzy sets in U. Then, a fuzzy soft set γ over U is defined by a function γ representing a mapping*

$$\gamma : X \to F(U) \text{ such that } \gamma(x) = \theta \text{ if } x \notin X$$

Here, for every $x \in X$, $\gamma(x)$ *is a fuzzy set over U and it is called fuzzy value set of parameter x-element of the fs-set. Thus, an fs-set γ over U can be represented by the set of ordered pairs*

$$\gamma = \{(x; \gamma(x)) : x \in X; \gamma(x) \in F(U)\}$$

Note that from now on the sets of all fs-sets over U will be denoted by FS(U).

TOPSIS Method

The operations within the TOPSIS process include: decision matrix normalization, distance measures, and aggregation operators [30]. For more detail of TOPSIS, we refer to the earlier studies [17], [32], [13]. The TOPSIS process is carried out as

Step 1. Constructing of decision matrix D.
Step 2. Creating of standard (normalized) decision matrix R.
Step 3. Creating the weighted normalized decision matrix V.
Step 4. Determining of positive ideal solution (PIS), A^+ and negative ideal solution (NIS), A^-.
Step 5. Calculating of separation measurements of positive ideal (S_i^+) and the negative ideal (S_i^-) solutions.
Step 6. Calculating of relative closeness (C_i^+) of alternatives to the ideal solution.
Step 7. Ranking the preference order.

3. COMBINING TOPSIS AND GREY RELATION METHOD

In this section, we combine TOPSIS and grey relation method to determine separation measurements for each parameter. Therefore, we calculate the separation measurements in step 7 by using formula in [31] defined by

$$r_{ij}^{+} = r(A^{+}(j), A_i(j)) = \frac{min_i min_j |A^{+}(j) - A_i(j)| + \zeta\ max_i max_j |A^{+}(j) - A_i(j)|}{|A^{+}(j) - A_i(j)| + \zeta\ max_i max_j |A^{+}(j) - A_i(j)|} \quad (1)$$

$$r_{ij}^{-} = r(A^{-}(j), A_i(j)) = \frac{min_i min_j |A^{-}(j) - A_i(j)| + \zeta\ max_i max_j |A^{-}(j) - A_i(j)|}{|A^{-}(j) - A_i(j)| + \zeta\ max_i max_j |A^{-}(j) - A_i(j)|} \quad (2)$$

here, $r(A^+(j);A_i(j))$ and $r(A^-(j);A_i(j))$ are the grey relation coefficients of each alternative to ideal and negative ideal solutions, respectively. For simplicity of representation, $|A^+(j) - A_i(j)|$ and $|A^-(j) - A_i(j)|$ will be presented Δ_i^+ *and* Δ_i^-, respectively.

After calculating $r(A^+(j), A_i(j))$ and $r(A^-(j), A_i(j))$, separation measurements will be calculated according to the following Formula

$$S_i^+ = \frac{1}{n}\sum_{j=1}^{n} r_{ij}^+ \quad , \quad for \quad i = 1, 2, \cdots, m \quad (3)$$

$$S_i^- = \frac{1}{n}\sum_{j=1}^{n} r_{ij}^- \quad , \quad for \quad i = 1, 2, \cdots, m. \quad (4)$$

Now we will give the operations of proposed method. The main procedure of this method is presented with the following steps:

Step 1. *Defining of problem*.

Step *2. Constructing of weighed fuzzy parameter matrix D with choosing linguistic rating from Table 1.*

Table 1. Linguistic Terms for Evaluation of Parameters.

Linguistic Terms	FVs
Very Good / Very Important (VG/VI)	0.98
Good / Important(G /I)	0.88
Fair / Medium(F/M)	0.50
Bad / Unimportant (B / UI)	0.25
Very Bad / Very Unimportant (VB/VUI)	0.05

Step 3. *Constructing of weighted normalized fuzzy parameter matrix R and forming weighed vector* $W = (W_1, W_2, ..., W_n)$.

