Towards a new mathematics for science

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Abstract

In this paper, it is argued that, despite its many strengths and undoubted value in science, mathematics also has some weaknesses as a handmaiden for science. There appears to be potential in developing a new mathematics for science (NMFS) which is designed to overcome those weaknesses and to meet more fully the needs of science.

As a background to the proposals: 1) The conclusions of a companion paper are noted—that mathematics is, fundamentally, a set of techniques for the compression of information and the application of those techniques; 2) That the effectiveness of mathematics in science is because it provides a means of achieving the compression of information which lies at the heart of science; 3) Since science and mathematics are products of the human intellect, it should not be surprising to find that the workings of the human mind has an influence on both of them; and 4) The significance of information compression in science and mathematics is in line with an abundance of other evidence for the significance of information compression in human learning, perception, and thinking.

Continuing the theme of information compression as a unifying principle, the *SP theory of intelligence* and its realisation in the *SP computer model* demonstrate how diverse aspects of intelligence may be modelled via information compression within the powerful framework of *multiple alignment*.

An NMFS is proposed, created as an amalgamation of mathematics as it is today with the SP system as it is today, including developments in both areas that are anticipated now. It is envisaged that, for reasons described in the paper, the proposed NMFS will overcome several of the apparent weaknesses in mathematics.

In several sections, there is discussion of some possible implications for science of the proposed NMFS, including: potential for the long-sought-after

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unification of quantum mechanics and general relativity; expansion of the concept of "object" in physics; ambiguity in human perception as an analogy for the concept of superposition in quantum mechanics; the phenomenon of discontinuous dependencies in natural languages as an analogy for nonlocality and entanglement in quantum mechanics; a "waterspout" interpretation for some of the two-slits experiments; potential advantages of the proposed NMFS in the realm of statistics; and its potential to be a vehicle for the representation of *all* kinds of scientific knowledge, with consequent benefits in the processing of that knowledge.

Keywords: information compression, multiple alignment, mathematics, quantum mechanics, relativity, representation of knowledge, artificial intelligence.

1 Introduction

As the title of this paper suggests, it is about the development of a form of mathematics that is designed to meet the needs of science.

Creating a new mathematics for science (NMFS) may seem perverse since mathematics has an exceptionally long history with many achievements, and it has shown its value repeatedly in many branches of science. And yet a long history with many achievements does not exclude the possibility that bad habits and blind spots may build up, and, as with any other human endeavour, it does not exclude the need for periodic review and revision in the light of new thinking and new insights.

A theme in these proposals is that: 1) Science is the product of the human intellect so it should not be surprising that features of human thinking should have an influence on scientific theories. 2) In much of science those kinds of influence may be ignored. But 3) in physics, and quantum mechanics in particular, it seems that aspects of human learning, perception and cognition can be important.

To anticipate a little, much of the thinking in this paper derives from the *SP theory of intelligence* and its realisation in the SP computer model, outlined in Appendix B.

In brief, the following main topics will be discussed:

- *Some apparent shortcomings of mathematics as a handmaiden for science* (Section 2). Although mathematics as it is today is remarkably powerful and successful, it has some weaknesses as a vehicle for expressing scientific knowledge and for the making of scientific inferences.

- *The significance of information compression in human cognition, in science, and in mathematics* (Section 3). As a background to what follows, and drawing on [59], this section outlines how human learning, perception, and thinking may be understood as compression of information; how science may be seen as, at root, the search for compression in the world;[1] and how mathematics may be understood as a set of techniques for compression of information and their application; and, for those reasons, why mathematics is so effective in science.

- *Towards a new mathematics for science* (Section 4). This section proposes the creation of an NMFS as an amalgamation of mathematics of today (with projected developments) with the SP system as it is today (with projected developments). This section describes how this development may help to overcome the previously-described weaknesses of mathematics as a handmaiden of science.

- *Some implications for science* (Section 5). This section describes how the proposed NMFS may provide new perspectives in several areas of science.

Owing to the close relationship between mathematics and logic, most of what is said about mathematics in this paper is likely also to apply to logic. There is relevant discussion in [49, Chapter 10].

2 Some apparent shortcomings of mathematics as a handmaiden for science

As was mentioned in the Introduction, the long history of mathematics, with its many achievements, does not guarantee that it is perfect. This section summarises what appear to be some weaknesses in mathematics as a servant for science.

2.1 Shortcomings of mathematics in the representation of knowledge and in artificial intelligence

Although areas of study like the representation of knowledge and artificial intelligence can, and often do, make use of mathematics, they are not, in themselves, normally regarded as part of mathematics. A possible exception is that there is increasing recognition of the close relationship between statistics and optimization as disciplines within mathematics and machine learning as a part of AI.[2]

[1] Here, and elsewhere in this paper, the word "world" will be used as a shorthand for the observable universe.

[2] See, for example, "Machine learning", *Wikipedia*, bit.ly/1SKDful, retrieved 2017-03-20: "Machine learning is closely related to (and often overlaps with) computational statistics,

Mathematics as the handmaiden of science would, almost certainly, be enhanced by the inclusion within it of insights gained and principles developed in the study of how different kinds of knowledge can or should be represented, and in the development of artificial intelligence.

As an example, artificial intelligence (and mainstream computer science) make extensive use of the "object-oriented" concept of *class-inclusion hierarchies*, with an associated form of inference called *inheritance of attributes*—but those ideas, which can be very useful, appear not to have been adopted in mathematics.

2.2 Comprehensibility

The fact that many people can find mathematics incomprehensible will, in many cases, simply reflect their interests or aptitudes, or insufficient training. But quantum mechanics is different. Even the most highly trained people, with a keen interest in the subject, can find quantum mechanics difficult to understand and can be uncomfortable with the exhortation that one should "shut up and calculate" [1, Locations 106 and 119].

The tentative view adopted here is that this situation arises from a weakness in mathematics rather than a weakness in the brain power of such luminaries as Albert Einstein.[3]

2.3 Under-expressiveness

At present, scientific knowledge is expressed with a combination of natural language, diagrams, drawings, static photographs, videos, computer programs, and mathematics. This works quite well but there seems to be a problem which is probably more to do with human psychology than anything else.

Because of its long history and its power and precision, mathematics may be seen to have a higher status than other means of representing scientific ideas. But while some ideas can be expressed quite neatly with mathematics, others do not sit well with the conventions that are available in mathematics as it is today. If one is inclined to rely on mathematics as the means of expressing the core of any scientific theory, some potentially useful ideas may get overlooked.

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which also focuses in prediction-making through the use of computers. It has strong ties to mathematical optimization, which delivers methods, theory and application domains to the field." See also the section headed "Relation to statistics" in the same source.

[3]Despite Einstein's great achievements, "He struggled with maths" [33, Location 1012], "[Quantum mechanics] seemed absurd to Einstein." [33, Location 1537], and "[Einstein] remains convinced that things could not be as strange as this theory proposed—and that, 'behind' it, there must be a further, more reasonable explanation." [33, Location 1554].

For example, concepts of heuristic search and backtracking are familiar in AI but are not easily expressed with mainstream mathematics.[4] For that reason, many AI researchers use computer programming as their preferred means of creating working models. This can yield insights that may be missed in research in which mathematics is seen as the preferred means of expressing the essentials of any theory.

2.4 Over-expressiveness

If may seem perverse to criticise mathematics on the one hand for being under-expressive (Section 2.3), and on the other hand, in this section, for being over-expressive. But it seems that those two weaknesses can co-exist.

The basic problem seems to be that, with mathematical notations, and also with computer programs, it is all too easy to create structures that have the hallmarks of mathematical rigour but are scientifically meaningless or wrong. Here, in the following subsections, are some apparent examples.

2.4.1 The financial crisis of 2008

Writing about the financial crisis of 2008, John Lanchester says that banks recruited many mathematical experts or "quants" who created mathematical computer programs that were too complicated to be understood by banking managers, and perhaps the quants themselves:

> "Banking is all about the management of risk; and in this central respect the bankers massively failed. They did so by relying on inaccurate mathematical models which they themselves didn't fully understand. ... The result was that they failed as utterly and completely as it is possible to fail." [23, p. 142].

In a similar vein, Steve Keen writes:

> "[One of the reasons for problems in economics] is the division of mainstream economists into effective 'castes', with only a tiny but exalted subset of the profession undertaking the detailed mathematical work needed to discover the weaknesses in the theory. The vast majority of economists believe that this high caste, the mathematical economists, did their work properly, and proved that the theory is internally consistent. The caste has indeed done its work properly, but it has proved

[4] A possible exception is when concepts of heuristic search and backtracking are cast in the form of simulated annealing.

> precisely the opposite: that the theory is consistent only under the most restrictive and specious of assumptions.
>
> "However, rather than taking the next logical step, and acknowledging that the foundations of economics are unsound and must therefore be changed, most mathematical economists are so wedded to this way of thinking, and so ignorant of the real world, that they instead invent some fudge to disguise the gaping hole they have uncovered in the theory." [22, Location 1961].

It seems that more than a few people in the financial services industries have been dazzled by the apparent rigour of mathematics and thereby led to make serious errors. As argued in Section 4, certain disciplines are needed to constrain what can be done when mathematics or computer programs are used as a means of expressing a scientific theory, or something like an economic or financial model which should have the rigour of a scientific theory.

2.4.2 String theory and multiverse theory

As described in Appendix D.4, there are concerns about string theory and multiverse theory.

The gist of the argument is that these mathematically-based theories are unfalsifiable because they are too general. For example, Peter Woit, writing about string theory, says (as quoted in Appendix D.4): "It predicts nothing about anything, since you can get pretty much any physics you want by appropriately choosing how to to make six of the ten dimensions invisible." [44, Location 1054].

As described in Appendix E, notions of falsifiability and unfalsifiability may be interpreted in terms of information compression or, equivalently, simplicity and power.

2.4.3 Hoax scientific papers

Mathematics features in some of the hoax scientific papers created by, for example, SCIgen.[5] and Mathgen.[6] An example from Mathgen is *Independent, Negative, Canonically Turing Arrows of Equations and Problems in Applied Formal PDE*,[7] which was accepted for publication in the journal *Advances in Pure Mathematics* in August 2012.[8]

This kind of thing suggests the need for more discipline in the evaluation of mathematics when it is applied to science, as discussed in Section 4.

[5] See bit.ly/1mdOwaZ.
[6] See bit.ly/14JP0NB.
[7] PDF, bit.ly/2kxVMyK.
[8] See bit.ly/2kTJe94.

2.4.4 Spaghetti programming, structured programming, and object-oriented programming

Computer programming is not the same as mathematics but it's relationship is sufficiently close for it to be discussed here.

Up until about 1970, the use of the infamous "go to" statements in computer programs allowed jumps from any part of a program to any other, a practice that often led to "spaghetti" programs with complex and tangled control structures that could be difficult to understand or to maintain [12].

These problems led to the introduction of the more disciplined "structured programming" (see, for example, [20]) with consequent benefits in terms of comprehensibility, maintainability and reductions in cost. And then, following the introduction of the *Simula* computer language [6], it became apparent that there were even more benefits in "object-oriented programming", something that is now prominent in most software engineering.

Mathematics has somehow avoided the "go to" statement and the worst of spaghetti programming. But, as indicated in Section 2.1, it may benefits from class-inclusion hierarchies and inheritance of attributes. the main features of object-oriented programming.

Also, it appears that, in both mathematics and computing, there is still more to be done to control over-expressiveness, as described in Section 4.5.

2.4.5 Infinities

A prominent feature of mainstream mathematics and other formal systems (including the SP system), and the subject of highly-original research by Georg Cantor, is the ability to describe things that are infinitely big or infinitely small.

This can be done quite simply with a recursive function like that shown in Figure 1. If the first line (`if (x <= 0) return ;`) is taken out, the function will, in principle, print out "hello, world" an infinite number of times, unless the computer crashes, or its power supply fails, or it runs out of ink or paper, and so on.

Although it can be fascinating to consider how infinities may come in different sizes, and paradoxes that arise from the concept of infinity, the notion of infinity can be problematic for science, as described in Appendix D.3.

```
void create\_redundancy(int x)
{
    if (x <= 0) return ;
    printf("hello, world; ") ;
    return create\_redundancy(x - 1) ;
}.
```

Figure 1: A simple recursive function showing how, via computing, it is possible to create repeated (redundant) copies of "hello, world". Reproduced from Figure 1 in [59].

3 The significance of information compression in human cognition, in science, and in mathematics

As a background to what follows, this section summarises arguments in [59]:

- *Human learning, perception, and thinking as compression of information.* In [59, Section 2] there is a summary of direct and indirect evidence that much of human learning, perception, and thinking may be understood as compression of information, in keeping with the fundamental importance of information compression in the SP system (Appendix B). If that is accepted, it should not be surprising to find that information compression is of central importance in both science and mathematics, since they are both products of the human intellect.

- *On the "mysterious" effectiveness of mathematics in science.* It is noted that the effectiveness of mathematics in science appears to some writers to be "mysterious" or "unreasonable".

- *Science as a search for compression in the world.* Then reasons are given for thinking that science is, at root, the search for compression in the world.

- *Mathematics as a set of techniques for compressing information, and their application.* At more length, several reasons are given for believing that mathematics is, fundamentally, a set of techniques for compressing information, and their application.

- *Mathematics assists in the compression of information in science.* From there, it is argued that the effectiveness of mathematics in science is because it provides a means of achieving the compression of information which lies at the heart of science.

- *The anthropic principle.* The anthropic principle provides an explanation of why we find the world—aspects of it at least—to be compressible.

Probably, [59] should be read in conjunction with the rest of this paper. There is related discussion in Appendix A.

4 Towards a new mathematics for science

If it is accepted, at least provisionally, that mathematics is largely about compressing information, it seems natural to consider whether or how that idea might lead to new insights, new avenues for research, and remedies for the apparent shortcomings of mathematics in the service of science, as outlined in Section 2.

Since, in what follows, the phrase "the proposed NMFS" is often needed, it will for the sake of brevity be shortened to "P-NMFS".

In broad terms, the suggested advantages of P-NMFS are: 1) It would expand the scope of mathematics with new ways of representing and processing knowledge. 2)It would introduce a new discipline into mathematics: quantitative evaluation of calculations in terms of information compression. This may help to guard against unfalsifiable theories and spurious applications of mathematics.

The suggestion here, to be developed in the rest of this section, is that:

- Since information compression is central in the organisation and workings of the previously-mentioned *SP theory of intelligence* and its realisation in the *SP computer model* (outlined in Appendix B); ...

