Dialectical Logic – Negation Of Classical Logic*

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Abstract

The division of zero by zero turned out to be a long lasting and not ending puzzle in mathematics and physics. An end of this long discussion is not in sight. In particular zero divided by zero is treated as indeterminate thus that a result cannot be found out. It is the purpose of this publication to solve the problem of the division of zero by zero while relying on the general validity of classical logic. According to classical logic, zero divided by zero is one.

Keywords

Indeterminate forms, Classical logic, Zero divided by zero

1. Introduction

Aristotle's unparalleled influence on the development of scientific knowledge in western world is documented especially by his contributions to classical logic too. Besides of some serious limitations of Aristotle's logic, Aristotle's logic became dominant and is an adequate basis for understanding science, since centuries. In point of fact, some authors are still of the opinion that Aristotle himself has discovered everything there was to know about classical logic. After all, classical logic, as such at least closely related to the study of objective reality, deals with absolutely certain inferences and truths. In general, classical logic describes the most general, the most simple, the most abstract laws of objective reality. Under conditions of classical logic, there is no uncertainty.

In contrast to classical logic, probability theory deals with uncertainties. This raises questions concerning whether there is an overlap between classical logic and probability theory at all. Without attempting to be comprehensive, it may help to sketch at least view words on this matter in this publication. Classical logic is at least closely allied with probability theory and vice versa. As such, classical logic has no meaning apart from probability theory and vice versa. It should therefore come as no surprise that there are trials to combine logic and probability theory within one and the same mathematical

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framework, denoted as dialectical logic. However, as already published, there are natural ways in which probability theory is treated as an extension of classical logic to the values between +0 and +1 where probability of an event is treated as its truth value. In this context, Fuzzy logic is of no use and already refuted [1]. In particular, the relationship between classical logic and probability theory [2] is the same as between Newtonian mechanic's and Einstein's special theory of relativity. The one passes over into the other and vice versa without any contradictions.

2. Material and methods

2.1. Definitions

DEFINITION 0. (NUMBER +0).
Let c denote the speed of light in vacuum, let ε_0 denote the *electric constant* and let μ_0 the *magnetic constant*. The number +0 is defined as the expression

$$\left(c^2 \times \varepsilon_0 \times \mu_0\right) - \left(c^2 \times \varepsilon_0 \times \mu_0\right) \equiv +1 - 1 \equiv +0 \tag{0}$$

DEFINITION 1. (NUMBER +1).
Let c denote the speed of light in vacuum, let ε_0 denote the *electric constant* and let μ_0 the *magnetic constant*. The number +1 is defined as the expression

$$c^2 \times \varepsilon_0 \times \mu_0 \equiv +1 \tag{1}$$

DEFINITION 2. (BERNOULLI TRIAL).
Let t denote a Bernoulli trial thus that

$$t = +1, \ldots, +N \tag{2}$$

DEFINITION 3. (THE SAMPLE SPACE).
Let ${}_{R}C_{t}$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_{0}x_{t}$ denote an event, a subset of the sample space ${}_{R}C_{t}$. Let ${}_{0}\underline{x}_{t}$ denote the negation of an event ${}_{0}x_{t}$, another, complementary subset of the sample space ${}_{R}C_{t}$. In general, we define the sample space ${}_{R}C_{t}$ as

$${}_{R}C_{t} \equiv \left\{ {}_{0}x_{t}, {}_{0}\underline{x}_{t} \right\} \tag{3}$$

or equally as

$${}_{0}x_{t} + {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} \tag{4}$$

In other words and according to quantum theory, the sample space ${}_{R}C_{t}$ at one certain Bernoulli trial t is in a state of superposition of ${}_{0}x_{t}$ and ${}_{0}\underline{x}_{t}$.

DEFINITION 4. (THE COMPLEX CONJUGATE ${}_{R}C_{t}{}^{*}$ OF THE SAMPLE SPACE ${}_{R}C_{T}$).

