Dark Matter and Rotation Curves.

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Abstract

In present paper we show that the rotation curves of our Galaxy, can be obtained directly from the baryon matter distribution function and there is no need to involve the concept of dark matter. Such parameters of the galaxy as its mass and the distribution of baryonic matter can be easily calculated from the observed rotation curves. Calculated total mass of the Galaxy is 35x10(10)M_sun.

Keywords: Dark Matter; Rotation Curves; Gravitational Potential; Mass of the Galaxy.

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1 Introduction

From the very beginning of its discovery in 1932 by Jaan Oort and confirmation made by Fritz Zwicky in 1933, the phenomena of Dark Matter (DM) was widely discussed. In present paper we consider the kinematics of the baryon component for our Galaxy.

To begin with, we would like to mention here some interesting facts.

- It is well known that the halo stars (the old population of the galaxy) move more slowly than the young population of the disk does. This fact clearly indicates an unsatisfactory explanation of the Rotation Curves (RC) as provoked by the presence of DM.

- An immense dispersion of the star velocities beginning at distances of the order of 10 Kpc, which can not be explained with a homogeneous, spherically distributed DM. Such a spread can be due only to the presence of a fine structure such as the arms of galaxy, which are baryonic by nature.

- Recently it was shown that there is a significant correlation between the features in the RC and the spiral structure of the baryonic disk. As the authors say: "The dark and baryonic mass are strongly coupled" [1], [2], [3], [4].

- This year the rotation curves for high redshifted galaxies were reported and it was clearly shown that a large fraction of massive high - redshifted galaxies are strongly baryon-dominated (see [5] and references therein).

All this clearly indicates the need for a more accurate simulation of the galactic baryon component.

In present brief letter we suggest analytical solution for RC of gravitationally bounded systems and argue that there is no need to involve the concept of DM to explain rotation curves of all known galaxies. Here we suggest an integral approach to the problem. Of course the same result can be obtained by the direct solution of the Poisson differential equation in cylindrical coordinate system. An interested reader can find this material here [6].

The article is organized as follows:

In part 2 we discuss RC produced by spherically distributed matter, we suggest an analytical solution for the density function under condition V=const, and compare it with those obtained with numerical simulations.

In part 3 we suggest analytical solution for RC in the case of cylindrically distributed matter. We apply our solution to describe RC of our Galaxy and find excellent agreement with the observed ones.

2 Rotational curves in the case of spherical symmetry

The case, when the density distribution has spherical symmetry deserves a separate consideration because of the DM spherical distribution, supposed in all numerical simulations. But to begin with, let's look at common misconceptions and a simple example.

It is commonly accepted in current literature that in the case of Newtonian dynamics, for the stars of a rotationally supported galaxies the following relation for the velocity should be satisfied: $V \propto R^{-1/2}$ [1] [7] [8]. However, as it will be shown below, this is not true in both cases of spherically and cylindrically distributed matter, because in reality this relation depends dramatically on the distribution of gravitating matter. In the case of cylindrical symmetry, we can illustrate this misunderstandings with the following very simple reasoning. Let's consider a massive sphere of unit thickness characterized by radius r. Let the observer be at the point located at a distance R from the sphere. Consider forces produced by the matter distributed on the nearest and the remote walls of the sphere and restricted by small solid angle Ω as the observer sees them. It is easy to show that in this case these two forces will be equal: $F_1 = F_2 = G\pi a^2 s_1/R^2$ and can be substituted by an effective force $F_e = G(m_1+m_2)/(R+r)^2$, produced by imaginary central mass $m_1 + m_2$ (here a is the radius of the circle clipped by solid angle Ω in the nearest wall and s_1 stay for the surface density of the sphere).

In the case of a cylindrically distributed matter, we can develop similar arguments, however, if our assumptions on the mass $m_1 = \pi a^2 s_1$ remain valid for the nearest mass (we suppose the solid angle be reasonable, the force remain to be $F_1 = G\pi a^2 s_1/R^2$, then for the second one it will not be so, because the remote mass m_2 will be formed by a thin strip instead of a complete circle clipped out on the sphere by Ω, as it was before. As a consequence the center of the effective mass shifts toward to the observer.

This trivial reasoning clearly shows that such a simple relation $V \propto R^{-1/2}$ can not be applied to the systems with cylindrical symmetry (strictly speaking, as we will show it below, it can not be applied even for spherical systems if the density function depends on distance and the observer is inside of such a system).

By taking into account that the so-called dark matter is thought to be spherically distributed, it is important to consider the spherically symmetric case in details.