- And since the SP theory, with the powerful concept of *multiple alignment*, has strengths in several aspects of intelligence [57] and has potential as a *universal framework for the representation and processing of diverse kinds of knowledge* (UFK) [53, Section III]; ...

- And in view of evidence that information compression is central in the organisation and workings of mathematics (Section 3); ...

- We may conclude that an NMFS, with advantages over 'ordinary' mathematics, may be created as some kind of amalgamation of the SP system with mathematics, each one as it is now or with developments that are anticipated now.

With regard to the last point, it may seem outlandish to incorporate a theory of intelligence in P-NMFS, but, as was noted in Section 2.1, there is increasing recognition already of, for example, the close relationship between machine learning as a branch of AI, and statistics and optimization as branches of mathematics.

With regard to amalgamation, it is true that mathematics may borrow ideas from the SP system, and *vice versa*, without the two things being combined. But the amalgamation may create a richer and more coherent structure:

- By introducing structures and processes from the SP system such as multiple alignment and unsupervised learning.
- By interpreting mathematical concepts, as far as possible, within the SP framework, and modifying the SP framework where necessary.
- By dropping structures and processes from one area where equivalent things can be done better with structures and processes from the other area.

The amalgamation may be seen to be a richer source of concepts than mathematics by itself or the SP system by itself, with or without borrowing, especially when information compression is seen as a bridge between the two.

The subsections below outline what the amalgamation might look like, emphasising features that are different from mainstream mathematics.

For the sake of brevity, mathematics as it is today, with developments that are anticipated now, will be abbreviated as "MT"; and the SP system as it is today, with developments that are anticipated now, will be "SP".

4.1 Probabilities

In view of the intimate connection between information compression and concepts of prediction and probability ([36, 38, 24], [59, Appendix D]), P-NMFS, like SP, would be fundamentally probabilistic. This should not be seen as a problem since it has been recognised for some time that the same is true of MT:

> "I have recently been able to take a further step along the path laid out by Gödel and Turing. By translating a particular computer program into an algebraic equation of a type that was familiar even to the ancient Greeks, I have shown that there is randomness in the branch of pure mathematics known as number theory. My work indicates that—to borrow Einstein's metaphor—God sometimes plays dice with whole numbers." [9, p. 80].

As indicated in this quotation, randomness in number theory is closely related to Gödel's incompleteness theorems. These are themselves closely related to the phenomenon of recursion, a feature of many formal systems (including SP), many of Escher's pictures, and much of Bach's music, as described in some detail by Douglas Hofstadter in *Gödel, Escher, Bach: An Eternal Golden Braid* [18].

4.2 Versatility in the representation of knowledge via the powerful concept of multiple alignment

With regard to apparent deficiencies of MT in the representation of knowledge (Section 2.1), a major strength of SP—which P-NMFS would share—is the potential of SP to be a *universal framework for the representation and processing of diverse kinds of knowledge* (UFK) [53, Section III].

The SP system has the potential to be a UFK because, although SP patterns are not very expressive in themselves, they come to life in the multiple alignment framework. Within that framework, they may serve in the representation of several different kinds of knowledge, including: the syntax of natural languages; class-inclusion hierarchies (with or without cross classification); part-whole hierarchies; discrimination networks and trees; if-then rules; entity-relationship structures [50, Sections 3 and 4]; relational tuples (*ibid.*, Section 3), and concepts in mathematics, logic, and computing, such as 'function', 'variable', 'value', 'set', and 'type definition' ([49, Chapter 10], [54, Section 6.6.1]).

With the addition of two-dimensional SP patterns to the SP computer model, there is potential for the SP system to represent such things as: photographs; diagrams; structures in three dimensions [51, Section 6.1 and 6,2]; and procedures that work in parallel [52, Sections V-G, V-H, and V-I, and Appendix C].

Since information compression is at the heart of SP—in the building of multiple alignments and in unsupervised learning—and since the techniques for the compression of information within SP are very general, there is reason to believe that the system may serve in the succinct representation of *any* kind of knowledge, not just the examples that have been listed in this section.

4.3 Versatility and adaptability in intelligence via the powerful concept of multiple alignment

With regard to apparent deficiencies of MT in artificial intelligence (Section 2.1), multiple alignment, as it has been developed in SP, has the potential, as noted in Appendix B, to be as significant for an understanding of human-like intelligence as is DNA for biological sciences: it could be the "double helix" of intelligence.

SP has proved to be versatile not only in the representation of knowledge (Section 4.2) but also in aspects of intelligence. The same would be true of P-NMFS.

There is more detail in the two subsections that follow.

4.3.1 How unsupervised learning may be achieved via the processing of multiple alignments

SP has strengths and potential in "unsupervised" learning of new knowledge, meaning learning without the assistance of a "teacher" or anything equivalent. As outlined in Appendix B.2, unsupervised learning is achieved in SP via the processing of multiple alignments to create Old patterns, directly and indirectly, from New patterns, and to build collections of Old patterns, called *grammars* which are relatively effective in the compression of New patterns.

Unsupervised learning appears to be the most fundamental form of learning, with potential as a foundation for other forms of learning such as reinforcement learning, supervised learning, learning by imitation, and learning by being told.

4.3.2 How other aspects of intelligence flow from the building of multiple alignments

By contrast with the way in which SP models unsupervised learning via the processing of already-constructed 'good' multiple alignments, other aspects of intelligence flow from the building of multiple alignments (Appendix B.1). These other aspects of intelligence include: analysis and production of natural language; pattern recognition that is robust in the face of errors in data; pattern recognition at multiple levels of abstraction; computer vision [51]; best-match and semantic kinds of information retrieval; several kinds of reasoning (more in Section 4.3.3, next); planning; and problem solving.

4.3.3 How several kinds of reasoning flow from the building of multiple alignments

In scientific research and in the applications of science, what is potentially one of the most useful attributes of SP is its versatility in reasoning, described in [49, Chapter 7] and [45, Section 10]. Strengths of the SP system in reasoning, derived from the building of multiple alignments, include: one-step 'deductive' reasoning; chains of reasoning; abductive reasoning; reasoning with probabilistic networks and trees; reasoning with 'rules'; nonmonotonic reasoning and reasoning with default values; Bayesian reasoning with "explaining away"; causal reasoning; reasoning that is not supported by evidence; the already-mentioned inheritance of attributes in class hierarchies; and inheritance of contexts in part-whole hierarchies. There is also potential for spatial reasoning [52, Section IV-F.1], and for what-if reasoning [52, Section IV-F.2].

4.4 Seamless integration of diverse kinds of knowledge and diverse aspects of intelligence

Again with regard to apparent deficiencies of MT in artificial intelligence (Section 2.1), there is clear potential for SP to provide seamless integration of diverse kinds of knowledge and diverse aspects of intelligence, in any combination. This is because diverse kinds of knowledge and diverse aspects of intelligence all flow from a single coherent source: SP patterns within the multiple alignment framework.

In this respect, there is a sharp contrast between SP and the majority of other AI systems, which are either narrowly specialised for one or two functions or, if they aspire to be more general, are collections or kluges of different functions, with little or no integration.[9]

This point is important because it appears that seamless integration of diverse kinds of knowledge and diverse aspects of intelligence, in any combination, are essential pre-requisites for human levels of fluidity, versatility and adaptability in intelligence.

4.5 A potential remedy for over- and under-expressiveness

It appears that the problems of under-expressiveness (Section 2.3) and over-expressiveness (Sections 2.4 to 2.4.5) are the same as, or closely related to, the problems of under-generalisation and over-generalisation described in [59, Appendix B]. And it appears that the remedy, in all cases, is the same: evaluate the given theory in terms of information compression as described in Appendix E.

In brief, this means that, for anything that aspires to be, or purports to be, a scientific theory or a scientific analysis that is expressed in terms of P-NMFS, it is necessary: 1) to identify the body of raw data, **I**, to which it relates; and 2) to provide a measure of information compression, comparing the size (in bits) of **I** in its original state with the size (in bits) of **I** when it has been compressed.

Here are some additional comments about over-expressiveness, relating to what was said before:

- *The financial crisis of 2008.* If, as a general rule, all bodies of mathematics used in the financial services industries were to be evaluated in terms of information compression as described in Appendix E, this might help to forestall another banking crisis like that of 2008.

[9] Although Allen Newell called for the development of "Unified Theories of Cognition" [27, 26], and researchers in "Artificial General Intelligence" are aiming for a similar kind of integration in AI, it appears that none of the resulting systems are fully integrated: "We have not discovered any one algorithm or approach capable of yielding the emergence of [general intelligence]." [15, p. 1].

- *String theory and multiverse theory.* The same kind of discipline may also help to head off blind alleys in scientific theorising, as string theory and multiverse theory appear to be.

- *Hoax scientific papers.* And if referees of academic papers were to apply the same disciplines to any mathematics that may be presented in submitted papers, this may help to weed out hoax scientific papers before they get published.

- *Spaghetti programming, structured programming, and object-oriented design.* As was described in Section 2.4.4, software engineering has, with increasing benefits, progressed from spaghetti programming, through structured programming, to object-oriented programming.

  The SP system (and thus P-NMFS) promises to go further, yielding all the benefits object-oriented programming (class-inclusion hierarchies, part-whole hierarchies, and associated kinds of inheritance), but with the additional benefit of seamless integration of the two kinds of hierarchy (Appendix B.1, last paragraph).

  More generally, SP promises seamless integration of the five variants of ICMUP described in [59, Section 5], the several forms of knowledge mentioned in Section 4.2, and the encoding of discontinuous dependencies as described in Section 5.8.

- *Infinities.* Although SP and P-NMFS, like other formal systems, would be capable of describing things that are infinitely big or infinitely small, and those kinds of infinities can be problematic in science (Section 2.4.5 and Appendix D.3), the evaluation of any scientific theory in terms of information compression (Appendix E) should have the effect of eliminating infinities. Although this has not yet been explored in detail, it seems likely that any infinity in a theory that is expressed in terms of SP or P-NMFS is likely to lead to a relatively poor score in terms of information compression.

5 Some implications for science

This section explores some of the ways in which P-NMFS, with features outlined on Section 4, may have an impact in aspects and areas of science.

5.1 Potential for the unification of quantum mechanics and general relativity

Since there would be potential with P-NMFS for the reinterpretation of both quantum mechanics and the theory of general relativity in terms of concepts from SP, there is a corresponding potential for the long-sought-after unification of quantum mechanics and general relativity. In general, the provision of multiple alignment as a UFK [53, Section III] is likely to facilitate the simplification and integration of concepts wherever that is feasible.

That said, an implication of the ideas in this paper is that we are unlikely ever to develop a "theory of everything" (Appendix D.1). It is likely that, quite often, we must be content with theories, in one area or in diverse areas, that are fully or partially independent of each other.

5.2 Information, waves, energy, and matter

If it accepted that science and mathematics are both fundamentally about compression of information (Section 3), it is pertinent to consider what the information is that is to be compressed.

Most directly and immediately the raw material for science (and for scientific applications of mathematics) is the information that we receive via our eyes, ears, nose, skin, and so on. But rather than attempting to develop different theories, one for each of visual, auditory, nasal, haptic, and other forms of information, it seems preferable to work with a more abstract concept of information, like that developed by Claude Shannon and others. Here are some tentative ideas:

- *Information as discriminable variations.* It seems that all forms of information, including those mentioned above, may be seen as discriminable variations in some substrate:
  - Visual information means patterns that vary in space (across the retina) and in time. Even something like a uniform monochromatic area that does not change with time may be seen to contain discriminable variations because sinusoidal light waves are needed for us to register the monochromatic colour.
  - Auditory information means variations in pressure in the air. As with vision, something like a constant pure note which, at first sight, appears not to contain any variation, cannot be heard without sinusoidal variations in pressure.
  - Similar things may be said about smells, touch sensations, and so on.

This idea is similar to Shannon's **??** concept of information, which may be described informally as a "measure of the number of possible alternatives for something" [33, Location 2565]: when alternatives are distributed across time or space, they become discriminable variations in some substrate.

- *Information as waves.* Another way of expressing the idea that information is discriminable variations in some substrate is to say that information is a wave or waves, most simply in one dimension but also in two dimensions and possibly more. This is true regardless of whether the wave is a classically curvy kind of analogue thing or whether it is represented via step functions as digital 0s and 1s.

- *The possible equivalence of information and energy.* Much of what has been said here about information and waves, may also be said about energy. Forms of energy such as light, heat and sound appear to be largely about variations in some substrate:

  > "Today, physicists commonly accept the idea that information can be used as a conceptual tool to throw light on the nature of heat." [33, Location 2637].

  Since it is accepted that different forms of energy may be converted, one into the other, it seems that the "energy-as-variations" idea may also apply to such things as potential energy and elastic energy where variations are less obvious.

  On the strength of the dictum that "If it looks like a duck, swims like a duck, and quacks like a duck, then it probably is a duck.", there seems to be a case for supposing that information and energy are the same thing. Notwithstanding some criticisms, the similarity between the concept of entropy in thermodynamics and the concept of entropy in information theory lends support to this view.

  A possible objection is that, with something like heat, the variations are largely random and we often perceive random variations to be uninformative. It is true that most people regard white noise as boring and uninteresting. But in terms of classical information theory, any such random signal is transmitting at or near the maximum for a given bandwidth.

- *Information, waves, energy, and matter.* From the foregoing, we may regard information, waves, and energy as essentially the same thing. Then if it accepted that, in accordance with Schrödinger's wave mechanics, the entire subatomic world may be seen to be made up of waves, it follows that the subatomic world may also be seen to be made up of "information-waves-energy".

More generally, since in accordance with Einstein's equation, $E = mc^2$, matter of any kind is equivalent to energy, we may regard information, waves, energy, and matter, as all the same thing. From the perspective of science-and-mathematics-as-compression-of-information, it seems reasonable to use the word *information* as a relatively short name for "information-waves-energy-matter".

There appear to be connections between this tentative conclusion, ideas about the role of information compression in cognition, mathematics, and science, discussed elsewhere in this paper, and some ideas about information and physics described in [33, Chapter 12], discussed briefly in Appendix C.

- *A fractal character to the world.* If information is the modelling clay from which everything else is constructed, we may guess that, for the most part, it has an analogue character without the step functions of digital information. We may also guess that, like a fractal pattern, we may in principle drill down as far as we like, resolving more and more detail as we go deeper.[10]

- *Limits on resolution.* Whether or not there are theoretical limits on how deep we can go, there will certainly be practical limits on resolution set by the limitations of available technologies for making observations, including the minimum sizes of the the basic building blocks for those technologies, such as the wave lengths of this or that form of radiation used to probe the structure of matter. And there may be limits on resolution imposed by scientists themselves, seeking to focus on higher-level regularities in data, without distractions from lower-level detail.