Step 4. *Constructing fuzzy decision matrices* D_k *for each decision makers and building of fuzzy average decision matrix* V .

Step 5. *Constructing of weighed fuzzy decision matrix V.*

Step 6. *Finding of fuzzy valued positive ideal solution (FVPIS)and fuzzy valued negative ideal solution (FVNIS).*

Step 7. *Calculating of the separation measurement* (S_i^+ , S_i^-) *for each parameter according to formula 3, 4.*

Step 8. *Calculating of the relative closeness* (C_i^+) *of alternative to the ideal solution.*

Step 9. Ranking the preference order.

4. APPLICATION

In this section, we have presented an application by using the algorithm of this new group decision making method. We can solve the following problem step by step as follows:

Step 1. Defining the problem.

Assume that a researcher group wants to decide a drug which is best for cancer. There are eight drugs which are the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$. The researchers take into consideration a set of parameters, $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$. The parameters x_i (i = 1, 2, 3, 4, 5, 6, 7) stand for "anti-inflammatory", "degreaser", "painkiller", "increasing kidney load", "increasing burden of liver", "negatively affect pregnancy" and "weaken the heart muscle", respectively. Then we can give the following examples.

Suppose that four researchers (decision-makers) to chose the best drug. Firstly, each researcher has to consider their own set of parameters. Then, they can construct their fuzzy soft sets. Next, by using TOPSIS on fuzzy soft set theory decision making method we select a candidate on the basis for the sets of decision makers parameters. Finally, they applies the following steps:

Assume that decision makers D_1, D_2, D_3 and D_4 construct fuzzy soft sets, respectively as follows,

$$\begin{aligned}
&\gamma_X^{(1)} = \\
&\Big\{(x_1, \{.9/u_1, 1/u_2, .5/u_3, .8/u_4, .7/u_5, .4/u_6, .6/u_7, .1/u_8\}), \\
&(x_2, \{.6/u_1, .5/u_2, .3/u_3, .1/u_4, .7/u_5, .2/u_6, .9/u_7, .8/u_8\}), \\
&(x_3, \{.5/u_1, .3/u_2, .9/u_3, 1/u_4, .8/u_5, .7/u_6, .7/u_7, .6/u_8\}), \\
&(x_4, \{.1/u_1, .8/u_2, .5/u_3, .2/u_4, .1/u_5, .5/u_6, .3/u_7, .4/u_8\}), \\
&(x_5, \{.5/u_1, .1/u_2, .2/u_3, .6/u_4, .1/u_5, .4/u_6, .5/u_7, .7/u_8\}), \\
&(x_6, \{.3/u_1, .2/u_2, .4/u_3, .5/u_4, .6/u_5, .1/u_6, .5/u_7, .2/u_8\}), \\
&(x_7, \{.4/u_1, .3/u_2, .3/u_3, .5/u_4, .1/u_5, .2/u_6, .5/u_7, .2/u_8\})\Big\}
\end{aligned}$$

$$\begin{aligned}
&\gamma_X^{(2)} = \\
&\Big\{(x_1, \{.1/u_1, .6/u_2, .4/u_3, .7/u_4, .8/u_5, .5/u_6, 1/u_7, .9/u_8\}), \\
&(x_2, \{.8/u_1, .9/u_2, .2/u_3, .7/u_4, .1/u_5, .3/u_6, .5/u_7, .6/u_8\}), \\
&(x_3, \{.6/u_1, .7/u_2, .7/u_3, .8/u_4, 1/u_5, .9/u_6, .3/u_7, .5/u_8\}), \\
&(x_4, \{.4/u_1, .3/u_2, .5/u_3, .1/u_4, .2/u_5, .5/u_6, .8/u_7, .1/u_8\}), \\
&(x_5, \{.7/u_1, .5/u_2, .4/u_3, .1/u_4, .6/u_5, .2/u_6, .1/u_7, .5/u_8\}), \\
&(x_6, \{.2/u_1, .5/u_2, .1/u_3, .6/u_4, .5/u_5, .4/u_6, .2/u_7, .3/u_8\}), \\
&(x_7, \{.2/u_1, .5/u_2, .2/u_3, .1/u_4, .5/u_5, .3/u_6, .3/u_7, .4/u_8\})\Big\}
\end{aligned}$$