Let ${}_{R}C_{t}{}^{*}$ denote the complex conjugate of the sample space ${}_{R}C_{t}$, the set of all the possible outcomes of a random experiment et cetera. In general, we define

$$ {}_{R}C_{t} \times {}_{R}C_{t}^{*} \equiv 1 \tag{5} $$

with the consequence that

$$ {}_{R}C_{t}^{*} \equiv \frac{1}{{}_{R}C_{t}} \tag{6} $$

DEFINITION 5. (THE EIGEN-VALUES OF ${}_{0}X_{T}$).

Under conditions of classical logic, ${}_{0}x_{t}$ can take only one of the values

$$ {}_{0}x_{t} \equiv \{+0,+1\} \tag{7} $$

DEFINITION 6. (THE EIGEN-VALUES OF ${}_{0}\underline{X}_{T}$).

Under conditions of classical logic, ${}_{0}\underline{x}_{t}$ can take only one of the values

$$ {}_{0}\underline{x}_{t} \equiv \{+0,+1\} \tag{8} $$

DEFINITION 7. (THE SIMPLE FROM OF THE NEGATION OF ${}_{0}X_{T}$).

Let ${}_{0}\underline{x}_{t}$ denote the negation of an event ${}_{0}x_{t}$. In general, we define the negation of an event ${}_{0}x_{t}$ as

$$ {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} - {}_{0}x_{t} \tag{9} $$

Scholium.

Under conditions of classical logic, ${}_{R}C_{t}=1$ and we obtain that ${}_{0}\underline{x}_{t}=1-{}_{0}x_{t}$. George Boole (1815-1864) himself reformulated already a rigorous algebraic concept of Aristotle's system of logic in his 1854 monograph *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities* [3]. The term 'Boolean algebra', an algebra of a two-valued logic, was first suggested by Sheffer in 1913 and honors this English mathematician. The first mathematically or algebraically formulation of the notion negation was provided to us by Georg Boole. In general, following Boole, negation in terms of algebra, can be expressed as ${}_{0}\underline{x}_{t}=1-{}_{0}x_{t}$. According to George Boole and Boolean algebra +0 denotes false and +1 denotes true. "*Hence the respective interpretations of the symbols 0 and 1 in the system of Logic are Nothing and Universe*" [3]. According to Boole, "*If x represent any class of objects, then will 1 - x represent the contrary or supplementary class of objects, i. e. the class including all objects which are not comprehended in the class x.*" [3]. Boole generalizes the contrary of x very precisely.

"*And, in general, whatever class of objects is represented by the symbol x, the contrary class will be expressed by 1 - x.*" [3]. According to Boole, ${}_0\underline{x}_t$, i. e. the contrary class of x, the negation of x, is expressed algebraically as $1-{}_0x_t$.

DEFINITION 8. (THE PROBABILITY OF ${}_0X_T$).
Let $\psi({}_0x_t)$ denote the eigen-function [4] as associated with the eigen-value ${}_0x_t$. Let ${}_RC_t^*$ denote the complex conjugate of the sample space ${}_RC_t$. Let $c({}_0x_t)$ denote the complex co-efficient as associated with the eigenvalue ${}_0x_t$ while satisfying some normalization condition. Let $c^*({}_0x_t)$ denote complex conjugate of the complex coefficient as associated with the eigenvalue ${}_0x_t$. Let $p({}_0x_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0x_t$ at one single Bernoulli trial t.

$$p({}_0x_t) \equiv \psi({}_0x_t) \times {}_RC_t^* \equiv c({}_0x_t) \times c^*({}_0x_t) \equiv \frac{E({}_0x_t)}{{}_0x_t} \tag{10}$$

DEFINITION 9. (THE EXPECTATION VALUE OF ${}_0X_T$).
Let $\psi({}_0x_t)$ denote an eigen-function as associated with an eigen-value ${}_0x_t$. Let ${}_RC_t^*$ denote the complex conjugate of the sample space ${}_RC_t$. Let $c({}_0x_t)$ denote the complex coefficient as associated with the eigenvalue ${}_0x_t$ while satisfying some normalization condition. Let $p({}_0x_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0x_t$ at one single Bernoulli trial t. Let $E({}_0x_t)$ denote the expectation value of an event ${}_0x_t$ at one single Bernoulli trial t. In general, we define

$$E({}_0x_t) \equiv p({}_0x_t) \times {}_0x_t \equiv c({}_0x_t) \times \psi({}_0x_t) \tag{11}$$