- *Most theories are emergent.* An implication of the idea that everything is made of waves with a fractal character so that the depth or detail that we may consider is constrained only by limits on the resolution of available technologies for making observations, is that most theories are emergent, in much the same way that the relatively neat equations for the gas laws (Boyle's Law, Charles's Law, etc) emerge from an underlying messiness and unpredictability implied by the kinetic theory of gases. Other possible examples are the concepts of time and space, which, in the theory of look quantum gravity,

[10] But see Appendix D.3 about the hazards of assuming too readily that anything in the world is infinite. Also, "The central prediction of loop theory is ... that space is not a continuum, it is not divisible ad infinitum, it is formed of 'atoms of space'." [33, Location 1818], and "... arbitrarily small chunks of space do not exist. There is a lower limit to the divisibility of space." (*ibid.* Location 1834).

emerge from the more fundamental concept of the quantum gravitational field [33, Chapter 7].

An implication of this view is that we should not be surprised if things get more complicated, not simpler, as we dig deeper. This point will be revisited in Section 5.9.

5.3 The need for approximations

It has been recognised for a long time that, in applying mathematics to science, it may be necessary to make approximations—and the same is likely to be true of P-NMFS. The main reasons for making approximations with P-NMFS are likely to be:

- *Computational complexity and heuristic search.* Assuming that P-NMFS embraces processes like the building of multiple alignments (Appendix B.1) and unsupervised learning in the SP framework (Appendix B.2), the making of approximations is an integral part of heuristic search in both of those processes.

  Apart from the building of multiple alignments or unsupervised learning, there may be other areas, particularly in AI, where computational complexities impose limits that may be practical or theoretical or both, that force us to retreat from what is theoretically ideal, and to be content with results that are "reasonably good".

- *Limits on resolution.* As mentioned under the heading "Limits on resolution" in Section 5.2, there will be practical limits on resolution in the gathering of raw data, and there may be limits imposed by scientists themselves, seeking to focus on higher-level regularities in data. Both these things imply approximations in the development and application of theories.

- *Generalisation.* A very important aspect of any kind of learning is generalisation from the data which is the basis for learning, without undergeneralisation or over-generalisation [59, Appendix B]. Generalisation is what allows us to make judgements about things which are similar to but not exactly the same as what we have seen before. Although generalisation is important, it may be seen as a form of approximation.

5.4 Sources of uncertainty in science

An implication of the perspective developed in this paper is that uncertainties, with corresponding probabilities, will be an inescapable feature of how we observe and theorise about natural phenomena. This section summarises what appear to be the main sources of uncertainty in science:

- *Good old-fashioned noise.* An implication of the ideas: 1) that everything is made of waves with a fractal character (Section 5.2); and 2) that all theories are emergent (*ibid.*); is that our observations will never fit any theory precisely and that there will always be unpredictability or *noise*, both auditory and otherwise, of the kinds that are familiar in the workings of cars, television sets, electric toothbrushes, and so on.[11]

- *Uncertainties arising from the making of approximations.* As outlined in Section 5.3, there are several reasons why, in science, it is often necessary to make approximations. Any such approximation will be a source of uncertainties.

- *Uncertainties associated with ill-defined entities.* The boundaries of something like a stone are reasonably clear. But the boundaries of something like a cloud or a forest can be much less clear, as can be seen from the windows of a plane as it flies into a cloud or out of it. It seems that this kind of fuzziness is most pronounced amongst quantum-level entities such as electrons, protons, neutrons, and the like.

- *Inferences.* Since science is not merely about describing the world but is also about making inferences about the world, and since many such inferences are normally uncertain, the making of inferences is a key source of uncertainty in science.

- *Uncertainties in mathematics and other formal systems.* In view of the randomness that has been shown to exist in number theory, noted in Section 4.1, and since similar conclusions are likely to apply to other formal systems that exhibit recursion, including SP, it seems that there is an unavoidable source of uncertainty in any body of science that relies at least partly on mathematics or some comparable formal system.

- *Observer effects.* A prominent feature of quantum mechanics is that, when we are trying to observe or make measurements of things that are very small, our sensing devices may easily have an impact on whatever it is that is the focus of interest. This is a further source of uncertainty in science.

- *Uncertainties arising from measurements of speed.* Since: 1) The measurement of the speed of an object requires measurements of position from at

[11] See also the comment about white noise in Section 5.2.

least two different time frames; and since 2) Abstraction of an object concept from raw data as described in Section 5.5 is a process that takes time; It is likely that there will be corresponding uncertainties about the speed of the object at any one position.

These sources of uncertainty appear to be different from those that are the basis for Heisenberg's uncertainty principle.

With the possible exception of observer effects, these sources of uncertainty apply at all levels of analysis in science, not merely at the sub-atomic level.

With regard to Einstein's famous dictum that "[God] does not play dice." [19, Location 6056], a tentative answer to the implied question is that we can never know. On the strength of the uncertainties described above, it seems that uncertainties in science arise largely, perhaps exclusively, from shortcomings in our means of making observations, or in our methods for analysing observations, or in our desire to make inferences. There seems to be some kind of glass wall that separates us from the "real" world.

5.5 The discovery and processing of objects

Objects, such as galaxies, stars, and planets, and, less certainly, molecules, atoms, and sub-atomic particles, have important roles in theoretical physics, and in related fields such as cosmology. But the way in which such objects are conceived in those fields—as theoretical primitives—contrasts sharply with the way in which objects are conceived in artificial intelligence, cognitive psychology, and indeed SP, where they are seen as relatively high-level concepts that must be abstracted or constructed from raw data.

5.5.1 Abstraction of perceptual objects from random-dot stereograms

Consider, for example, how a perceptual object may be abstracted from a random-dote stereogram, as described in [21]. In brief, each of the two images shown in Figure 2 is a random array of black and white pixels, with no discernable structure, but they are related to each other as shown in Figure 3: both images are the same except that a square area near the middle of the left image is further to the left in the right image.

When the images in Figure 2 are viewed with a stereoscope, projecting the left image to the left eye and the right image to the right eye, the central square appears gradually as a discrete object suspended above the background. Although this illustrates how stereoscopic vision aids depth perception—a subject of some interest in its own right—the main interest here is on how we see the central square as a discrete object. There is no such object in either of the two images

Figure 2: A random-dot stereogram from [21, Figure 2.4-1], reproduced with permission of Alcatel-Lucent/Bell Labs.

1	0	1	0	1	0	0	1	0	1
1	0	0	1	0	1	0	1	0	0
0	0	1	1	0	1	1	0	1	0
0	1	0	Y	*A*	*A*	*B*	*B*	0	0
1	1	1	X	*B*	*A*	*B*	*A*	0	1
0	0	1	X	*A*	*A*	*B*	*A*	1	0
1	1	1	Y	*B*	*B*	*A*	*B*	0	1
1	0	0	1	1	0	1	1	0	1
1	1	0	0	1	1	0	1	1	1
0	1	0	0	0	1	1	1	1	0

1	0	1	0	1	0	0	1	0	1
1	0	0	1	0	1	0	1	0	0
0	0	1	1	0	1	1	0	1	0
0	1	0	*A*	*A*	*B*	*B*	X	0	0
1	1	1	*B*	*A*	*B*	*A*	Y	0	1
0	0	1	*A*	*A*	*B*	*A*	Y	1	0
1	1	1	*B*	*B*	*A*	*B*	X	0	1
1	0	0	1	1	0	1	1	0	1
1	1	0	0	1	1	0	1	1	1
0	1	0	0	0	1	1	1	1	0

Figure 3: Diagram to show the relationship between the left and right images in Figure 2. Reproduced from [21, Figure 2.4-3], with permission of Alcatel-Lucent/Bell Labs.

individually. It exists purely in the *relationship* between the two images, and seeing it means matching one image with the other and unifying the parts which are the same.

Seeing the central object means finding a 'good' match between relevant pixels in the central area of the left and right images, and likewise for the background. Here, a good match is one that yields a relatively high level of information compression. Since there is normally an astronomically large number of alternative ways in which combinations of pixels in one image may be aligned with combinations of pixels in the other image, it is not normally feasible to search through all the possibilities exhaustively.

As with the building of multiple alignments (Appendix B.1), it is necessary to use heuristic techniques, building solutions in stages and weeding out low-scoring partial solutions at each stage. This can give results that are "reasonably good" but are not guaranteed to be theoretically ideal. One such method for the analysis of random-dot stereograms has been described by Marr and Poggio [25]. An example showing how the SP computer model may produce a plausible result with a one-dimensional analogue of a random-dot stereogram is described in [51, Section 5, Figure 9].

5.5.2 More about the abstraction of perceptual objects

Random-dot stereograms demonstrate rather clearly how something that we see as a discrete object-like entity—the central square in the example above—must necessarily be the product of a relatively complex process of analysis, and it should not be regarded as a theoretical primitive. Of course, random-dot stereograms are highly artificial, but there are reasons to believe that processes for the recognition of real-world objects or object-like entities, by natural or artificial systems, are likely to be at least as complex:

- *The discovery of words in natural language.* Words are widely-regarded as natural elements of speech but an examination of speech as it may appear on an oscilloscope screen or in a sound spectrogram shows that we speak in "ribbons", without any consistent physical marker between one word and the next.

  It as been shown via computational modelling, in more than one study (see, for example, [29, 11, 46]), that much of the word structure of language may be discovered by statistical analysis, similar in many respects to the way in which the central object may be abstracted from a random-dot stereogram, without the need for physical markers between one word and the next, without the assistance of any pre-supplied dictionary, and without the need for a "teacher" or anything equivalent.

Here, the statistical analysis is essentially a process of searching for repeating patterns which, collectively, via a parsing of the raw data, yield relatively high levels of compression of the data. New patterns are built up hierarchically by pairwise joining of smaller patterns including atomic symbols.

- *The discovery of word-like entities by SP.* Similar results have been obtained with the SP computer model, discovering word-like entities in language-like texts via unsupervised learning [49, Chapter 9]. As with the earlier work, the essence of the process is the development of one or more *grammars*, each one comprising a collection of SP patterns that, together, yield high levels of information compression. But the overall organisation of the model is rather different, with multiple alignment as the central organising principle instead of the hierarchical pairwise building of patterns.

- *Motion as an aid in the discovery of objects.* Apart from stereoscopic vision or the statistical analysis of the structure of language, an important aid to the discovery of an object concept in vision is motion of the object against its background [51, Section 5.2]. Here, successive frames in a stream of visual information play the parts of the left and right images in a stereoscopic display. As with a random-dot stereogram, the matching and unification of patterns, revealing differences between frames, provides a means of picking out a moving object as a discrete entity, distinct from its background. Animals that rely on camouflage for protection gain most benefit by staying still. If or when such an animal moves, it becomes much more easily seen (*ibid.*).

- *Objects in three dimensions.* As noted in Section 5.5.1, viewing the world with two eyes aids the perception of depth in vision. But, in addition, a knowledge of the three-dimensional form of an object may be built up by stitching together overlapping views of the object from several different angles, in much the way that a panoramic digital photograph of a scene may be created by stitching together neighbouring but overlapping views of the scene [51, Sections 6.1 and 6.2].

In addition to the evidence from random-dot stereograms (Section 5.5.1), these examples lend support to the view that objects are not "objectively" present in the world. They are constructs of the human brain or of artificial brain-like systems, created via a relatively complex process of compressing information. Implications of this view are discussed in several other sections.

5.6 Motion, speed, velocity, momentum, and acceleration

An important concern of physics and cosmology is, amongst other things, the *motion* of any given object (relative to some other object or to some kind of background), and, more specifically, its *speed*, *velocity*, *momentum*, and *acceleration*.

As we have seen (Section 5.5.2), motion can itself be an important means of isolating an object from its surroundings via the matching and unification of patterns. But those processes which allow an object to be isolated as a discrete entity may also provide a means of assessing is speed, velocity, and so on.

There are potential benefits in reinterpreting existing concepts of objects and their motions, speeds, and so on, in terms of relevant concepts from SP. However, there is work still to be done in developing relevant concepts within the framework of SP (Appendix B.10). Some preliminary ideas are described in [51, Section 5.3, Figure 10].

Probably, the most effective means of developing these and related questions about how aspects of the world could or should be encoded within SP is, first, to solve some residual problems in how the system achieves unsupervised learning [45, Section 3.3], and then use the improved processes for learning to discover how the system itself may encode motion, speed, and so on. That in itself may suggest further refinements in the learning processes.

5.7 Superposition

> "The idea of superposition is not unique to quantum mechanics but is a general property of all waves. Imagine watching someone dive into an empty swimming pool. You will see the ripples travel outwards along the surface of the water as simple undulations all the way to the other end of the pool. This is in stark contrast to the state of the water when the pool is full of people swimming and splashing about. The turbulent shape of its surface is now due to the combined effect of many disturbances and is achieved by adding them all together. This process of adding different waves together is known as superposition." [1, Location 1025].[12]

A potentially useful analogy for the concept of superposition in quantum mechanics is ambiguity in the realm of human perception. Consider, for example, what happens when we hear a short portion of sound that, using a simplified

[12] This description of superposition is distinctly less puzzling than it is in the idea of a "qubit" as a superposition of 0 and 1. In the first case, there are two or more waves that can be added together in the normal way. With a qubit, it is less clear how 0 and 1 may be added together in the manner that is understood in quantum mechanics unless they are both waves.

version of the phonetic alphabet, may be written: '`ae i s k r ee m`'. We may hear this as "ice cream" or "I scream" and may perhaps swither between the two interpretations as we review what we have heard.

This ambiguity can be modelled with SP as shown in Figure 4. The two multiple alignments shown are the two best multiple alignments created by the SP computer model with '`ae i s k r ee m`' as a New pattern shown in row 0, and a collection of appropriate Old patterns like those in rows 1 to 3 of the two multiple alignments, one pattern per row.

```
0          ae i s        k r ee m        0
           |  | |        | | |  |
1          |  | |    N 0 k r ee m #N     1
           |  | |    |            |
2 NP 2 A   |  | | #A N            #N #NP 2
       |   |  | | |
3      A 0 ae i s #A                     3

(a)
```

```
0          ae i        s k r ee m              0
           |  |        | | | |  |
1          |  |    V 0 s k r ee m #V           1
           |  |    |              |
2 S 0 NP   |  | #NP V              #V ADV #ADV #S 2
      |    |  | |
3     NP 1 ae i #NP                            3

(b)
```

Figure 4: A multiple alignment showing the two best parsings of the phoneme sequence 'texttae i s k r ee m' created by the SP computer model. Reproduced, with permission, from Figure 5.3 in [49].