$$\gamma_X^{(3)} = \Big\{(x_1, \{.4/u_1, .7/u_2, .8/u_3, .5/u_4, 1/u_5, .9/u_6, .1/u_7, .6/u_8\}), (x_2, \{.2/u_1, .7/u_2, .1/u_3, .3/u_4, .5/u_5, .6/u_6, .8/u_7, .9/u_8\}), (x_3, \{.7/u_1, .8/u_2, 1/u_3, .9/u_4, .3/u_5, .5/u_6, .6/u_7, .7/u_8\}), (x_4, \{.5/u_1, .1/u_2, .2/u_3, .5/u_4, .8/u_5, .1/u_6, .4/u_7, .3/u_8\}), (x_5, \{.4/u_1, .1/u_2, .6/u_3, .2/u_4, .1/u_5, .5/u_6, .7/u_7, .5/u_8\}), (x_6, \{.1/u_1, .6/u_2, .5/u_3, .4/u_4, .2/u_5, .3/u_6, .2/u_7, .5/u_8\}), (x_7, \{.2/u_1, .1/u_2, .5/u_3, .3/u_4, .3/u_5, .4/u_6, .2/u_7, .5/u_8\})\Big\}$$

$$\gamma_X^{(4)} = \Big\{(x_1, \{.5/u_1, 1/u_2, .9/u_3, 1/u_4, .6/u_5, .4/u_6, .7/u_7, .8/u_8\}), (x_2, \{.3/u_1, .5/u_2, .6/u_3, .8/u_4, .9/u_5, .2/u_6, .7/u_7, .1/u_8\}), (x_3, \{.9/u_1, .3/u_2, .5/u_3, .6/u_4, .7/u_5, .7/u_6, .8/u_7, 1/u_8\}), (x_4, \{.5/u_1, .8/u_2, .1/u_3, .4/u_4, .3/u_5, .5/u_6, .1/u_7, .2/u_8\}), (x_5, \{.2/u_1, .1/u_2, .5/u_3, .7/u_4, .1/u_5, .5/u_6, .4/u_7, .1/u_8\}), (x_6, \{.4/u_1, .2/u_2, .3/u_3, .2/u_4, .5/u_5, .1/u_6, .6/u_7, .5/u_8\}), (x_7, \{.3/u_1, .3/u_2, .4/u_3, .2/u_4, .5/u_5, .2/u_6, .1/u_7, .5/u_8\})\Big\}$$

Step 2. Weighed fuzzy parameter matrix $D = \lfloor d_{ij} \rfloor_{m\times n}$ constructing as below,

$$D = \begin{bmatrix} 0.50 & 0.50 & 0.88 & 0.98 & 0.98 & 0.25 & 0.88 \\ 0.25 & 0.25 & 0.50 & 0.88 & 0.98 & 0.25 & 0.50 \\ 0.50 & 0.25 & 0.88 & 0.98 & 0.98 & 0.50 & 0.88 \\ 0.25 & 0.50 & 0.98 & 0.98 & 0.98 & 0.50 & 0.98 \end{bmatrix}$$