DEFINITION 10. (THE EXPECTATION VALUE OF ${}_0\underline{X}_T$).
Let $p({}_0\underline{x}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{x}_t$ at one single Bernoulli trial t. Let $E({}_0\underline{x}_t)$ denote the expectation value of an event ${}_0\underline{x}_t$ at one single Bernoulli trial t. In general, we define

$$E({}_0\underline{x}_t) \equiv p({}_0\underline{x}_t) \times {}_0\underline{x}_t \tag{12}$$

DEFINITION 11. (THE PROBABILITY OF THE SAMPLE SPACE).
The sample space (Ω or ${}_RC_t$) of an experiment is the set of all possible outcomes for that experiment with the consequence that the sum of the probabilities of the distinct outcomes within a sample space is equal to 1. In general, we define

$$p({}_0x_t) + p({}_0\underline{x}_t) \equiv p({}_RC_t) \equiv p(\Omega) \equiv 1 \tag{13}$$

DEFINITION 12. (THE VARIANCE OF A SINGLE EVENT).
Let $\sigma({}_0x_t)$ denote the variance of an eigen-value ${}_0x_t$ at one [4] certain Bernoulli trial t. In general, we define the variance of an eigen-value ${}_0x_t$ at one certain Bernoulli trial t as

$$\begin{aligned}\sigma({}_0x_t)^2 &\equiv E({}_0x_t - E({}_0x_t))^2\\ &= E({}_0x_t{}^2) - E({}_0x_t)^2\\ &= ({}_0x_t \times {}_0x_t)\times p({}_0x_t)\times(1-p({}_0x_t))\\ &\equiv E({}_0x_t)\times({}_0x_t)\times(1-p({}_0x_t))\end{aligned} \tag{14}$$

DEFINITION 13. (THE INNER OR LOGICAL CONTRADICTION).
Let $\Delta({}_0x_t)^2$ denote the logical contradiction squared as associated with an eigen-value ${}_0x_t$. In general we define

$$\Delta({}_0x_t)^2 \equiv \frac{\sigma({}_0x_t)^2}{({}_0x_t \times {}_0x_t)} \equiv p({}_0x_t)\times(1-p({}_0x_t)) \tag{15}$$

Scholium.
Under conditions of classical logic, there are no logical contradictions and we do obtain that $\Delta({}_0x_t)^2 = p({}_0x_t)\times(1-p({}_0x_t))=0$. As soon as $\Delta({}_0x_t)^2\neq 0$, we are under conditions of dialectical logic or probability theory and we will obtain an inner contradiction.

DEFINITION 14. (CONJUNCTION).
Let $p({}_0A_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0A_t$ at one single Bernoulli trial t. Let $p({}_0\underline{A}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{A}_t$ at one single Bernoulli trial t. In general it is $p({}_0A_t) + p({}_0\underline{A}_t) = 1$. Let $p({}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_RB_t$ at one single Bernoulli trial t. Let $p({}_R\underline{B}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_R\underline{B}_t$ at one single Bernoulli trial t. In general it is $p({}_RB_t) + p({}_R\underline{B}_t) = 1$. The sample space is a compound sample space while the probability is still equal to 1. The conjunction, denoted by the sign $\cap$, is defined (under conditions of mutual independence) by the expression

$$p({}_0A_t \cap {}_RB_t) \equiv p({}_0A_t)\times p({}_RB_t) = \frac{E({}_0A_t \cap {}_RB_t)}{({}_0A_t)\times({}_RB_t)} \tag{16}$$

where $p({}_0A_t \cap {}_RB_t)$ denotes the joint probability of ${}_0A_t \cap {}_RB_t$ while $E({}_0A_t \cap {}_RB_t)$ denotes the joint expectation value at one single Bernoulli trial t.

DEFINITION 15. (DISJUNCTION).