Consider now what happens when we augment the New pattern with some disambiguating context in, for example, a pattern like '`ae i s k r ee m l ae w d l i`'. In this case, the ambiguity disappears as can be seen in the multiple alignment shown in Figure 5 (a), a multiple alignment showing the "I scream" interpretation, without any rival interpretation. Likewise, there is just one "ice cream" interpretation, shown in Figure 5 (b), when 'texttae i s k r ee m i z k o l d' is supplied to the SP computer model with an appropriate collection of Old patterns.

```
0          ae i         s k r ee m          l ae w d l i         0
           |  |         | | | |  |          | |  | | | |
1          |  |         | | | |  |    ADV 0 l ae w d l i #ADV    1
           |  |         | | | |  |     |                  |
2 S 0 NP   |  | #NP V   | | | |  | #V ADV                #ADV #S 2
      |    |  |  |  |   | | | |  | |
3     |    |  |  |  V 0 s k r ee m #V                            3
      |    |  |  |
4     NP 1 ae i #NP                                              4

(a)
```

```
0              ae i s        k r ee m             i z         k o l d       0
               |  | |        | | |  |             | |         | | | |
1              |  | |        | | |  |             | |     A 2 k o l d #A     1
               |  | |        | | |  |             | |     |           |
2 S 1 NP       |  | |        | | |  |    #NP VB   | | #VB A           #A #S 2
      |        |  | |        | | |  |     |  |    | |  |
3     |        |  | |    N 0 k r ee m #N   |  |    | |  |                    3
      |        |  | |    |            |   |  |    | |  |
4     NP 2 A   |  | | #A N            #N #NP |    | |  |                    4
           |   |  | | |                      |    | |  |
5          A 0 ae i s #A                     |    | |  |                    5
                                             |    | |  |
6                                            VB 0 i z #VB                   6

(b)
```

Figure 5: (a) A multiple alignment showing the best parsing found by the SP computer model for the phoneme sequence '`ae i s k r ee m`' when it is included within the larger sequence '`ae i s k r ee m l ae w d l i`'. (b) A multiple alignment showing the best parsing of the phoneme sequence '`ae i s k r ee m`' when it is included within the larger sequence '`ae i s k r ee m i z k o l d`'. Reproduced, with permission, from Figure 5.4 in [49].

The analogy with superposition as that is understood in quantum mechanics is not exact but it is suggestive. When there is relatively little information available, there may be a "superposition" of two possible interpretations of the ambiguous pattern '`ae i s k r ee m`'. But when more information is available (in the form of disambiguating context), perhaps corresponding to what happens in a quantum mechanics experiment when a detector has been put in place, then the "superposition" disappears and only one of the two possibilities is observed.

One possible objection to the forgoing is that there is nothing to show how the pattern 'textttae i s k r ee m' might be analysed into two or more components in a manner that is comparable with Fourier analysis. But similar results may be obtained with a pattern like '`A X B Y C Z`' which may be analysed into '`A B C`' and '`X Y Z`', somewhat like the analysis of a wave pattern created via superposition into the wave patterns from which it was derived.

Another possible objection is that with superposition in quantum mechanics there is nothing like the human brain or AI system that may interpret data in the manner described above. In answer to this objection, any kind of non-human and non-AI detector may exhibit comparable results. When information is limited, there may be ambiguity in the sense that the detector may flip unpredictably from one interpretation to another, but when fuller information is supplied, any such ambiguity may be resolved.

5.8 Nonlocality, entanglement, multiple alignment, and discontinuous dependencies

There is little doubt that the phenomena of "nonlocality" and "entanglement" are genuine features of the world and not merely some "weirdness" in quantum mechanics which may, at some stage, be explained away:

> "Today, quantum nonlocality and entanglement are no longer the subject of philosophical debate. They are accepted as crucial features of the quantum world. Indeed, entanglement of many particles could lead to the development of a whole new technology not even dreamed of by the quantum pioneers." [1, Location 1274].

With the related concept of superposition, nonlocality and entanglement may be described thus:

> "[Superposition means that] a quantum particle can be in a combination of two or more states at the same time, while [nonlocality] says that two quantum particles ... can somehow remain in touch with each other however far apart they are. [Combining these two ideas], the

> idea of two dice remaining in (nonlocal) contact with each other how ever far apart they are is known as entanglement." (*ibid.* Locations 1229–1235).

A potentially useful analogy for nonlocality and entanglement is illustrated in the multiple alignment shown in Figure 6. In this example, the sentence *The winds from the west are strong* is identified as a sentence (defined by the pattern '`<S> ... </S>`' in column 10), and parsed into constituents such as a noun phrase ('`<NPa> ... </NPa>`' in column 7), a qualifying phrase ('`<Q> ... </Q>`' in column 4), and so on.

```
0        1      2        3       4        5      6       7        8      9      10       11       12       13

                                                                                <S>
                                                                                <Num> ----------- <Num>
                                                                                                  PL
                                                                                </Num> ---------- </Num>
                                                                                <NPb> -- <NPb>
                                                         <NPa> ------------------------- <NPa>
                                                         <D> ---- <D>
                                                                  d1
the ------------------------------------------------------------- the
                                                         </D> --- </D>
                                                 <N> --- <N>
                                                 pln -------------------------------------------- pln
winds ------------------------------------------ winds
                                                 </N> -- </N>
                                                         </NPa> ------------------------ </NPa>
                                 <Q> --------------------------------------------------- <Q>
                         <PP> -- <PP>
                         pp2
from ------------------- from
                         </PP> - </PP>
                <NPa> ---------- <NPa>
                <D> --------------------- <D>
                                          d1
the ------------------------------------- the
                </D> -------------------- </D>
         <N> -- <N>
         sngn
west --- west
         </N> - </N>
                </NPa> --------- </NPa>
                                 </Q> -------------------------------------------------- </Q>
                                                                                </NPb> - </NPb>
                                                                         <V> -- <V>
                                                                         plv -------------------- plv
are -------------------------------------------------------------------- are
                                                                         </V> - </V>
                                                                                <A> ---------------------- <A>
                                                                                                           a1
strong --------------------------------------------------------------------------------------------------- strong
                                                                                </A> --------------------- </A>
                                                                                </S>

0        1      2        3       4        5      6       7        8      9      10       11       12       13
```

Figure 6: One of the two best multiple alignments created by the SP computer model with a store of Old patterns like those shown in columns 1 to 12, one pattern per column (representing grammatical structures, including words), and a New pattern (representing a sentence to be parsed), shown in column 0. The other best multiple alignment created by the SP computer model is the same as the one shown but with the Old patterns in a slightly different order across the columns.

For present purposes, the key point of interest is that, within sentences like this, there is a syntactic "dependency" between the plural subject of the sentence (*the winds*) and the plural main verb (*are*). The rule here is that, in English at least, if the subject is plural then the main verb must be plural, and if the subject is singular then the main verb must be singular. This kind of dependency is often described as "discontinuous" because it can jump over intervening structure such as *from the west*, and there appears to be no limit on how big that intervening structure may be.

Amongst the several ways in which discontinuous dependencies may be represented, one of the simplest is via the multiple alignment construct. This can be seen in column 12 in Figure 6 where the symbol '`pln`' is aligned with a matching symbol within the noun-phrase structure for *the winds*, the symbol '`plv`' is aligned with a matching symbol within the verb-phrase structure for *are strong*.

The fact that '`pln`' and '`plv`' both appear in one pattern (the one shown in column 12) is what marks the dependency between the two. This dependency is a recurrent feature of English. It is anticipated that, when SP is more fully developed, it will be able, via unsupervised learning, to discover that kind of recurrent pattern for itself, instead of it being provided ready-made in the store of Old patterns as was the case with the example in Figure 6.

Of course, computational linguistics is not the same as theoretical physics. But insights gained in the first area may have traction in the second. It seems possible that a non-local dependency between, for example, two entangled electrons, such that one has a clockwise spin while the other has a counter-clockwise spin, may be understood in a manner that is similar to our understanding of the phenomenon of syntactic dependencies in natural languages. In both cases:

- There is a correlation between the two elements of the dependency.
- The dependency may bridge arbitrarily large amounts of intervening structure.
- There is a kind of 'instant' communication in the sense that, if we know one element of a dependent pair, we know immediately what the other should be.

A possible objection is that, with natural languages, it is possible for people to make grammatical errors by saying things like *The winds from the west is strong*, but there seems to be no counterpart in physics.

One possible answer is that, while people may make errors in speaking or writing, such errors normally arise in the processes of writing or speaking and are not normally part of a person's considered knowledge of the language.

Another possible answer is that any differences of this kind between linguistics and physics is merely statistical—to do with differences in the strengths of relevant correlations.

5.9 A "waterspout" interpretation for some of the two-slits experiments

The famous two-slits experiment and its variations can indeed seem 'weird'. Here, a tentative "waterspout" interpretation of what is going on will be described, which may help to make things less baffling. The account here will draw on Jim Al-Khalili's excellent description of the basic experiment and its variations, and why they can be so puzzling [1, Chapter 1].

The basic experiment itself goes like this:

> "First, a beam of light is shone on a screen with two narrow slits in it that allow some of the light to pass through to a second screen where an interference pattern is seen. This is a sequence of light and dark bands that are due to the way the separate light waves emerging from the two slits spread out, overlap and merge before hitting the back screen. Where two wave crests (or troughs) meet they combine together to form a higher crest (or lower trough) that corresponds to more intense light and hence a bright band on the screen. But where a crest of one wave corresponds with a trough of the other they cancel out resulting in a dark patch. In between these two extremes some light survives and there is a gradual blending in of the pattern on the screen. It is therefore only because light behaves as a wave washing through both slits simultaneously that the interference pattern appears." [1, Chapter 1].

If the experiment is repeated with grains of sand, dropped through two slits in a horizontal screen, we end up with two piles of sand and no evidence of interference or other wave-like behaviour. But things are different if we repeat the experiment with atoms:

> "A special apparatus—let us call it an atomic gun for want of a better name—fires a beam of atoms at a screen with two appropriately narrow slits. On the other side, the second screen is treated with a coating that shows up a tiny bright spot wherever a single atom hits it. ... First, we run the experiment with just one slit open. Not surprisingly, we get a spread of light spots on the back screen behind the open slit. ...[We may suppose that] some of the atoms may just be bumping off

> the edges of the slit rather than going cleanly through and this might account for the spread. ... Next, we open the second slit and wait for the spots to appear on the screen. ... you would naturally guess that it would look like the two piles of sand. ... [But], surprise, surprise, atoms just don't behave like this. Instead, we see an interference pattern of light and dark fringes just as we did with light. The brightest part of the screen, believe it or not, is in the centre where we would not expect many atoms to be able to reach! (*ibid.* Locations 164–185).

To add to the puzzlement:

> "... not only has the double-slit experiment been performed with atoms, but it has also been done by firing individual atoms one at a time! That is, only when we see the flash of light on the back screen signalling the arrival of the atom do we fire the next one, and so on. There is only ever one atom travelling through our apparatus at any one time. Each atom that manages to get through the slits leaves a tiny localized spot of light somewhere on the screen. ... What we see is quite incredible. The spots gradually build up on the screen and light bands of an interference pattern slowly emerge where there is a high density of spots. In between these bands are dark regions where no or very few atoms land. ... To recap, each atom fired from the gun leaves it as a tiny 'localized' particle and arrives at the second screen also as a particle, as is evident from the tiny flash of light when it arrives. But in between, as it encounters the two slits, there is something mysterious going on, akin to the behaviour of a spread-out wave that gets split into two components, each emerging from a slit and interfering with the other on the other side. How else can we rationalize the way the atom has to be aware of both slits at the same time?" (*ibid.* Locations 197–210).

Furthermore:

> "With a detector in place that records which slit each atom passes through, the interference pattern disappears. It is as though the atoms do not wish to be caught in the act of going both ways at once, and only travel through one slit or the other. Two bands form on the screen adjacent to the slits as a result of particle-like behaviour, similar to what happens with the sand. With the detector turned off we now have no knowledge of the route taken by each atom. Now that their secret is safe, the atoms revert to their mysterious wave-like behaviour

and the interference pattern comes back! ... It is only when the atom is being watched that it remains as a particle throughout. Clearly the act of observing the atom is crucial." (*ibid.* Locations 249–254).

And:

> "In what are known as 'delayed choice' experiments it is possible to have a detector in place and only switch it on after the atom has gone through the slits. ... With modern high-speed electronics the detector can be close enough to one of the slits to be able to tell whether the atom had come through it, and yet it need only be switched on after the atom, behaving like a spread-out wave, has emerged from both the slits, but before it reaches the detector. Surely now it is too late for the atom to suddenly decide to behave like a localized particle that has only passed through one of the slits. Apparently not. In such experiments, the interference pattern is nevertheless found to disappear." (*ibid.* Locations 261–268).

Here, in outline, is an alternative, and perhaps less puzzling, "waterspout" view of what is going on with atoms in the two-slits experiments:[13]

- *Smooth interference patterns from coherent light in the basic experiment are what we would expect to see.* With the coherent light that is used in the basic two-slits experiment, we would expect to see the smooth interference patterns that are in fact observed.

- *Atoms may be, fundamentally, waves.* Instead of supposing that atoms can ever be like grains of sand, let us suppose, in accordance with the suggestions in Section 5.2, that they *always* behave like waves.[14]

- *Each atom may be a zone of relatively high energy within a wave.* While atoms may be, fundamentally, waves, we may suppose that they are not exactly like the coherent light which features in the basic two-slits experiment. Instead, we may suppose that each atom is a zone within a wave in which there is a relatively high concentration of energy, perhaps in the form of a relatively high frequency, or a relatively high amplitude, or both those things.

[13] Similar things may be said about molecules [1, Locations 313–340]

[14] Strictly speaking, in accordance with Section 5.2, grains of sand are also waves which cannot be recognised as discrete objects without a probabilistic analysis as described in Section 5.5.

- *The creation of a 'waterspout'.* When an atom hits the second screen in the two-slits experiment, we may suppose that what happens is comparable in complexity with what happens when ordinary waves on the sea crash into a rocky shore (in accordance with the idea that all theories are 'emergent' (Section 5.2), meaning that any given theory is founded on something more complex underneath).

  With ordinary waves in the sea, it can happen that, where there is a particularly large wave, or where a configuration of rocks concentrates the energy in the wave, or both those things, a spout of water can be sent up which is much higher than normal. With the two-slits experiment we may suppose that something like this will happen wherever there is a relatively high concentration of energy, as would be the case with an atom, and we may suppose that the 'waterspout' is itself a zone in which energy is even more concentrated.