Step 3. Weighted normalized fuzzy parameter matrix and weighed vector W are obtained as follow,

$$R = \begin{bmatrix} 0.63 & 0.63 & 0.53 & 0.51 & 0.50 & 0.32 & 0.53 \\ 0.32 & 0.32 & 0.30 & 0.46 & 0.50 & 0.32 & 0.30 \\ 0.63 & 0.32 & 0.53 & 0.51 & 0.50 & 0.63 & 0.53 \\ 0.32 & 0.63 & 0.59 & 0.51 & 0.50 & 0.63 & 0.59 \end{bmatrix}$$

$$W = [0.140\ 0.140\ 0.144\ 0.146\ 0.147\ 0.140\ 0.144]$$

Step 4. Fuzzy decision matrices can be constructed by decision makers as follows,

$$D_1 = \begin{bmatrix} 0.9 & 0.6 & 0.5 & 0.1 & 0.5 & 0.3 & 0.4 \\ 1.0 & 0.5 & 0.3 & 0.8 & 0.1 & 0.2 & 0.3 \\ 0.5 & 0.3 & 0.9 & 0.5 & 0.2 & 0.4 & 0.3 \\ 0.8 & 0.1 & 1.0 & 0.2 & 0.6 & 0.5 & 0.5 \\ 0.7 & 0.7 & 0.8 & 0.1 & 0.1 & 0.6 & 0.1 \\ 0.4 & 0.2 & 0.7 & 0.5 & 0.4 & 0.1 & 0.2 \\ 0.6 & 0.9 & 0.7 & 0.3 & 0.5 & 0.5 & 0.5 \\ 0.1 & 0.8 & 0.6 & 0.4 & 0.7 & 0.2 & 0.2 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0.1 & 0.8 & 0.6 & 0.4 & 0.7 & 0.2 & 0.2 \\ 0.6 & 0.9 & 0.7 & 0.3 & 0.5 & 0.5 & 0.5 \\ 0.4 & 0.2 & 0.7 & 0.5 & 0.4 & 0.1 & 0.2 \\ 0.7 & 0.7 & 0.8 & 0.1 & 0.1 & 0.6 & 0.1 \\ 0.8 & 0.1 & 1.0 & 0.2 & 0.6 & 0.5 & 0.5 \\ 0.5 & 0.3 & 0.9 & 0.5 & 0.2 & 0.4 & 0.3 \\ 1.0 & 0.5 & 0.3 & 0.8 & 0.1 & 0.2 & 0.3 \\ 0.9 & 0.6 & 0.5 & 0.1 & 0.5 & 0.3 & 0.4 \end{bmatrix}$$

$$D_3 = \begin{bmatrix} 0.4 & 0.2 & 0.7 & 0.5 & 0.4 & 0.1 & 0.2 \\ 0.7 & 0.7 & 0.8 & 0.1 & 0.1 & 0.6 & 0.1 \\ 0.8 & 0.1 & 1.0 & 0.2 & 0.6 & 0.5 & 0.5 \\ 0.5 & 0.3 & 0.9 & 0.5 & 0.2 & 0.4 & 0.3 \\ 1.0 & 0.5 & 0.3 & 0.8 & 0.1 & 0.2 & 0.3 \\ 0.9 & 0.6 & 0.5 & 0.1 & 0.5 & 0.3 & 0.4 \\ 0.1 & 0.8 & 0.6 & 0.4 & 0.7 & 0.2 & 0.2 \\ 0.6 & 0.9 & 0.7 & 0.3 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$D_4 = \begin{bmatrix} 0.5 & 0.3 & 0.9 & 0.5 & 0.2 & 0.4 & 0.3 \\ 1.0 & 0.5 & 0.3 & 0.8 & 0.1 & 0.2 & 0.3 \\ 0.9 & 0.6 & 0.5 & 0.1 & 0.5 & 0.3 & 0.4 \\ 1.0 & 0.8 & 0.6 & 0.4 & 0.7 & 0.2 & 0.2 \\ 0.6 & 0.9 & 0.7 & 0.3 & 0.1 & 0.5 & 0.5 \\ 0.4 & 0.2 & 0.7 & 0.5 & 0.5 & 0.1 & 0.2 \\ 0.7 & 0.7 & 0.8 & 0.1 & 0.4 & 0.6 & 0.1 \\ 0.8 & 0.1 & 1.0 & 0.2 & 0.1 & 0.5 & 0.5 \end{bmatrix}$$