Let $p({}_0A_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0A_t$ at one single Bernoulli trial t. Let $p({}_0\underline{A}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{A}_t$ at one single Bernoulli trial t. In general it is $p({}_0A_t) + p({}_0\underline{A}_t) = 1$. Let $p({}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_RB_t$ at one single Bernoulli trial t. Let $p({}_R\underline{B}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_R\underline{B}_t$ at one single Bernoulli trial t. In general it is $p({}_RB_t) + p({}_R\underline{B}_t) = 1$. The sample space is a compound sample space while the probability is still equal to 1. The disjunction, denoted by the sign $\cup$, is defined (under conditions of mutual independence) by the expression

$$p\left({}_0A_t \cup {}_RB_t\right) = \frac{E\left({}_0A_t \cap {}_RB_t\right)}{\left({}_0A_t\right)\times\left({}_RB_t\right)} + \frac{E\left({}_0\underline{A}_t \cap {}_RB_t\right)}{\left({}_0\underline{A}_t\right)\times\left({}_RB_t\right)} + \frac{E\left({}_0A_t \cap {}_R\underline{B}_t\right)}{\left({}_0A_t\right)\times\left({}_R\underline{B}_t\right)} \tag{17}$$

where $E({}_0A_t \cap {}_RB_t)$ denotes the joint expectation value at one single Bernoulli trial t. In other words, it is

$$p\left({}_0A_t \cup {}_RB_t\right) = p\left({}_0A_t\right) + p\left({}_RB_t\right) - p\left({}_0A_t \cap {}_RB_t\right) \tag{18}$$

2.2. Methods

2.2.1. Thought Experiments

Thought experiments [5] play a central role both in natural sciences and in the philosophy and are valid devices of the scientific [6] investigation. One of the most common features of thought experiments is that thought experiments can be taken to provide evidence in favor of or against a theorem, a theory et cetera. In particular, there have been attempts to define a "thought experiment", still there is no standard definition for thought experiments and the term is loosely characterized. More precisely, general acceptance of the importance of thought experiments can be found in almost all disciplines of scientific inquiry and are going back at least two and a half millennia and have practiced since the time of the Pre-Socratics [7]. A surprisingly large majority of impressive examples of thought experiments can be found in physics among some of its most brilliant practitioners are Galileo, Descartes, Newton and Leibniz [5]. Many famous physical publications have been characterized as thought experiments and include Maxwell's demon, Einstein's elevator (and train, and stationary lightwave), Heisenberg's microscope, Schrödinger's cat et cetera. Thought experiments are conducted for diverse reasons in a variety of areas and are equally common in pure, applied and in experimental mathematics.

2.2.2 Counterexamples

The relationship between an axiom and a conclusion derived in a technically correct way from such an axiom determines the validity of such a conclusion. In particular, it is impossible for an axiom to be true and a conclusion derived in a technically correct way from the same axiom to be false. A conclusion derived in a technically correct way must follow with strict necessity from an axiom and must be free of contradictions. In point of fact, a logical contradiction is not allowed in this context.
A *counterexample [8]* is a simple and valid proof technique which philosophers and mathematicians use extensively to disproof a certain philosophical or mathematical [9] position or theorems as wrong and as not generally valid by showing that it does not apply in a certain single case. By using counterexamples researchers may avoid going down blind alleys and stop losing time, money and effort.

2.3. Axioms

There have been many attempts to define the foundations of logic in a generally accepted manner. However, besides of an extensive discussion in the literature it is far from clear whether the truth as such is a definable notion. As generally known, axioms and rules of a publication have to be chosen carefully especially in order to avoid paradoxes and inconsistency. Thus far, for the sake of definiteness and in order to avoid paradoxes the theorems of this publication are based on the following axiom.

2.2.1. Axiom I (Lex identitatis. Principium Identitatis. Identity Law)

In general, it is

$$+1 \equiv +1 \tag{19}$$

3. Results

3.1. Theorem (The addition of probabilities)

Let $p({}_0A_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0A_t$ at one single Bernoulli trial t. Let $p({}_0\underline{A}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{A}_t$ at one single Bernoulli trial t. In general it is $p({}_0A_t) + p({}_0\underline{A}_t) = 1$. Let $p({}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_RB_t$ at one single Bernoulli trial t. Let $p({}_R\underline{B}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_R\underline{B}_t$ at one single Bernoulli trial t. In general it is $p({}_RB_t) + p({}_R\underline{B}_t) = 1$.

CLAIM.