It is well known that for point - like source the gravitational potential can be written as

$$du = \frac{Gdm(r)}{r} \quad . \tag{1}$$

Let's consider spherically symmetrical density $\sigma(\rho)$ in $[g \cdot cm^{-3}]$. In this case the mass differential can be expressed as

$$dm = \sigma\rho^2 \sin\theta d\rho d\theta d\phi \quad . \tag{2}$$

For this reason we have the following potential function for a spherically symmetric system:

$$du = \frac{G\sigma\rho^2 \sin\theta d\rho d\theta d\phi}{\sqrt{R^2 + \rho^2 - 2R\rho\cos\theta}} \quad , \tag{3}$$

where R is the distance from the observer to the origin of the sphere under consideration.

Performing elementary integration with respect to the angles θ and ϕ, we obtain:

$$du = \frac{4\pi G\sigma(\rho)}{R}\rho^2 d\rho, \tag{4}$$

or

$$u = \frac{4\pi G}{R}\int_0^R \sigma(\rho)\rho^2 d\rho, \tag{5}$$

which gives us for the constant density the following relation for the square velocity:

$$V^2 = R\frac{du}{dR} = 4\pi G\sigma R^2, \tag{6}$$

i.e. in this particular case we have $V \propto R$, instead of $V \propto R^{-1/2}$. This simple example we suggest just to stress again that RC are dependent dramatically on the density distribution function which, in turn, changes during the galaxy evolution.

In the case of an arbitrary density distribution, as one can see from the relation (5), in the Newtonian dynamics and for spherical symmetry, the velocity in general case does not obey the expected trivial relation $V \propto R^{-1/2}$ mentioned by authors cited above. Actually we should chose very special form of the density function to comply relation $V \propto R^{-1/2}$, or to reach more interesting case of constant velocity.

Now let's consider how should density depends on distance to maintain condition $V = const.$ in the case of spherically distributed matter.

The DM density function (which could produce approximately constant RC in a certain range of distances) is widely discussed and applied to model DM haloes, see for example [9], [10], [11] and references therein. Unfortunately to date, such density functions are calculated only from numerical simulations. We will not offer here an exhaustive description of all such functions, but mention just two most commonly recognized. The first were suggested by Navarro, Frenk and White [9] and has the form

$$\sigma_{NFW}(r) = \frac{\sigma_0}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}. \tag{7}$$

Another function were obtained recently by Di Cintio et al. [12], [13] and was compared with the first one in [11]. As it was stressed in [11], the function of Di Cintio et al. much better approximates the observer RC. This density distribution of DM can be represented as:

$$\sigma_{DC14}(r) = \frac{\sigma_0(\frac{r}{r_s})^{|\gamma|}}{(1+(\frac{r}{r_s})^{\alpha})^{\frac{\beta-\gamma}{\alpha}}}. \tag{8}$$

In the case of DM dominated galaxy, this distribution is characterized by the following values of the parameters $\gamma \approx -0.88$, $\alpha \approx 1.4$, $\frac{\beta-\gamma}{\alpha} \approx 2.6$.

It should be emphasized again that these functions were obtained from numerical simulations on the basis of the requirement to describe an approximately constant velocity in a strictly defined range of distances, in order to approximate RCs of real galaxies. Let's calculate analytically the density function which corresponds to constant velocity. By taking into account that

$$V^2\,(R) = R\frac{du}{dR}, \tag{9}$$

and requiring that $V = const$, we immediately obtain from the eq. (5) the following differential equation for the density $\sigma(r)$:

$$\sigma'(R)R^3 + 2\sigma(R)R^2 = \frac{V^2}{4\pi G}. \tag{10}$$

Or by normalizing the variable for convenience of comparison $x = r/r_s$, we have:

$$\sigma'(x)x^3 + 2\sigma(x)x^2 = \frac{V^2}{4\pi G r_s^2}. \tag{11}$$

Integrating (10), we obtain

$$\sigma(x) = \frac{V^2}{4\pi G r_s^2} \frac{\ln(\frac{r}{r_s})}{(\frac{r}{r_s})^2}. \tag{12}$$

This is exact expression for the density function, which comply condition $V = const$ over the entire range of distances. As can be seen, expressions (7), (8) and (12) have similar behavior for the argument greater than one.

3 Rotation curve in the case of cylindric symmetry

Now consider a thin disk of size $a = 25Kpc$ and thickness $2b = 6Kpc$. Let dm be a point - like mass inside of the disk and r is the distance from the mass to observer (which also is located inside of the disk at distance R from the center of galaxy). Than the distance observer - point-like mass can be written as:

$$r^2 = z^2 + R^2 + \rho^2 - 2R\rho\cos\varphi, \tag{13}$$

where ρ , z and φ are cylindrical coordinates of the mass under consideration.