and fuzzy average decision matrix build as

$$V = \begin{bmatrix} 0.48 & 0.48 & 0.68 & 0.38 & 0.45 & 0.25 & 0.28 \\ 0.83 & 0.65 & 0.53 & 0.50 & 0.20 & 0.38 & 0.30 \\ 0.65 & 0.30 & 0.78 & 0.33 & 0.43 & 0.33 & 0.35 \\ 0.75 & 0.48 & 0.83 & 0.30 & 0.50 & 0.43 & 0.28 \\ 0.78 & 0.55 & 0.70 & 0.35 & 0.23 & 0.45 & 0.35 \\ 0.55 & 0.33 & 0.70 & 0.40 & 0.40 & 0.23 & 0.28 \\ 0.60 & 0.73 & 0.60 & 0.40 & 0.43 & 0.38 & 0.28 \\ 0.60 & 0.60 & 0.70 & 0.25 & 0.45 & 0.38 & 0.40 \end{bmatrix}$$

Step 5. Weighed fuzzy decision matrix $\mathcal{V}$ is constructed as below,

$$\mathcal{V} = \begin{bmatrix} 0.07 & 0.07 & 0.10 & 0.06 & 0.07 & 0.04 & 0.04 \\ 0.12 & 0.09 & 0.08 & 0.07 & 0.03 & 0.05 & 0.04 \\ 0.09 & 0.04 & 0.11 & 0.05 & 0.06 & 0.05 & 0.05 \\ 0.11 & 0.07 & 0.12 & 0.04 & 0.07 & 0.06 & 0.04 \\ 0.11 & 0.08 & 0.10 & 0.05 & 0.03 & 0.06 & 0.05 \\ 0.08 & 0.05 & 0.10 & 0.06 & 0.06 & 0.03 & 0.04 \\ 0.08 & 0.10 & 0.09 & 0.06 & 0.06 & 0.05 & 0.04 \\ 0.08 & 0.08 & 0.10 & 0.04 & 0.07 & 0.05 & 0.06 \end{bmatrix}$$

Step 6. Fuzzy valued positive ideal solution (FV-PIS) and fuzzy valued negative ideal solution (FV-NIS) can be obtained as below,

$A^+ = \{A^+(1) = 0.12, A^+(2) = 0.10, A^+(3) = 0.12,$
$A^+(4) = 0.04, A^+(5) = 0.03, A^+(6) = 0.03, A^+(7) = 0.04\}$

$A^- = \{A^-(1) = 0.07, A^-(2) = 0.04, A^-(3) = 0.08,$
$A^-(4) = 0.07, A^-(5) = 0.07, A^-(6) = 0.06, A^-(7) = 0.06\}$

Step 7. Grey relation coefficient of each alternative to the ideal $r(A^+(j);A_i(j))$ and the negative ideal $r(A^-(j), A_i(j))$ solution can be obtained as below. We will briefly denote $p = min\,\Delta_i^+$, $q = min\,\Delta_i^-$, $P = max\,\Delta_i^+$, $Q = max\,\Delta_i^-$, $p^* = min\{min\,\Delta_i^+\}$, $q^* = min\{min\,\Delta_i^-\}$, $P^* = max\{max\,\Delta_i^+\}$ and $Q^* = max\{max\,\Delta_i^-\}$.