- *A 'waterspout' is not an atom.* Assuming that things work roughly as described, it is not hard to imagine that the metaphorical waterspout would leave a relatively discrete mark on the sensitive screen. But, contrary to the assumption that such a mark is caused by the impact of a bullet-like atom ("Despite an atom being a tiny localized particle (after all, each atom hits the screen at a single point)" [1, Location 190]), we may suppose that it is an artifact, caused by the interaction of a wave and the second screen, which has the appearance of having been produced by the impact of a bullet-like atom.

- *Each atom may pass through both slits, even when atoms are fired one at a time.* When atoms are fired, one at a time, at the double-slit screen, it seems that each one, as a wave, would pass through both slits. As with any wave passing through the two slits, there would be interference on the other side. But, since the wave of any one atom would be relatively concentrated in space and time, it seems likely that there would be a narrower range of interference fringes than with coherent light, perhaps with only one dominant crest that lasts for a relatively short time. The 'waterspout' from this crest would create a flash of light which may be interpreted as "the atom" hitting the screen. Nevertheless, interference associated with each atom, together with likely variations amongst the several atoms, would allow a range of interference fringes to be built up over time that is similar to interference fringes with coherent light.

- *A detector, switched on, may be enough to disturb interference patterns.* In versions of the experiment with atoms, and with one or more detectors for atoms that are switched on throughout the experiment, or delayed until each atom has passed through the first screen, it seems possible that the disappearance of interference patterns is simply the result of disturbance by the presence of a detector that is switched on—in much the same way that the 'beats' from two notes that are very close in pitch but not identical can be disturbed by small changes in the arrangements; or, indeed, like the quantum phenomenon of *decoherence*: how "large quantum systems [can be] dramatically sensitive to uncontrolled external influences" [1, Location 1390].

- *Contrast with quantum mechanics.* In two respects, the interpretation of two-slits experiments described here contrasts with how these experiments are often interpreted in quantum mechanics:
  - The interpretation described here—that an atom is always a wave but may, when it interacts with a sensitive screen, create an artifact that has the appearance of a small bullet-like entity—contrasts with the quantum mechanics view that quantum-scale particles do not exist until they are observed.[15]
  - Instead of supposing that there is a "collapse of the wave function" when the atom is detected, the scheme described above may be seen as a process of interpretation, broadly similar to those described in Section 5.7 but performed by an inanimate screen rather than a human or other animal.

5.10 Statistical theory

Within MT, statistical theory is well established and has proved its worth in many applications in science and other areas. But of course there is always room for improvement.

Potential advantages of SP, either on its own or as part of P-NMFS, include:

- The strengths of SP in the making of predictive inferences and the calculation of associated probabilities (Sections 4.1 and 4.3.3) flow directly from the central role of information compression in the workings of SP, and because of

[15] "Electrons don't always exist. They exist when they interact. They materialize in a place when they collide with something else. ... When nothing disturbs it, an electron does not exist in any place." [33, Location 1331–1339]; and "[A quantum-scale object's] position and velocity, its angular momentum and its electrical potential, and so on, acquire reality only when it collides—'interacts'—with another object. It is not just its position which is undefined, as Heisenberg had recognized: no variable of the object is defined between one interaction and the next." [33, Location 1365].

the intimate relation between information compression and concepts of prediction and probability (Appendix B.8). In effect, compression of incoming data via unsupervised learning in SP achieves a thorough statistical analysis of those data.

- The multiple alignment concept has proved to be a powerful vehicle for several kinds of probabilistic reasoning (Section 4.3.2), and for their seamless integration in any combination (Section 4.4). Collectively, these several kinds of reasoning, working together, have potential as a powerful aid to statistical inference.

- The SP system has proved to be an effective alternative to Bayesian reasoning, including reasoning in Bayesian networks and an alternative to Pearl's analysis [30, Section 2.2.4] of the phenomenon "explaining away" ([45, Section 10.2], [49, Section 7.8]).

- In addition to its strengths in learning patterns of association, SP, via unsupervised learning, has strengths and potential to learn entire structures, including the potential to learn 3D structures ([51, Sections 6.1 and 6.2]). In its strengths and potential for the learning of structures, it goes beyond mainstream statistics.

5.11 All-or-nothing and probabilistic computations

Notwithstanding Gregory Chaitin's discovery that, at a deep level, there is randomness in number theory (Section 4.1), much of mathematics and logic has an all-or-nothing flavour that assumes that, with calculations like $2 + 2 = 4$ and deductions like *If all humans are mortal and Socrates is human, then Socrates is mortal*, the conclusions are certain, without any qualification.

Although that kind of certainty is often useful, it does not sit well with the view that mathematics is fundamentally a set of techniques for compressing information, and their application [59, Section 3], and that mathematics, for that reason, has a probabilistic dimension (Appendix B.8).

P-NMFS, with an acceptance that mathematics is fundamentally about compression of information, is likely to provide a more satisfactory perspective in which there will, in certain situations, be inferences with probabilities that are 0 or 1 or very close to those values (see, for example, [45, Section 10.1.3] or [49, Section 7.7.3]) but, very often, there will also be inferences with probabilities that are not close to 0 or 1.

This kind of amalgamation of probabilistic and non-probabilistic forms of reasoning—flowing from the view that information compression is a fundamental feature of mathematics, logic, and computing—is arguably more satisfactory than forms of "probabilistic logic" in which traditional concepts of probability and of logic are bolted together without any deeper integration.

This is not merely a matter of theoretical purity. There are many areas of science which are likely to benefit from smooth integration of probabilistic and non-probabilistic forms of reasoning.

5.12 Uniformity in the representation of knowledge

As noted in Section 2.3, scientific knowledge is at present expressed with a combination of natural language, diagrams, drawings, static photographs, videos, computer progrms, and mathematics. This has worked well for many years but there are potential advantages with P-NMFS.

In view of the versatility of SP in the representation of diverse kinds of knowledge and its potential as a UFK (Sections 4.2 and 4.3), there is potential for P-NMFS to be a vehicle for the representation of *all* kinds of scientific knowledge, via the use of SP patterns that are processed within the multiple alignment framework.

It may seem strange, for example, to replace drawings of botanical specimens or of anatomical parts of animals with representations via a new version of mathematics,[16] but SP and thus P-NMFS that incorporates SP has clear potential with computer vision and the related field of computer graphics, as discussed in [51]. There is potential not only for 2D representations but also for the representation of 3D structures, as discussed in [51, Section 6.1 and 6.2].

Of course it would be excessively labour-intensive for such representations to be created by people in the manner of computer programming. Fortunately, there is potential for SP to create SP representations automatically from photographs and videos, with such inputs treated as New information.

The potential benefits of a uniform format for scientific knowledge are chiefly in the processing of that knowledge, as described in Sections 5.12.1 and 4.3.3.

5.12.1 Unsupervised learning in science

An important reason for representing different kinds of scientific knowledge with a uniform system is that it is likely to facilitate the discovery, via unsupervised learning, of significant patterns, associations or structures that may otherwise remain obscure.

A simple example, discussed in [53, Section III-A.1], is how the association between thunder and lightning would be relatively hard for an artificial system to

[16] A notable early example of the application of mathematics to the description of botanical and zoological forms is D'Arcy Thompson's book *On Growth and Form* [41].

discover if those two things were, on various occasions, referred to by a variety of names or represented with different formalisms or formats.

Since people have little difficulty in learning that thunder is normally preceded by lightning, we may guess that there is some uniformity in how those two things are represented in our brains, despite the fact that thunder is something we hear while lightning is something we see.

5.13 "Shut up and calculate" versus the DONSVIC principle

There is an apparent conflict between the DONSVIC principle (Appendix B.4) and the exhortation, to anyone who is trying to make sense of quantum mechanics in terms of non-mathematical concepts, that they should "Shut up and calculate" [1, Locations 106 and 119].

DONSVIC implies that one should be able to understand quantum mechanics in terms that are natural and intuitive, while "Shut up and calculate" implies that, with quantum phenomena, that aspiration is futile and that the mathematics of quantum mechanics is all that matters.

In this connection, the word "natural" probably needs to be qualified. Phenomena such as non-locality and entanglement (discussed in Section 5.8) can seem seem unnatural when they are first encountered. But this may be attributed to the fact that they are not things that are normally encountered in everyday life rather than any intrinsic difficulty in understanding what they mean. The tentative view adopted here is that the concept of naturalness in the DONSVIC principle may apply to things like nonlocality and entanglement when one's initial unfamiliarity with the concepts has worn off.

But a different conclusion probably applies to quantum mechanics as a whole. As was noted in Section 2.2, the absence of a satisfactory natural interpretation of the mathematics of quantum mechanics may reflect a weakness in mainstream mathematics rather than a weakness in the brain power of people like Albert Einstein.

P-NMFS as described in Section 4 may, if it has been well developed, yield an interpretation of the phenomena which are the subject of quantum mechanics which is: 1) intuitive and natural, in accordance with the DONSVIC principle; and 2) as accurate as the mathematics in today's quantum mechanics in the calculation of predictions. As noted above, phenomena such as nonlocality and entanglement appear to be genuine (Section 5.8), so if P-NMFS is working properly, it should accommodate them.

In support of this view is the following somewhat speculative foray into cognitive psychology. It seems that new concepts in science may, quite often, take shape in forms that do not directly match the conventions of mainstream mathematics. Here are three putative examples:

- Michael Faraday developed his ideas without a knowledge of mathematics and James Clerk Maxwell translated them into mathematical form.

  > "Without knowing mathematics, [Faraday] writes one of the best books of physics ever written, virtually devoid of equations. He sees physics with his mind's eye, and with his mind's eye creates worlds." [33, Location 623].

  and

  > "Maxwell quickly realizes that gold has been struck with [Faraday's] idea. He translates Faraday's insight, which Faraday explains only in words, into a page of equations. These are now known as Maxwell's equations. They describe the behaviour of the electric and the magnetic fields: the mathematical version of the 'Faraday lines'." [33, Location 677].

- Charles Darwin described his theory of evolution by natural selection with words and pictures. To this day, it is still normally described in the same way.

- It seems that Albert Einstein's ideas were generally developed first in non-mathematical form and only later cast into mathematics:

  > "Einstein had a unique capacity to imagine how the world might be constructed, to 'see' it in his mind. The equations, for him, came afterwards; ... For Einstein, the theory of general relativity is not a collection of equations: it is a mental image of the world arduously translated into equations." [33, Location 1025].

It seems possible that P-NMFS, by incorporating SP concepts of knowledge representation and intelligence, may provide the means of representing and processing scientific concepts in forms that are more in accord with the intuitions of some leading scientists than is conventional mathematics.

6 Conclusion

In this paper, it is argued that, despite its many strengths and undoubted value in science, mathematics also has some weaknesses as a handmaiden for science (Section 2). It has, for example: some shortcomings in the representation of knowledge and in AI kinds of processing; under-expressiveness can be a problem; and there are several problems with over-expressiveness, including problems with infinities, and problems with mathematically-based scientific theories that appear to be unfalsifiable.

To overcome the afore-mentioned weaknesses in mathematics, development of P-NMFS may help to meet the needs of science more closely.

In its essentials, the proposal here is that P-NMFS may be created as an amalgamation of mathematics as it is today with SP as it is today, including developments in both areas that are anticipated now. It is envisaged that the amalgamation would incorporate concepts from SP such as multiple alignment and unsupervised learning, that concepts in mathematics would be interpreted within the SP framework as far as possible, that SP may be modified where necessary, and that structures and processes from one area would be dropped where equivalent things can be done better with structures and processes from the other area.

In this paper, it is envisaged that P-NMFS, like SP, will be fundamentally probabilistic, that it will exhibit versatility in the representation of knowledge and versatility in aspects of intelligence including unsupervised learning, that it will provide for the seamless integration of diverse kinds of knowledge and diverse aspects of intelligence, and that it will overcome the previously-mentioned problems of under-expressiveness and over-expressiveness. In particular, P-NMFS will provide a means of evaluating scientific theories in terms of information compression as described in Appendix E.

In several sections, there is discussion of some possible implications for science of the NMFS proposal and the thinking behind it:

- Since there would be potential with P-NMFS for the reinterpretation of both quantum mechanics and the theory of general relativity in terms of concepts from SP, there is a corresponding potential for the long-sought-after unification of quantum mechanics and general relativity. But it is unlikely that anyone will ever develop a "theory of everything", and it is likely that, quite often, it will be necessary for us to accept two or more theories, in one area or in diverse areas, that are fully or partially independent of each other.

- Without adopting the solipsistic view that each person's ideas are the only things that he or she can be sure of, with the possibility that the external world might not exist, the view adopted here is that there is a world outside ourselves but that our knowledge of it is necessarily filtered through our senses, sensing devices, methods of analysis including AI, and the 'cognitive machinery' of our brains.

These filters can have a considerable impact on the shape of our theories. More specifically, this paper proposes that *multiple alignment*, within SP, may be a *universal framework for the representation and processing of diverse kinds of knowledge* (UFK) [53, Section III], and that, potentially, all scientific knowledge, including knowledge of physics, may be cast in that mould.

- The raw material for science may be seen to be information in the form of waves which, for reasons given in Section 5.2, may encompass "energy" and "matter". It seems that all such information has a fractal character meaning that, in principle, there is no limit to the level of detail we may consider. But there will be practical limits set by our sensing technologies. And scientists may sometimes choose to set higher limits in order to study higher-level regularities in the world.

  An implication of the idea that there is, in principle, no limit to the level of detail we may consider, is that most theories are "emergent" in the same way that the relatively simple gas laws emerge from the relative complexity implied by the kinetic theory of gases.

- In the application of mathematics in science, there has often been a need to make approximations, and it is likely that the same will be true of P-NMFS, for reasons outlined in Section 5.3.

- Sources of uncertainty in science include: "noise" meaning unpredictability in data (the non-redundant information that is left after our best compression technique(s) have been applied); uncertainties arising from the making of approximations; the making of scientific inferences; uncertainties related to Gödel's incompleteness theorems; observer effects; and uncertainties arising from the measurement of speed.

- The way in which objects in physics and cosmology are conceived as theoretical primitives contrasts sharply with the way in which objects are conceived in artificial intelligence and cognitive science, where they are seen as relatively high-level concepts that must be abstracted or constructed from raw data. SP has strengths and potential in that kind of abstraction.

- Similar things may be said about concepts like motion, speed, velocity, momentum, and acceleration.

- A potentially useful analogy for the concept of superposition in quantum mechanics is ambiguity in the realm of human perception. This has been illustrated with examples from the SP computer model.