Let's split the potential formed by the mass in point of observer, into longitudinal and tangential components $u = u_{\|} + u_{\perp}$.

It is clear that tangential one will not affect on the RC. For longitudinal component we have

$$u_{\|} = u\frac{R - \rho\cos\varphi}{r}, \tag{14}$$

But for the point - like mass the potential is $u = Gm/r$, so

$$du_{\|} = Gdm\frac{R - \rho\cos\varphi}{r^2}, \tag{15}$$

or in it's complete form

$$u_{\|} = G\int_0^R \int_{-b}^b \int_0^{2\pi} \frac{\sigma(\rho, z)(R - \rho\cos\varphi)\rho d\rho d\varphi dz}{z^2 + R^2 + \rho^2 - 2R\rho\cos\varphi}. \tag{16}$$

Integrating over φ we obtain:

$$u_{\|} = \frac{2\pi G}{R}\int_0^R \int_0^b \sigma(\rho, z)\rho d\rho dz, \tag{17}$$

In order to follow further, we need the density distribution function be defined. To the best of our knowledge, the most used one to describe observed galaxies, is the Miyamoto - Nagai density function [14]. Unfortunately, this function is not convenient for analytic calculations and therefore we approximate it by a factorized function of the same degree that allows integration:

$$\sigma(\rho, z) = \frac{10^{10} M_\odot}{(\gamma t^2+1)^{3/2}} \sum_k \frac{\alpha_k}{(\beta_k x^2+1)^{3/2}}. \tag{18}$$

Here γ, α_k , β_k are parameters of approximation and dimensionless variables are $x = \rho/a$ and $t = z/b$. In this case the integration over z can be carried out easily:

$$I = \int_0^b \frac{1}{(\gamma t^2+1)^{3/2}} dz = \frac{bt}{\sqrt{\gamma t^2+1}} \Big|_0^1 = \frac{b}{\sqrt{\gamma+1}}, \tag{19}$$

and we obtain

$$u_\| = \frac{\eta b}{R\sqrt{\gamma+1}} \int_0^R \sum_k \frac{\alpha_k}{(\beta_k x^2+1)^{3/2}} \rho d\rho, \tag{20}$$

where we introduce the constant $\eta = 2\pi G 10^{10} M_\odot$. Integration over ρ gives

$$u_\| = \frac{\eta a^2 b}{R\sqrt{\gamma+1}} \sum_k \frac{\alpha_k}{\beta_k} \left[1 - \frac{1}{(\beta_k \frac{R^2}{a^2}+1)^{1/2}}\right]. \tag{21}$$

This is gravitational potential produced by galaxy in the point R. The required RC can be found now both from (20) and from (21).

$$V^2 = \frac{\eta a^2 b}{R\sqrt{\gamma+1}} \sum_k \frac{\alpha_k}{\beta_k} \left[1 - \frac{\frac{3}{2}\beta_k \frac{R^2}{a^2}+1}{(\beta_k \frac{R^2}{a^2}+1)^{3/2}}\right]. \tag{22}$$

This expression was obtained for the density function (18) characterized by the parameters γ, α_k and β_k, and it can be used immediately to plot RC. As an example let's calculate the RC for our Galaxy, the density function for which has been most thoroughly studied. It's density function was suggested by Miyamoto and Nagai (MN) [14] and contains two terms that describe the inner, central part of the Galaxy and disk, respectively. To compare our density function (18) with the MN one, we left two terms in the sum in the expression (18). We normalized our density function in a way that makes it easier to compare it to the MN one. These two functions are shown in Fig.1. As can be seen, the approximation proposed by us is in excellent agreement with the MN function in the most important distance region $1 < R < 25 Kpc$.

Now, we can plot RC for our Galaxy using expression (22). The result is shown in Fig.2 for five-terms approximation of the density function (which correspond to the core, main bulge, disk and thin disk), in comparison with the