PIS									
	u_1	u_2	u_3	u_4	u_5	u_6	u_7	p	P
Δ_1^+	.05	.03	.02	.02	.04	.01	0	0	.05
Δ_2^+	0	.01	.04	.03	0	.02	0	0	.04
Δ_3^+	.03	.06	.01	.01	.03	.02	.01	0	.06
Δ_4^+	.01	.03	0	0	.04	.03	0	0	.04
Δ_5^+	.01	.02	.02	.01	0	.03	.01	0	.03
Δ_6^+	.04	.05	.02	.02	.03	0	0	0	.05
Δ_7^+	.04	0	.03	.02	.03	.02	0	0	.04
Δ_8^+	.04	.02	.02	0	.04	.02	.02	0	.04
p^*								0	
P^*									.06

From the above chart, all grey relational coefficients can be calculated by Eq. (1). In the example, $\zeta = 0.5$. For ideal solution, the entire results for the grey relational coefficient is shown in matrix as

$$r(A^+(j), A_i(j)) = \begin{bmatrix} 0.38 & 0.50 & 0.60 & 0.60 & 0.43 & 0.75 & 1.00 \\ 1.00 & 0.75 & 0.43 & 0.50 & 1.00 & 0.60 & 1.00 \\ 0.50 & 0.33 & 0.75 & 0.75 & 0.50 & 0.60 & 0.75 \\ 0.75 & 0.50 & 1.00 & 1.00 & 0.43 & 0.50 & 1.00 \\ 0.75 & 0.60 & 0.60 & 0.75 & 1.00 & 0.50 & 0.75 \\ 0.43 & 0.38 & 0.60 & 0.60 & 0.50 & 1.00 & 1.00 \\ 0.43 & 1.00 & 0.50 & 0.60 & 0.50 & 0.60 & 1.00 \\ 0.43 & 0.60 & 0.60 & 1.00 & 0.43 & 0.60 & 0.60 \end{bmatrix}$$

and

NIS									
	u_1	u_2	u_3	u_4	u_5	u_6	u_7	q	Q
Δ_1^-	0	.03	.02	.01	0	.02	.02	0	.03
Δ_2^-	.05	.05	0	0	.04	.01	.02	0	.05
Δ_3^-	.02	0	.03	.02	.01	.01	.01	0	.03
Δ_4^-	.04	.03	.04	.03	0	0	.02	0	.04
Δ_5^-	.04	.04	.02	.02	.04	0	.01	0	.04
Δ_6^-	.01	.01	.02	.01	.01	.03	.02	.01	.03
Δ_7^-	.01	.06	.01	.01	.01	.01	.02	.01	.06
Δ_8^-	.01	.04	.02	.03	0	.01	0	0	.04
q^*								0	
Q^*									.06

Similarly, for negative ideal solution, we have the entire results for the grey elational coefficient is shown in matrix as

$$r(A^-(j), A_i(j)) = \begin{bmatrix} 1.00 & 0.50 & 0.60 & 0.75 & 1.00 & 0.60 & 0.60 \\ 0.38 & 0.38 & 1.00 & 1.00 & 0.43 & 0.75 & 0.60 \\ 0.60 & 1.00 & 0.50 & 0.60 & 0.75 & 0.75 & 0.75 \\ 0.43 & 0.50 & 0.43 & 0.50 & 1.00 & 1.00 & 0.60 \\ 0.43 & 0.43 & 0.60 & 0.60 & 0.43 & 1.00 & 0.75 \\ 0.75 & 0.75 & 0.60 & 0.75 & 0.75 & 0.50 & 0.60 \\ 0.75 & 0.33 & 0.75 & 0.75 & 0.75 & 0.75 & 0.60 \\ 0.75 & 0.43 & 0.60 & 0.50 & 1.00 & 0.75 & 1.00 \end{bmatrix}$$

After calculating $r(A^+(j), A_i(j))$ *and* $r(A^-(j), A_i(j))$, the connection between i^{th} alternative and reference number sequence is calculated according to formula 3, 4. Then S_i^+ *and* S_i^-, for $i \in \{1, 2, 3, 4, 5, 6, 7, 8\}$, we have

$$\begin{aligned}
S_1^+ &= 0.6086, & S_1^- &= 0.7214\\
S_2^+ &= 0.7543, & S_2^- &= 0.6486\\
S_3^+ &= 0.5971, & S_3^- &= 0.7071\\
S_4^+ &= 0.7400, & S_4^- &= 0.6371\\
S_5^+ &= 0.7071, & S_5^- &= 0.6057\\
S_6^+ &= 0.6443, & S_6^- &= 0.6714\\
S_7^+ &= 0.6614, & S_7^- &= 0.6686\\
S_8^+ &= 0.6086, & S_8^- &= 0.7186
\end{aligned}$$