- A potentially useful analogy for nonlocality and entanglement in quantum mechanics is the phenomenon of discontinuous dependencies in natural languages. This has been illustrated with an example from the SP computer model.

- Section 5.9 describes a "waterspout" interpretation for some of the two-slits experiments. Here, a "waterspout" is a concentration of energy, like a waterspout that may be created when waves in the sea crash into a rocky shore. It is a possible artifact in the two-slits experiments, created when an electromagnetic wave interacts with a sensitive screen. Interpreting each spot on the screen as an artifact may be preferable to its interpretation as the result of a particle hitting the screen.

- Potential advantages of SP in the realm of statistics may be seen: in the way that compression of incoming data via unsupervised learning achieves, in effect, a thorough statistical analysis of those data; in the way that the multiple alignment concept has proved to be a vehicle for several kinds of probabilistic reasoning and for their seamless integration in any combination; in that SP has proved to be an effective alternative to Bayesian reasoning, including reasoning in Bayesian networks; and in the way that SP can learn structures as well as statistical associations and correlations.

- SP provides a potentially useful amalgamation of probabilistic and non-probabilistic forms of reasoning, all flowing from the central role in the system of information compression.

- In view of the versatility of SP in the representation of diverse kinds of knowledge, there is potential for P-NMFS to be a vehicle for the representation of *all* kinds of scientific knowledge. The potential benefits of a uniform format for different kinds of knowledge are chiefly in the processing of that knowledge, especially in the automatic or semi-automatic creation of theory.

- P-NMFS may, if it has been well developed, yield an interpretation of the phenomena which are the subject of quantum mechanics which is intuitive and natural, and also as accurate as the mathematics in today's quantum mechanics. That expectation may apply to phenomena such as nonlocality and entanglement that can seem strange at first.

A Human cognition, science and mathematics

A little reflection shows that science and mathematics, as products of the human mind, must necessarily be influenced by the psychology of learning, perception, and thinking, meaning cognitive psychology. That influence appears to be strongest in physics, especially quantum mechanics.

This appendix expands a little on the brief remarks in Section 3 about the relationships amongst human cognition, science, and mathematics:

- Instead of a Platonist view that mathematical entities are "... numinous and transcendent entities, existing independently of both the phenomena they order and the human mind that perceives them." [17, pp. 95–96], mathematics may be seen as a psychological phenomenon, the product of the human intellect, and a toolkit for the analysis of scientific observations and for scientific theorising.

- The importance of information compression in science and mathematics (Section 3) is in line with the abundance of other evidence for the importance of information compression in human learning, perception, and cognition ([48, 58], [49, Chapter 2]).

- There is no need to adopt the solipsistic view that our ideas are the only things we can be sure of and that the external world might not exist.[17]

- The view adopted in this paper is that there is a world outside ourselves but that our knowledge of it is necessarily filtered through our senses, through any artificial sensing devices that we may use, through methods of analysis that we may apply (including artificial intelligence), and through our brains. These filters, particularly the last, can have a considerable impact on the shape of our theories and beliefs about the external world.

- More specifically, the conjecture, supported by evidence, that *multiple alignment*, within SP, may be a *universal framework for the representation and processing of diverse kinds of knowledge* (UFK) [53, Section III], implies that all scientific knowledge, including knowledge of physics, may be cast in that mould.

  Notice that this is not the kind of "catch all" theory that explains everything and nothing (Appendix E). This is because it may be falsified with a

[17]It appears that, in rejecting solipsism, we may also be rejecting an important part of the "Copenhagen" interpretation of quantum mechanics: "... by giving the observer ... a privileged status, the Copenhagen interpretation denies the existence of an objective reality that exists in the absence of observation." [1, Location 1829]

demonstration that one or more kinds of knowledge cannot in principle be accommodated within the multiple alignment framework, as it is now or with projected future developments. And it may at some stage become feasible to evaluate the theory in terms of information compression.

- In this paper, aspects of physics where aspects of human cognition may be seen to have an important role have been described in Sections 5.5, 5.6, 5.7, and 5.8.

B Outline of the SP theory

The *SP theory of intelligence* and its realisation in the *SP computer model* is the product of a long programme of research, seeking to simplify and integrate observations and concepts across artificial intelligence, mainstream computing, mathematics, and human learning, perception and cognition, with information compression as a unifying theme.

The name "SP" is short for *Simplicity* and *Power*. This is because information compression is central in the workings of SP and information compression may be seen to be a process of maximising the *simplicity* of a body of information, **I**, by extracting redundancy from **I**, whilst retaining as much as possible of its non-redundant descriptive *power*. This may be seen to be equivalent to Occam's Razor [59, Section 2.1].

The SP system is described most fully in [49] and more briefly in [45]. Details of related papers, many of them about potential applications of the system, may be found on www.cognitionresearch.org/sp.htm.

The main sources of inspiration for the SP theory are:

- Research by Fred Attneave (see, for example, [2]), Horace Barlow (see, for example, [3, 4]), and others, showing the importance of information compression in the workings of brains and nervous systems. Some relevant evidence is presentedd in [59] (see also [48], [49, Chapter 2]).

- A programme of research developing computer models of language learning (summarised in [47]) in which the importance of information compression became increasingly clear.

- Ray Solomonoff's research on *Minimum Length Encoding*, and *Algorithmic Probability Theory* [36, 37, 38]. In [37], he argues that the great majority of problems in science and mathematics may be seen as either "machine inversion" problems or "time limited optimization" problems, and that both kinds of problem can be solved by inductive inference via the principle of minimum length encoding.

Key features of SP are:

- *Patterns and symbols.* In SP, all kinds of information or knowledge are represented as *patterns*, meaning arrays of atomic *symbols* in one or two dimensions. Here, a 'symbol' is simply a mark that may be matched with any other symbol to determine whether it is the same or different. No other result is permitted. At present the SP computer model works only with one-dimensional patterns but it is envisaged that it will be generalised to work with two-dimensional patterns.

- *The SP system as a brain-like system.* The SP system is conceived as a brain-like system that receives *New* patterns via its senses and stores some or all of them in compressed form as *Old* patterns.

- *Information compression via the matching and unification of patterns.* The SP system is dedicated to lossless *information compression via the matching and unification of patterns* (ICMUP), as outlined in [59, Section 5].

- *Lossless compression of information via the building and processing of multiple alignments.* More specifically, SP is dedicated to lossless information compression via the building and processing of *multiple alignments*, as outline in Appendices B.1 and B.2.

- *Optional discarding of non-redundant information.* As already mentioned, SP is designed to achieve lossless compression of information, retaining all non-redundant information and discarding only redundant information. But for some applications, users may if they wish achieve lossy compression of information by discarding some non-redundant information.

Other distinctive features of SP are described in Appendices B.3 to B.11.

B.1 The building of multiple alignments

Within SP, there are two key processes, described in this subsection and the one that follows.

When the system has a reasonably large set of Old patterns in store, then, when each New pattern is received, it is likely to be incorporated in a *multiple alignment* like the one shown in Figure 7.

```
0                 1                2                  3              4              5                  6

                  <species>
                  acris
                  <genus> ---------------------------------------------------------------------------- <genus>
                  Ranunculus ------------------------------------------------------------------------- Ranunculus
                                                                                    <family> --------- <family>
                                                                                    Ranunculaceae ---- Ranunculaceae
                                                                     <order> ------ <order>
                                                                     Ranunculales - Ranunculales
                                                      <class> ------ <class>
                                                      Angiospermae - Angiospermae
                                   <phylum> --------- <phylum>
                                   Plants ----------- Plants
                                   <feeding>
has_chlorophyll ------------------ has_chlorophyll
                                   photosynthesises
                                   <feeding>
                                   <structure> ------ <structure>
                                                      <shoot>
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hairy ----------- hairy
</stem> --------- </stem> -------------------------- </stem>
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                  compound
                  palmately_cut
                  </leaves> ------------------------ </leaves>
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                                                                                    all_parts_free
                                                                                    </arrangement>
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                  not_reflexed
                  </sepals> ------------------------------------------------------- </sepals>
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                                                                                                       five
                                                                                    </number> -------- </number>
                  <colour> -------------------------------------------------------- <colour>
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                  </colour> ------------------------------------------------------- </colour>
</petals> ------- </petals> ------------------------------------------------------- </petals> -------- </petals>
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numerous -------------------------------------------------------------------------- numerous
</stamens> ------------------------------------------------------------------------ </stamens>
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                                                                                    ovary
                                                                                    style
                                                                                    stigma
                                                                                    </pistil>
                                                                                    </hermaphrodite>
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                                                      </shoot>
                                                      <root>
                                                      </root>
                                   </structure> ----- </structure>
<habitat> ------- <habitat> ------ <habitat>
meadows --------- meadows
</habitat> ------ </habitat> ----- </habitat>
                  <common_name> -- <common_name>
                  Meadow
                  Buttercup
                  </common_name> - </common_name>
                                   <food_value> ----------------------------------- <food_value>
                                                                                    poisonous
                                   </food_value> ---------------------------------- </food_value>
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                                                                     </order> ----- </order>
                                                                                    </family> -------- </family>
                  </genus> --------------------------------------------------------------------------- </genus>
                  </species>

0                 1                2                  3              4              5                  6
```

Figure 7: The best multiple alignment created by the SP computer model, with a set of New patterns (in column 0) that describe some features of an unknown plant, and a set of Old patterns, including those shown in columns 1 to 6, that describe different categories of plant, with their parts and sub-parts, and other attributes. Reproduced from Figure 16 in [45], with permission.

Here, the concept of multiple alignment in SP, which has been borrowed and adapted from a similar concept in bioinformatics, means an alignment of two or more sequences of symbols so that matching symbols are brought into line. In SP, one of the sequences (sometimes more than one) is a New pattern which by convention is shown in column 0. All the other sequences, one in each column of the multiple alignment, are Old patterns.

For each New pattern, SP is designed to create or discover one or more multiple alignments that allow the New pattern to be encoded economically in terms of the Old patterns. How the encoding is done is described in [49, Section 3.5] and [45, Section 4,1]. When 'good' multiple alignments have been found, the code pattern for the best of them—which represents a compressed version of the New pattern—may be added to the store of Old patterns.[18] There may be a case for adding code patterns for the second or third best multiple alignments as well.

As in bioinformatics, the process of creating or discovering 'good' multiple alignments is complex, because, normally, there are astronomically large numbers of possible multiple alignments for any given set of sequences. Most of them are much less tidy and plausible than the one shown in Figure 7. To overcome the complexity, it is necessary to use heuristic techniques, building each multiple alignment in stages and weeding out low-scoring multiple alignments at each stage. This can give results that are "reasonably good" but are not guaranteed to be theoretically ideal.

The multiple alignment shown in Figure 7 may be seen to achieve the effect of recognising an unknown plant, with features shown in column 0, as a Meadow Buttercup (species *acris*, with features shown in column 1). At the same time, the unknown plant is recognised as belonging to the genus *Ranunculus* (column 6), to the family Ranunculaceae (column 5), to the order Ranunculales (column 4), and so on. Different features are stored at different levels.

This example shows how, within the multiple alignment framework, it is possible to represent a class-inclusion hierarchy, as described in the last paragraph, and also a part-whole hierarchy: '`not_reflexed`' (column 1), '`sepals`' (columns 1 and 5), '`flowers`' (columns 5 and 3), '`structure`' (columns 3 and 2), and '`phylum`' (column 2). And the example shows how those two kinds of hierarchy may be integrated smoothly, without awkward incompatibilities.

B.2 Unsupervised learning

Unsupervised learning in SP may be seen to be achieved in stages:

[18]The SP71 computer model does not in fact add code patterns to its store of Old patterns. But it is likely that later versions of the SP computer model will do that.

- *Storing New patterns directly.* In the earliest phase, when there are relatively few Old patterns in store, unsupervised learning will be largely a process of adding New patterns directly to the store of Old patterns, normally with the addition of two or more system-generated symbols as described below. Thus, for example, a New pattern like '`c o m e f o r b r e a k f a s t`' may be stored as something like '`A 1 c o m e f o r b r e a k f a s t #A`', where '`A`', '`1`', and '`#A`', are "identification" or "ID" symbols, for use by SP at some later stage, for the creation of code patterns.

- *Building multiple alignments.* At later stages, when there is a reasonably large number of Old patterns in store, it will become possible for the system to build multiple alignments like the one shown in Figure 7. When there are relatively few Old patterns in store, the best multiple alignments created by the system will be relatively simple. With larger numbers of Old patterns in store, it is likely that the best multiple alignments created by the system will be relatively complex.

- *The creation of grammars.* As the system starts to build multiple alignments, two other aspects of learning will come into play:

  - The system begins to create Old patterns from partial matches between patterns. For example, a partial match between '`c o m e f o r b r e a k f a s t`' and '`c o m e f o r l u n c h`' may lead to the creation of patterns like '`c o m e f o r`', '`b r e a k f a s t`', and '`l u n c h`'. Similar things may happen with code patterns that have been added to the store of Old patterns.
  - The system begins to create *grammars*—where each grammar is simply a collection of Old patterns—which are relatively good at encoding New patterns in terms of Old patterns. As with the building of multiple alignments, there is normally an astronomically large number of possible grammars and so it is necessary to use heuristic techniques to find one or two that are relatively good. As with the building of multiple alignments, this means the use of heuristic search: building grammars in stages and, at each stage, weeding out the low-scoring grammars. As before, this can give results that are "reasonably good" but are not guaranteed to be theoretically ideal.

When the repository of Old patterns is initially empty, what is normally the most interesting product of learning from a given body of information (**I**) (comprising New patterns) is the grammar (**G**) which yields a relatively small encoding (**E**) of **I** in terms of **G**. Of the two main results, **G** and **E**, it is normally **G** which is of most interest. This represents a distillation of the "essence" of **I**, meaning the recurrent features and structures in **I**.

It is anticipated that, when unsupervised learning in the SP computer model is more fully developed, it will exploit all five of the variants of ICMUP described in [59, Section 5.1]: chunking-with-codes, schema-plus-correction, run-length coding, class-inclusion hierarchies with inheritance of attributes, and part-whole hierarchies with inheritance of contexts. It is also anticipated that it will be able to encode discontinuous dependencies in structure, as described in Section 5.8.

B.3 Distinctive features and advantages of SP

Other features of SP are summarised in this and the following subsections.

Distinctive features of SP and its advantages compared with AI-related alternatives are described in [57]. In particular, Section V of the paper describes 13 problems with deep learning in artificial neural networks and how, in SP, those problems may be overcome.