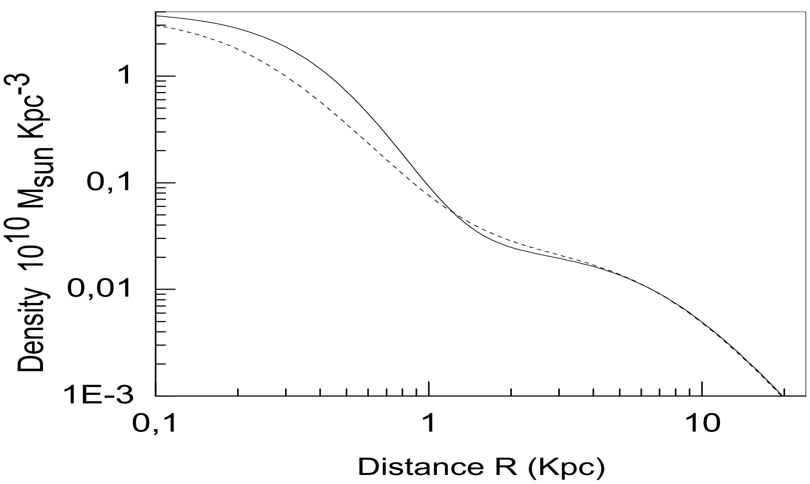

Figure 1: Density as a function of distance R. The density of Miyamoto - Nagai (solid line), and density determined by the expression (18) (dashed curve)

observed RC [15] (are shown by open squares). The parameters values of the model are $\gamma = 30$, $\alpha_1 = 0.007$, $\beta_1 = 2.5$, $\alpha_2 = 0.1$, $\beta_2 = 20$, $\alpha_3 = 0.35$, $\beta_3 = 100$, $\alpha_4 = 13.0$, $\beta_4 = 20000$, $\alpha_5 = 13.0$, and $\beta_5 = 900000$. As can be seen, the obtained RC perfectly coincides with the observational data. So we can conclude that DM is not needed to explain rotation curves, and this is the baryon density distribution, who is responsible for the RC shapes. Moreover, by using eqs. (18) and (22) we can reconstruct the baryon density distribution from observed RC. To conclude this part we would like to suggest evaluation for the total mass of our Galaxy. For the density distribution model used for better fit of the observed RC presented in Fig.3 (solid line) we obtain, integrating (18):

$$M_G = \frac{4\pi a^2 b 10^{10} M_\odot}{\sqrt{\gamma + 1}} \sum_k \frac{\alpha_k}{\beta_k}(1 - \frac{1}{\sqrt{\beta_k + 1}}), \tag{23}$$

and the baryon disk mass of our Galaxy is $M_G = 35.9 \cdot 10^{10} M_\odot$ which agrees with the Oort limit.

So we can conclude that we do not need dark matter to explain the rotation curves of our galaxy.

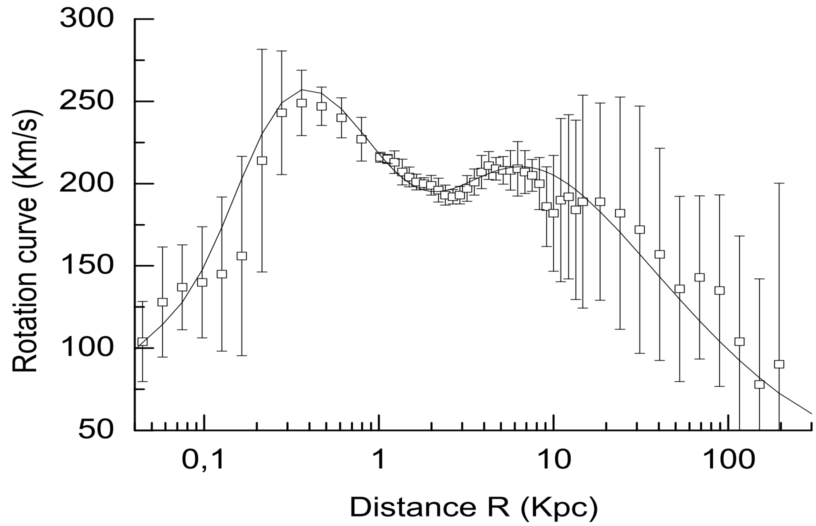

Figure 2: Rotation curves for our Galaxy obtained with density function (18) , compared with observed one [15].

4 Conclusions

The main results of the paper can be summarized as follows:

1) We argue that RC obtained for pure baryon component does depend on the distribution function and does not necessarily depends on distance as $V \propto R^{-1/2}$, as is claimed in papers [1], [7], [8].

2) We obtained an analytic expression for the density function of spherically distributed matter, satisfying the following condition: $V = const$ and compared it with those found for DM from numerical simulations of rotation curves produced by DM.

3) On the basis of new factorized density function, which excellently fits the Miyamoto-Nagai one, we obtained a general expression for the Galactic RC. We applied this expression to describe the RC of our Galaxy and find an excellent coincidence with observations.

4) We calculate the baryon disk mass of our Galaxy. Our evaluation is $M_G = 35.9 \cdot 10^{10} M_\odot$.

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