In general, it is

$$p({}_0A_t)+p({}_RB_t)=p({}_0A_t\cup{}_RB_t)+p({}_0A_t\cap{}_RB_t)=1-(1-p({}_RB_t))+1-p({}_0\underline{A}_t) \quad (20)$$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1=+1 \quad (21)$$

Multiplying this equation by $p({}_0A_t)$, we obtain

$$1\times p({}_0A_t)=1\times p({}_0A_t) \quad (22)$$

or

$$p({}_0A_t)=p({}_0A_t) \quad (23)$$

Adding $p({}_0\underline{A}_t)$, it is

$$p({}_0A_t)+p({}_0\underline{A}_t)=p({}_0A_t)+p({}_0\underline{A}_t)=1 \quad (24)$$

which is equivalent with

$$p({}_0A_t)+p({}_0\underline{A}_t)=1 \quad (25)$$

Rearranging equation, we obtain

$$p({}_0A_t)=1-p({}_0\underline{A}_t) \quad (26)$$

Adding $p({}_RB_t)$, it is

$$p({}_0A_t)+p({}_RB_t)=p({}_RB_t)+1-p({}_0\underline{A}_t) \quad (27)$$

which is equivalent with

$$p({}_0A_t)+p({}_RB_t)=1-(1-p({}_RB_t))+1-p({}_0\underline{A}_t) \quad (28)$$

or with

$$p({}_0A_t)+p({}_RB_t)=p({}_0A_t\cup{}_RB_t)+p({}_0A_t\cap{}_RB_t)=1-(1-p({}_RB_t))+1-p({}_0\underline{A}_t) \quad (29)$$

QUOD ERAT DEMONSTRANDUM.

3.2. Theorem (Inclusive Or)

Let $p({}_0A_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0A_t$ at one single Bernoulli trial t. Let $p({}_0\underline{A}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{A}_t$ at one single Bernoulli trial t. In general it is $p({}_0A_t) + p({}_0\underline{A}_t) = 1$. Let $p({}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_RB_t$ at one single Bernoulli trial t. Let $p({}_R\underline{B}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_R\underline{B}_t$ at one single Bernoulli trial t, it is $p({}_RB_t) + p({}_R\underline{B}_t) = 1$. Let $p({}_0A_t \cup {}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with (inclusive) disjunction.

CLAIM.

In general, under condition of independence, it is

$$p({}_0A_t \cup {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = 1 \tag{30}$$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \tag{31}$$

Multiplying this equation by $p({}_0A_t \cup {}_RB_t)$, we obtain

$$1 \times p({}_0A_t \cup {}_RB_t) = 1 \times p({}_0A_t \cup {}_RB_t) \tag{32}$$

or

$$p({}_0A_t \cup {}_RB_t) = p({}_0A_t \cup {}_RB_t) \tag{33}$$

Under conditions of independence it follows that

$$p({}_0A_t \cup {}_RB_t) = p({}_0A_t) + p({}_RB_t) - p({}_0A_t \cap {}_RB_t) = p({}_0A_t) + p({}_RB_t) - \left(p({}_0A_t) \times p({}_RB_t)\right) \tag{34}$$

or an equivalent relation as

$$p({}_0A_t \cup {}_RB_t) = 1 - \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) \tag{35}$$

At the end, we obtain

$$p({}_0A_t \cup {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = 1 \tag{36}$$

QUOD ERAT DEMONSTRANDUM.

In general, the equation above can be rewritten as

$$p({}_0A_t \cup {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = p({}_0A_t) + p({}_RB_t) - p({}_0A_t \cap {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = 1 \tag{37}$$