B.4 The DONSVIC principle

In connection with unsupervised learning, an important principle in SP is DONSVIC: "The discovery of natural structures via information compression" [45, Section 5.2]. The key idea here is that when techniques for information compression are designed to achieve relatively high levels of information compression, and when they are applied in such a way that they do in fact achieve relatively high levels of information compressiom, the resulting structures are likely to be ones that people will see as being natural.

This principle draws on the belief, supported by evidence ([48], [49, Chapter 2], [58]), that the human brain is one of the most effective systems for the compression of information that is known to science, and that the way we conceptualise the world, 'naturally', flows from the highly-effective ways in which we compress information.

Of course, the word "natural" is not precise. And it may need to be qualified in the light of concepts such as those associated with quantum mechanics (Section 5.13).

B.5 Integration of five basic techniques for the compression of information

An elegant feature of the multiple alignment concept is that it supports all five of the variants of ICMUP described in [59, Section 5.1]. It also provides for the seamless integration of these five techniques, in any combination. With the SP computer model, these things can be demonstrated in multiple alignments with user-supplied patterns, but more work is needed for these techniques to be fully realised via unsupervised learning.

B.6 Modelling commonsense reasoning and commonsense knowledge in SP

Several problems for AI systems in the modelling of commonsense reasoning and commonsense knowledge are described by Ernest Davis and Gary Marcus in [10]. How these problems may be solved via SP is described in a draft paper [55].

B.7 Versatility and adaptability in the representation of knowledge and in aspects of intelligence

As is outlined in Sections 4.2, 4.3, and 4.4, SP exhibits versatility in the representation of diverse kinds of knowledge, versatility in diverse aspects of intelligence, including unsupervised learning, and seamless integration of diverse kinds of knowledge and diverse aspects of intelligence in any combination.

These strengths of SP flow largely from the central role in the system of the powerful concept of multiple alignment. This concept, as it has been developed in SP, has the potential to be the "double helix" of intelligence, to be as significant for an understanding of human-like intelligence as is DNA for biological sciences.

B.8 The SP system is fundamentally probabilistic

The intimate relation between information compression and concepts of prediction and probability [36, 38, 24] means that SP is fundamentally probabilistic. For that reason, it is easy to calculate absolute and relative probabilities for multiple alignments and for inferences that may be drawn from multiple alignments ([49, Section 3.7], [45, Section 3.7]).

By contrast with Bayesian networks, for example, which store their statistical knowledge as tables of conditional probabilities, which can become very large, SP stores all its statistical knowledge very much more simply as a frequency of occurrence for each SP pattern, and uses that information for the calculation of absolute and relative probabilities, if or when required. How Bayesian networks may be modelled with SP is described in [49, Section 7.8] and [45, Section 10.2].

B.9 Software engineering and SP

Although it is not obvious at first sight, SP, via multiple alignment, may model several different features of programming languages, with corresponding potential in software engineering [54, Section 6.6]. Features that may be modelled include: *procedure*, *function*, or *subroutine*, with or without *parameters*; *variables*, *values*, and *types*; *conditional statements*; *repetition of procedures* via recursion; the *integration of 'programs' and 'data'*; and *object-oriented design*. There is also potential for *the processing of parallel streams of information* as described in [52, Sections V-G, V-H, and V-I, and Appendix C].

B.10 Unfinished business

As described in [45, Section 3.3], there there are still things to do in the development of the SP computer model. In brief, with updates:

- The system needs to be generalised to process *two-dimensional patterns* as well as 1D patterns.
- There is a need to understand better how *perceptual features in speech and in visual images* may be abstracted from raw data.
- Two main weaknesses need to be remedied in the processes for unsupervised learning: although the system learns the highest levels of abstraction, it cannot learn intermediate levels; and the system does not learn discontinuous dependencies in structure.
- A better understanding is needed of how the system may be applied to the representation and processing of numbers.
- A better understanding is needed of how the system may be applied to the representation and processing of concepts such as 'motion', 'speed', and 'acceleration' [51, Section 5.3], and other concepts that have not yet been examined in the SP programme of research.
- The SP computer model needs a more 'friendly' user interface.
- Since SP, like other AI systems, is intrinsically 'hungry' for computer power, especially with unsupervised learning, there are potential benefits in porting the existing model on to an existing high-performance computer and applying parallel processing where possible. Although SP can be demanding for computer power, there are reasons to believe that a mature version of SP is likely to be very much less demanding than, for example, systems for deep learning in artificial neural networks [34]. There is relevant evidence and arguments in [53, Sections VIII and IX] and [57, Sections IV and V-E].

B.11 SP-neural

There appears to be potential to translate key concepts in the SP theory—chiefly the representation of knowledge with SP patterns, and the building of multiple alignments—into equivalent concepts expressed in terms of neurons and their interconnections and inter-communications ([56], [49, Chapter 11]). These ideas, called *SP-neural*, are radically different from "artificial neural networks" that are popular in computer science, including artificial neural networks that feature in research on "deep learning". In particular, mechanisms for learning in SP, including SP-neural, are quite different from mechanisms for learning in artificial neural networks [57, Section V-D].

Figure 8 shows how a structure of class-inclusion relations and part-whole relations may be represented by *pattern assemblies*—the neural equivalent of SP patterns in the non-neural SP theory—and by connections between pattern assemblies.

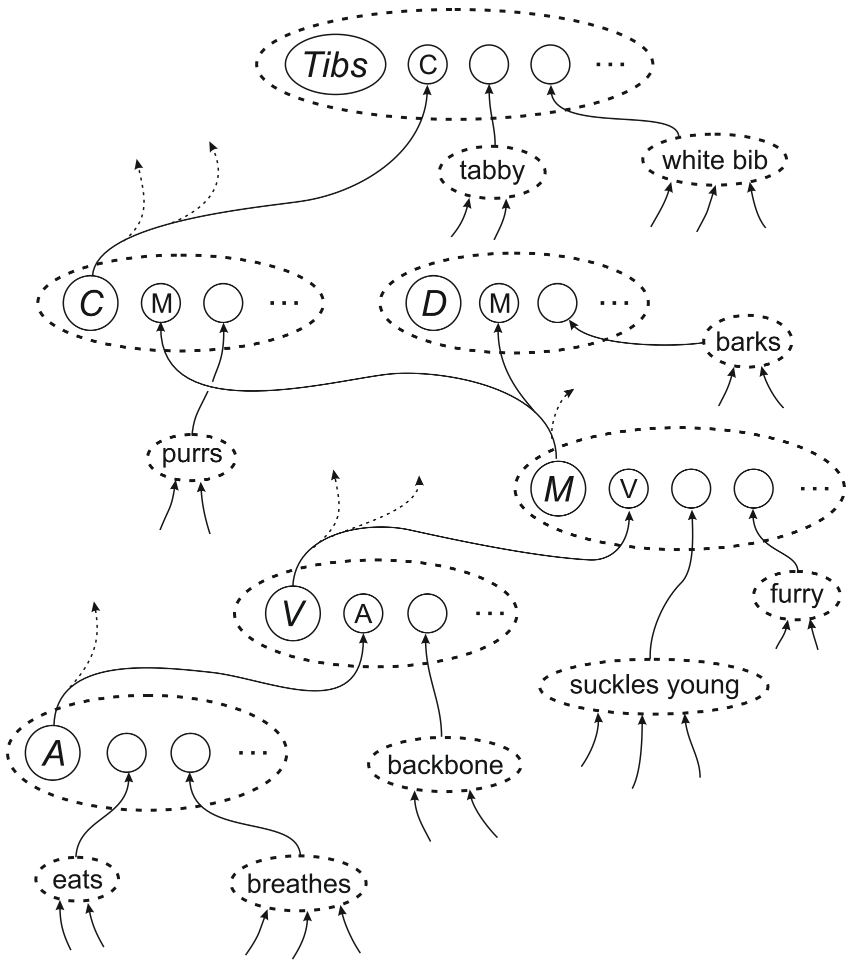

Figure 8: An example showing schematically how SP-neural can represent class-inclusion relations and part-whole relations. *Key*: 'C' = cat, 'D' = dog, 'M' = mammal, 'V' = vertebrate, 'A' = animal, '...' = further structure that would be shown in a more comprehensive example. Pattern assemblies—the neural equivalent of SP patterns—are surrounded by broken lines and each neuron is represented by an unbroken circle or ellipse. Lines with arrows show connections between pattern assemblies and the flow of sensory signals. Connections between contiguous neurons within each pattern assembly are not marked. ID-neurons—the neural equivalent of ID-symbols in the non-neural SP theory—are larger than the others and are labelled with oblique letters. All other neurons are C-neurons, the neural equivalent of C-symbols. Each of these may be seen as a neural reference to the cell assembly from which it receives signals. Reproduced with permission from Figure 11.6 in [49].

C Information and physics

In his excellent book for non-specialists about quantum gravity, Carlo Rovelli writes:

> "Many scientists suspect today that the concept of 'information' may turn out to be a key for new advances in physics. Information is mentioned in the foundations of thermodynamics, the science of heat, the foundation of quantum mechanics and in other areas besides, with the word quite often used very imprecisely." [33, Location 2558].

This suggests that there may be some convergence between current thinking amongst physicists about the role of information in theories of physics and what has been said in this paper about information (Section 5.2) and, throughout this paper, about the role of information compression in cognition, mathematics, and science.

In the light of what Rovelli says elsewhere in [33], some tentative points of similarity and difference are described in the following subsections.

C.1 The nature of information

As was noted in Section 5.2, the concept of information adopted by Rovelli (a "measure of the number of possible alternatives for something" [33, Location 2565], in accordance with Shannon's [35] concept of information) is similar to the concept of information adopted in this paper.

C.2 Information and communication

> "Why is the notion of information useful, perhaps even fundamental, to understanding the world? For a subtle reason: because it measures the ability of one physical system to communicate with another physical system." [33, Locations 2583–2590].

This "communication" aspect to information chimes with Shannon's concept of information, described in [35], as illustrated by the title of that book: *The Mathematical Theory of Communication*.

Here, there seems to be some discrepancy with the concept of information adopted in the SP programme of research, where it has not so far appeared necessary to bring a concept of communication into play. But see Appendix C.3.

C.3 Correlations and redundancy

> "The world isn't ... just a network of colliding atoms: it is also a network of correlations between sets of atoms, a network of real reciprocal information between physical systems." [33, Location 2614].

In this quote and elsewhere in [33, Chapter 12], the word "correlation" appears to mean much the same as the word "redundancy" in this paper and in other writings about information.

Readers who are familiar with the writings of Lewis Carroll will know that the words *And as in uffish thought he stood,* are to be followed by *The Jabberwock, with eyes of flame, Came whiffling through the tulgey wood, And burbled as it came!* We can say that these two parts of the Jabberwocky poem are "correlated" with each other because the first part is normally followed by the second part. But equally we can say that there is redundancy, meaning repetition of information, in the way that the two parts are frequently repeated together in the many copies of *Through the Looking-Glass*, and in the many other places where the poem appears.

If, in relevant contexts, the words "correlation" and "redundancy" mean the same thing, this may help to bridge the possible divide noted in Appendix C.2.

D This idea must die

This appendix considers some of the issues which are discussed in a collection of papers, *This idea must die* [7], concentrating on those which are most relevant to the main themes of this paper.

D.1 A theory of everything, and unification

The notion of a *theory of everything*:

> '... would be based on a parsimonious set of underlying mathematizable universal principles that integrate and explain all the fundamental forces of nature, from gravity and electromagnetism to the weak and strong nuclear forces, incorporating Newton's laws, quantum mechanics, and general relativity. Fundamental quantities like the speed of light, the dimensionality of spacetime, and the masses of the elementary particles would all be predicted, and the equations governing the origin and evolution of the universe through to the formation of galaxies and beyond would be derived—and so on." [43, Locations 278–281].

In his paper, Geoffrey West gives credit for the great progress that has been made towards the goal of developing a theory of everything, but points out, quite plausibly, that in this context the word "everything" is too strong: "Where's life, where are animals and cells, brains and consciousness, love and hate, etc, etc?" (*ibid.*, Location 288). The gist of his argument is that there is far too much complexity in the world for it all to be described or predicted by "'just' turning the crank of increasingly complicated equations and computations" which are "presumed, in principle, to be soluble to any sufficient degree of accuracy." (*ibid.*, Location 291).

In the same vein, Marcelo Gleiser writes:

> "The trouble starts when we take [the notion of unification of theories] too far and search for the *Über*-unification, the Theory of Everything, the arch-reductionist notion that all forces of nature are merely manifestations of a single force. This is the idea that needs to go." [14, Locations 313–316].

and

> "The impulse to unify it all runs deep in the souls of mathematicians and theoretical physicists, from the Langlands program to superstring theory. But here's the rub: Pure mathematics isn't physics. The power of mathematics comes precisely from its detachment from physical reality. A mathematician can create any universe she wants and play all sorts of games with it. A physicist can't; his job is to describe nature as we perceive it. (*ibid.*, Locations 322–326).

To be clear, it is *not* proposed that P-NMFS, as outlined in this paper, would be a theory of everything, or the beginnings of such a theory. What is proposed here is the beginnings of a framework for the creation or expression of theories. With development, the multiple alignment framework may facilitate the integration or "unification" of what may otherwise be inconsistent or incompatible theories (such as quantum mechanics and the theory of general relativity).

But it is likely that a comprehensive unification of all scientific theories will remain for ever out of reach, with some relatively successful theories where there are islands of redundancy and predictability, and with large seas of unpredictability in the areas in between. And even within the islands of redundancy and predictability there will be a persistent need to make approximations (as described in Section 5.3), with corresponding limits on the accuracy or reliability of predictions.

Nevertheless, the integration or unification of ideas is to be welcomed wherever it can be found.

D.2 Simplicity

A. C. Grayling writes:

> "Simplicity is a desideratum in science, and the quest for it is a driver in the task of effecting reductions of complex phenomena to their components. It lies behind the assumption that there must be a single force in nature, of which the gravitational, electroweak, and strong nuclear forces are merely manifestations; and this assumption in turn is an instance of the general view that there might ultimately be a single kind of thing (or stuff or field or as-yet-undreamt-of phenomenon) out of which variety springs by means of principles themselves fundamental and simple.
>
> "Compelling as the idea of simplicity is, there's no guarantee that nature itself has as much interest in simplicity as those attempting to describe it. ..." [16, Locations 358–364].

Yes, notwithstanding Isaac Newton's slightly whimsical suggestion that "Nature is pleased with simplicity" [28, p. 320], it's not nature that has an interest in simplicity, it is, as A. C. Grayling says, something that is of interest to those attempting to describe nature. So if some aspect of nature is intrinsically complex, then a good scientific description of it would also be complex, with the possible exception of approximations that we may choose to make (Section 5.3).