Step 8. Relative closeness of alternatives to the ideal solution is calculated as follows,

$$C_1^+ = \frac{S_1^-}{S_1^- + S_1^+} = \frac{0.7214}{0.7214 + 0.6086} = 0.542$$

Similarly,

$$C_2^+ = 0.462, \ C_3^+ = 0.542, \ C_4^+ = 0.463, C_5^+ = 0.461, \ C_6^+ = 0.510, \ C_7^+ = 0.503, \ C_8^+ = 0.541$$

Step 9. Ranking the preference order is $u_5 < u_2 < u_4 < u_7 < u_6 < u_8 < u_3 = u_1$.

5. CONCLUSION

In this paper, we have presented an application using new group decision making method [14]. Then, we combine the GRA based on the concepts of TOPSIS to evaluate and select the best alternative. The degree of grey relation between every alternative and FV-PIS, FV-NIS is calculated. An illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness. As can be seen from the examples given above, u_1, u_3 drugs seem to be more useful for cancer through eight different drugs. However, considering the side effects of the drug u_1, u_3 give the greatest harm to the patient. The drug u_5 provide less benefit and more damage in terms of benefits, but u_1, u_3 drug is less damage and more benefit to the body. This method can be successfully worked. It can be applied to decision making problems of many fields that contain uncertainty. Finally, the approach should be more comprehensive in the future to solve the related problems and a large number of examples could be recommended for test in future studies.

REFERENCES / KAYNAKLAR

[1] Ali, M. I., Feng, F., Liu, X., Min, W. K. and Shabir, M., On some new operations in soft set theory, Comput. Math. Appl., 57, (2009), 1547-1553.

[2] Brans, J. P., Vinvke, P. and Mareschal, B., How to select and how to rank projects: the PROMETHEE method, European Journal of Operation Research 24, (1986), 228-238.

[3] Boran, F. E., Genc, S., Kurt, M. and Akay, D., A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method, Expert Systems with Applications, 36(8), (2009), 11363-11368.

[4] Chen, C. T., Extensions of the TOPSIS for group decision making under fuzzy environment, Fuzzy Sets and Systems 114, (2000), 1-9.

[5] Chen, W. H., Distribution system restoration using the hybrid fuzzy-grey method. IEEE Transactions on Power Systems, 20,(2005), 199-205.

[6] Chi, P. and Liu, P., An extended TOPSIS method for the multi-attribute decision making problems on interval neutrosophic set, Neutrosophic Sets and Systems 1, (2013), 63-70.

[7] Çağman, N. and Enginolu, S., Soft set theory and uni-int decision making, European Journal of Operational Research, 207, (2010), 848-855.

[8] Çağman, N., Contributions to the theory of soft sets, Journal of New Results in Science, 4, (2014), 33-41.

[9] Deng, J. L., Control problems of grey systems, Systems and Controls Letters 5, (1982), 288-294.

[10] Deng, J., Grey System Fundamental Method, Huazhong University of Science and Technology, Wuhan, China, (1985).

[11] Deng, J., Grey System Book, Science and Technology Information Services, Windsor, (1988).

[12] Deng, J., Introduction to grey theory system, The Journal of Grey System 1 (1), (1989), 1-24.

[13] Eraslan, S., A Decision Making Method via TOPSIS on Soft Sets, Journal of New Results in Science,(8), (2015), 57-71.