And of course *simplicity* by itself may easily lead to over-simple theories. Hence the need for it to be in balance with descriptive or explanatory *power* (Section 3 and Appendix E).

D.3 Infinity

Despite the elegance and mathematical importance of Georg Cantor's work on infinities, the concept of infinity can be problematic for science. Max Tegmark writes:

> "The assumption that something truly infinite exists in nature underlies every physics course I've ever taught at MIT—and, indeed, all of modern physics. But it's an untested assumption, which begs the question: is it actually true?" [40, Location 804].

He goes on to say:

> "There are in fact two separate assumptions: 'infinitely big' and 'infinitely small.' By infinitely big, I mean that space can have infinite

volume, that time can continue forever, and that there can be infinitely many physical objects. By infinitely small, I mean the continuum—the idea that even a liter of space contains an infinite number of points, that space can be stretched out indefinitely without anything bad happening, and that there are quantities in nature that can vary continuously. The two assmptions are closely related, because inflation, the most popular explanation of our Big Bang, can create an infinite volume by stretching continuous space indefinitely.

"The theory of inflation has been spectacularly successful and is a leading contender for a Nobel Prize. It explains how a subatomic speck of matter transformed into a massive Big Bang, creating a huge, flat, uniform universe, with tiny density fluctuations that eventually grew into today's galaxies and cosmic large-scale structure—all in beautiful agreement with precision measurements from experiments such as the *Planck* and the BI-CEP2 experiments. But by predicting that space isn't just big but truly infinite, inflation has also brought about the so-called measure problem, which I view as the greatest crisis facing modern physics. Physics is all about predicting the future from the past, but inflation seems to sabotage this. When we try to predict the probability that something particular will happen, inflation always gives the same useless answer: infinity divided by infinity. The problem is that whatever experiment you make, inflation predicts there will be infinitely many copies of you, far away in our infinite space, obtaining each physically possible outcome; and despite years of teeth-grinding in the cosmology community, no consensus has emerged on how to extract sensible answers from these infinities. So, strictly speaking, we physicists can no longer predict anything at all!

"This means that today's best theories need a major shakeup by retiring an incorrect assumption. Which one? Here's my prime suspect: ∞.

"A rubber band can't be stretched indefinitely, because although it seems smooth and continuous, that's merely a convenient approximation. It's really made of atoms, and if you stretch it too far, it snaps. If we similarly retire the idea that space itself is an infinitely stretchy continuum, then a big snap of sorts stops inflation from producing an infinitely big space and the measure problem goes away. Without the infinitely small, inflation can't make the infinitely big, so you get rid of both infinities in one fell swoop—together with many other problems plaguing modern physics, such as infinitely dense black-hole singularities and infinities popping up when we try to quantize gravity." (*ibid.*, Locations 804–827).

Tegmark acknowledges that "infinity is an extremely convenient approximation for which we haven't discovered convenient alternatives." (*ibid.*, Location 830), but warns that "Despite their seductive allure, we have no direct observational evidence for either the infinitely big or the infinitely small." (*ibid.*, Location 837).

In a similar vein, writing about the elimination of the concept of infinity in the theory of quantum gravity, Rovelli writes:

> "The infinitely small no longer exists. The infinities which plague conventional quantum field theory, predicated on the notion of a continuous space, now vanish, because they were generated precisely by the assumption, physically incorrect, of the continuity of space. The singularities which render Einstein's equations absurd when the gravitational field becomes too strong also disappear: they are only the result of neglecting the quantization of the field." [33, Location 2130].

Again, Jim Al-Khalili writes:

> "An obvious question, with history as our guide, is whether the electron and quarks are indeed fundamental, or made of yet smaller pieces like Russian dolls. The honest answer is: we don't know! All we can say is that with the best experiments we are able to do today, there is no hint of deeper structure." [1, Locations 2478–2484].

In short, we should not lose sight of the empirical foundations of any theory, and we should take care that inferences that derive from any theory do not run too far ahead of their empirical underpinnings, as discussed in Appendix E.

D.4 String theory and multiverse theory

Some of the contributors to [7] express reservations about string theory and multiverse theory:

- Frank Tipler writes:

  > "Since string theorists have failed to propose *any* way to confirm string theory experimentally, string theory should be retired, today, now." [42, Location 1002].

- Peter Woit writes:

> "The idea of unifying physics by positing strings moving in ten space/time dimensions as fundamental entities was born in 1974 and became the dominant paradigm for unification from 1984 on. After forty years of research and tens of thousands of papers, what we've learned is that this is an empty idea. It predicts nothing about anything, since you can get pretty much any physics you want by appropriately choosing how to to make six of the ten dimensions invisible." [44, Locations 1054–1055].

- Paul Steinhardt writes:

> "A pervasive idea in fundamental physics and cosmology that should be retired: the notion that we live in a multiverse in which the laws of physics and the properties of the cosmos vary randomly from one patch of space to another. According to this view, the laws and properties within our observable universe cannot be explained or predicted, because they're set by chance. Different regions of space too distant to ever be observed have different laws and properties, according to this picture. Over the entire multiverse there are infinitely many distinct patches. Among these patches, in the words of Alan Guth, 'anything that can happen will happen; in fact, it will happen an infinite number of times.' Hence, I refer to this concept as a Theory of Anything.
>
> "Any observation or combination of observations is consistent with a Theory of Anything. No observation or combination of observations can disprove it. Proponents seem to revel in the fact that the theory cannot be falsified. The rest of the scientific community should be up in arms, since an unfalsifiable idea lies beyond the bounds of normal science. Yet, except for a few voices, there has been surprising complacency about (and in some cases grudging acceptance of) a Theory of Anything as a logical possibility. ..." [39, Locations 893–903].

- And Gregory Benford writes:

> "... it's hard to devise an elegant cosmological theory that yields directly the small cosmological constant we observe. Some solve this problem by invoking the anthropic principle, and thus multiverses of some sort. But this ventures near a violation of another form of the elegance standard, Occam's razor. Imagining a vast sea of multiverses, with us arising in one where conditions produce

> intelligent beings, seems excessive to many of us. It invokes a plentitude we can never see. The scientific test of multiverse cosmology is whether it leads to predictable consequences.
>
> "Can multiverses converse with one another? That would be a way of verifying the basis of such theories. Most multiverse models say there's no possible communication between the infinitude of multiverses. Brane theory, though, comes from models where no force law operates between branes except gravitation. Perhaps someday an instrument like LIGO, the Laser Interferometer Gravitational-wave Observatory, can detect such waves from branes. But is it elegant to shift confirmation onto some far future technology? Sweeping dust under a rug seems inelegant to me." [5, Locations 5371–5378].

These concerns relate to the issue of how scientific theories should be evaluated, discussed in Appendix E.

E The formation and evaluation of scientific theories

What is envisaged here is that, at some stage, SP and P-NMFS may provide a means, via unsupervised learning, for the automatic or semi-automatic creation of scientific theories.

An important part of any such process, and also in traditional methods for the formation of theories, is distinguishing 'good' theories from 'bad' ones. Here are brief notes on two influential views about how scientific theories can or should be evaluated:

- *Karl Popper and falsifiability.* In *The Logic of Scientific Discovery* [32], and in *Conjectures and Refutations* [31], Karl Popper argues that scientific theories cannot be confirmed, only falsified. Although these ideas have been criticised, they serve as a welcome antidote to such things as Freud's theory of dreams [13] which was constructed in such a way that it can always be made to fit any evidence, meaning that it can never be falsified. For that reason, Popper argues that such theories should not be regarded as scientific.

- *Occam's Razor.* As we have seen (Section 3), it is widely accepted that any good scientific theory should conform to Occam's Razor, meaning that it should combine *simplicity* with explanatory or descriptive *power*. This may be seen to be equivalent to the lossless compression of a body of observational data, **I**, with the qualification that the results of compression may be divided into two parts: a 'grammar' **G**, and an 'encoding' of **I** in terms of **G**, which we may call **E**, and normally we would be more interested in **G**, the 'theory' of **I**, than in **E**.

It would take us too far afield in this paper to discuss the matter in detail. In brief, the hypothesis favoured here is that notions of falsification may be seen at work within a conceptual framework like SP in which information compression is centre-stage.

In support of that hypothesis, the way in which the appearance of one black swan can, with 100% certainty, invalidate the generalisation that "All swans are white" is somewhat like the way that, within the SP framework, the news that "Tweety is a penguin" can lead, with 100% certainty, to the conclusion that "Tweety cannot fly", by contrast with the conclusion that "Tweety can fly (with probably 0.66)", based on the knowledge that "Tweety is a bird"—and without the additional knowledge that "Tweety is a penguin" ([45, Section 10.1], [49, Section 7.7]).

Although falsifiability is useful in distinguishing good scientific theories from bad ones, it has not eliminated scientists' reliance on somewhat vague "gut feelings" about this or that theory, or on such imprecise notions as "beauty" or "elegance" in a theory, and it has not eliminated the way in which judgments may be coloured by emotional attachments to one's own theories, or by success or otherwise in winning publicity for a given theory.

There is a pressing need for something more precise and disciplined, and this is where SP may help. There is potential with SP for the quantitative evaluation of any given scientific theory, **T**, in terms of information compression:

- *With fully-automatic unsupervised learning.* Although, with the current SP computer model, there is still unfinished business in the development of unsupervised learning (Appendix B.10), there is clear potential for the fully-automated unsupervised learning of scientific theories [54, Section 6.10.7].

  Any **T** would be delivered with a measure of information compression, comparing the size, S_I, (in bits) of the raw data (**I**) which provides the basis for **T** with the size, S_{IC}, (in bits) of those data when they have been compressed (**IC**). What has proved to be a useful measure is a ratio of the two: S_{IC}/S_I.[19]

  In comparing one theory with another, measures of information compression as just described would be most useful if rival theories are derived from the

[19] In the SP computer model, S_{IC} is the measure of success that is used in unsupervised learning [49, Section 9.2.6.1]. This is satisfactory because, for any given run of the program, S_I is constant.

same **I**. But even if rival theories are derived from different **I**s, measures of information compression may still prove useful in comparing theories, especially if there is overlap amongst different **I**s, the larger the better.

- *With traditional methods for the development of theories, or with semi-automatic unsupervised learning.* Without the fully automatic unsupervised learning of scientific theories, traditional methods would still be available for the development of scientific theories, and there is potential for the use of such methods in conjunction with semi-automatic unsupervised learning. Either way, there is potential for the quantitative evaluation of fully- or partially-developed theories, along the lines outlined above. For any given body of mathematics, or any given computer program, it would be necessary to show that it can achieve substantial compression of information with one or more representative bodies of data.

E.1 Working with data that has been encoded

A potential complication with this scheme is that the reference body of data for any theory, **I**, may not be "raw" data in the sense that it has been drawn directly from the environment. For various reasons, it may have been encoded already in terms of some theory or encoding scheme. In cases like that: 1) it may be necessary to decode **I** before it is used for the creation of **T** via unsupervised learning, or before it is used in the evaluation of a given theory in terms of information compression; or 2) It may sometimes make sense to use **I** in its encoded form as the basis for unsupervised learning or for the evaluation of a given theory that has **I** in its encoded form as its reference data. These and related issues are discussed more fully in [53, Section IV-B].

E.2 Striking a balance between over-general and over-specific theories

Quantitative measures of information compression should help to steer us away from weak theories, including ones like these:

- *Over-general theories.* Some theories are too general to be useful, exhibiting over-generalisation as described in [59, Appendix B]. They may be admirably *simple* but they are lacking in descriptive or explanatory *power*: they explain nothing because they explain everything. For example, a rather unhelpful 'theory' of knowledge would be that all kinds of knowledge may be represented with 0s and 1s. This is true but it gives us no handle on such knowledge-related issues as how to use knowledge in reasoning, how to recognise things, and so on. As described in Section 2.4.2 and Appendix D.4, string theory and multiverse theory may be over-general theories of this kind.

- *Over-specific theories.* Some theories are too specific to be useful, exhibiting under-generalisation as described in [59, Appendix B]. These are theories that, to a large extent, re-describe the observations they are meant to explain. They deliver little or nothing in terms of *simplicity* and, because there is little or no generalisation, they are weak in terms of descriptive or explanatory *power*.

In the light of the observations in [59, Appendix B], it seems likely that over-general and over-specific theories will both score worse in terms of information compression than theories that strike a balance between generality and specificity.

E.3 Hypothetical constructs

The emphasis here on the compression of raw data may suggest that unsupervised learning in SP is merely a form of statistical analysis that is incapable in principle of forming the kinds of hypothetical construct—such as 'electrons', 'photons', and the like—that have proved to be so useful in the development of science.

But although SP can discover correlations and associations between things, it is designed also to discover object-like entities such as words [49, Section 9.4], it has potential to discover 3D structures [51, Sections 6.1 and 6.2], and it is anticipated that, when some residual problems in the SP computer model have been solved [45, Section 3.3], it will be able to discover such structures as class-inclusion hierarchies and part-whole hierarchies.

In short, there is potential for the system to create or discover hypothetical constructs, as would be needed in any system for the automatic or semi-automatic creation of new theories.

F Quantum mind, quantum cognition, and the SP theory

Discussions in this paper about human learning, perception, and cognition, SP, and quantum mechanics may suggest, on a superficial reading, that what is proposed is merely a variant of ideas about "quantum mind"[20] or "quantum cognition".[21]

[20]See, for example, "Quantum mind", *Wikipedia*, bit.ly/2nmtgoN, retrieved 2017-03-15

[21]See, for example, "Quantum cognition", *Wikipedia*, bit.ly/2mYHg74, retrieved 2017-03-15

The point of this short appendix is to emphasise that the proposals in this paper are radically different:

- *Quantum mind* theories propose that "quantum mechanical phenomena, such as quantum entanglement and superposition, may play an important part in the brain's function and could form the basis of an explanation of consciousness".[22]

- *Quantum cognition* theories "[apply] the mathematical formalism of quantum theory to model cognitive phenomena such as information processing by the human brain, language, decision making, human memory, concepts and conceptual reasoning, human judgment, and perception.[23]

- By contrast, the *SP theory of intelligence* proposes that artificial intelligence, mainstream computing, mathematics, and human learning, perception, and cognition may, to a large extent, be understood as information compression via the matching and unification of patterns, and, more specifically, via a concept of *multiple alignment*, borrowed and adapted from bioinformatics. And in this paper it is proposed that there are potential benefits in developing P-NMFS from an amalgamation of mathematics with the SP system and that, within that new framework, there is potential for new syntheses in science, including new thinking about the phenomena that are the subject of quantum mechanics.

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