[14] Eraslan, S. and Karaaslan, F., A Group Decision Making Method Based on TOPSIS Under Fuzzy Soft Environment, Journal of New Theory, (3), (2015), 30-40.

[15] Feng, C. M. and Wang, R. T., Considering the financial ratios on the performance evaluation of highway bus industry, Transport Reviews 21 (4), (2001), 449 467.

[16] Hou, J., Grey relational analysis method for multiple attribute decision making in intuitionistic fuzzy setting, Journal of Convergence Information Technology, 10 (5), (2010).

[17] Hwang, C. L. and Yoon, K., Multiple attribute decision making: Methods and applications. New York: Springer-Verlag, (1981).

[18] Jiang, B. C., Tasi, S. L., and Wang, C. C., Machine vision-based grey relational theory applied to IC marking inspection. IEEE Transactions on Semiconductor Manufacturing, 15, (2002), 531-539.

[19] Lai Y. J., Liu, T. Y. and Hwang, C. L., TOPSIS for MODM, European Journal of Operational Research 76 (3), (1994), 486 500.

[20] Lin, C. T., Chang, C. W., and Chen, C. B., The worst ill-conditioned silicon wafer machine detected by using grey relational analysis. International Journal of Advanced Manufacturing Technology, 31, (2006), 388-395.

[21] Maji, P. K., Biswas, R. and Roy, A. R., Fuzzy soft sets, Journal of Fuzzy Mathematics, 9(3), (2001), 589-602.

[22] Maji, P. K., Biswas, R. and Roy, A. R., Soft set theory, Computers and Mathematics with Applications, 45, (2003), 555-562.

[23] Majumdar, P. and Samanta, S. K., Generalised fuzzy soft sets, Computers and Mathematics with Applications, 59(4),(2010), 1425-1432.

[24] Molodtsov, D. A., Soft set theory-first results, Computers and Mathematics with Applications, 37(1), (1999), 19-31.

[25] Mora'n, J., Granada, E., M'guez, J. L., and Porteiro, J., Use of grey relational analysis to assess and optimize small biomass boilers. Fuel Processing Technology, 87, (2006), 123-127.

[26] Olson, D. L., and Wu, D., Simulation of fuzzy multiattribute models for grey relationships. European Journal of Operational Research, 175, (2006), 111-120.

[27] Opricovic, S., Tzeng, G. H., Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS, European Journal of Operation Research 156, (2004), 445-455.

[28] Roy, B., The outranking approach and the foundations of ELECTRE methods, Theory Decision, 31, (1991), 49-73.

[29] Roy, A. R. and Maji, P. K., A fuzzy soft set theoretic approach to decision making problems, Journal of Computational and Applied Mathematics, 203(2), (2007), 412-418.

[30] Shih, H. S., Shyur, H. J. and Lee, E. S., An extension of TOPSIS for group decision making, Mathematical and Computer Modelling 45, (2007), 801-813.

[31] Tzeng, G. H. and Tasur, S. H., The multiple criteria evaluation of grey relation model, The Journal of Grey System 6 (2), (1994), 87-108.

[32] Yoon, K., A reconciliation among discrete compromise situations, Journal of Operational Research Society 38, (1987), 277-286. doi:10.1057/jors.1987.44.

[33] Yoon, K. and Hwang, C. L., Multiple Attribute Decision Making: An Introduction, Sage, Thousand Oaks, CA, (1995).

[34] Zadeh, L. A., Fuzzy Sets, Information and Control, 8, (1965), 338-353.

[35] Zhou, X., Li, Q. and Guo, L., On generalized interval-valued fuzzy soft sets, Journal of Applied Mathematics, vol. (2012), Article ID 479783, 18 pages.

[36] Zhu, P., Wen, Q., Operations on Soft Sets Revisited, Journal of Applied Mathematics, (2013), 1-7.

[37] Wu, H., A comparative study of using grey relational analysis in multiple attribute decision making problems. Quality Engineering, 15, (2002), 209- 217.