3.3. Theorem (Law Of Excluded Middle. Principium tertii exclusi)

Something at a certain Bernoulli trial t is either true or false but not both. Still, something, which is true at the Bernoulli trial t can be false at a Bernoulli trial t + x. *Example*. Let it be *true that* Sir Isaac Newton is *alive* at the year t=1700. Furthermore, let it be *true that* Sir Isaac Newton is *not alive* at the year t=2017. It is easy to see that both each other excluding states are true, which is a contradiction with the consequence that the principle of bivalence should be rejected. Similar and other early arguments against bivalence (Aristotle's sea battle argument) ignored the relation of bivalence to time. Thus far, various multi-valued logics have been developed, among them quantum logic, which lacks bivalence. Still, there are conditions where there is no third between two, *tertium non datur*. Thus far, let $p({}_0A_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0A_t$ at one single Bernoulli trial t. Let $p({}_0\underline{A}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_0\underline{A}_t$ at one single Bernoulli trial t. In general it is $p({}_0A_t) + p({}_0\underline{A}_t) =1$. Let $p({}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_RB_t$ at one single Bernoulli trial t. Let $p({}_R\underline{B}_t)$ denote the truth value, the degree of certainty, the probability as associated with an event ${}_R\underline{B}_t$ at one single Bernoulli trial t, it is $p({}_RB_t) + p({}_R\underline{B}_t) =1$. Let $p({}_0A_t \cup {}_RB_t)$ denote the truth value, the degree of certainty, the probability as associated with (inclusive) disjunction.

CLAIM.
In general, the *law of excluded middle* can be expressed by the formula

$$p({}_0A_t \cup {}_RB_t) = p({}_0A_t) + p({}_RB_t) = 1 \tag{38}$$

PROOF.
Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \tag{39}$$

Multiplying this equation by $p({}_0A_t \cup {}_RB_t)$, we obtain

$$1 \times p({}_0A_t \cup {}_RB_t) = 1 \times p({}_0A_t \cup {}_RB_t) \tag{40}$$

or

$$p({}_0A_t \cup {}_RB_t) = p({}_0A_t \cup {}_RB_t) \tag{41}$$

which can be rearranged as

$$p({}_0A_t \cup {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = p({}_0A_t) + p({}_RB_t) - p({}_0A_t \cap {}_RB_t) + \left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = 1 \tag{42}$$

Under conditions where

$$\left(\left(1 - p({}_0A_t)\right) \times \left(1 - p({}_RB_t)\right)\right) = 0 \tag{43}$$

and where

$$p({}_0A_t \cap {}_RB_t) = 0 \tag{44}$$

it is

$$p\left({}_{0}A_{t} \cup {}_{R}B_{t}\right)+0=p\left({}_{0}A_{t}\right)+p\left({}_{R}B_{t}\right)-0+0=1 \tag{45}$$

or

$$p\left({}_{0}A_{t} \cup {}_{R}B_{t}\right)=p\left({}_{0}A_{t}\right)+p\left({}_{R}B_{t}\right)=1 \tag{46}$$

QUOD ERAT DEMONSTRANDUM.

3.4. Theorem (Zero Divided By Zero Is Not Equal To Zero)

Let a set of integers consists of zero (0), the positive natural numbers and the negative natural numbers. Let X denote a set of integers thus that X={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Further, let Y denote a set of integers thus that Y={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Let the set X be independent of the set Y and vice versa. Let $f(y_t)$ denote a function which returns one single value out of Y at an single experiment t. Let $f(x_t)$ denote a function which returns one single value out of X at an single experiment t. Let $f(y_t)$ be independent of $f(x_t)$.

CLAIM/THEOREM.
In general, it is

$$\frac{f\left(y_{t}\right)-f\left(x_{t}\right)}{f\left(y_{t}\right)-f\left(x_{t}\right)} \neq 0 \tag{47}$$

PROOF BY CONTRADICTION.
A proof by contradiction is based on the law of non-contradiction. According to the rules of a proof by contradiction, we are starting this proof by assuming that *the opposite of our above claim is true*. A proof by assuming the opposite is not allowed to lead to a logical contradiction. Thus far, we assume that the claim above is not true; the opposite of our theorem above is true. It is generally valid that

$$\frac{f\left(y_{t}\right)-f\left(x_{t}\right)}{f\left(y_{t}\right)-f\left(x_{t}\right)}=0 \tag{48}$$

Now we perform a single thought experiment t=1. At this single experiment, we obtain the value $f(y_1)=3$ and $f(x_1)= 2$. Rearranging the equation above, we obtain

$$\frac{3-2}{3-2}=\frac{1}{1}=1=0 \tag{49}$$

a logical contradiction. This single experiment is enough to provide strict evidence that the term $((f(y_t)-f(x_t))/(f(y_t)-f(x_t)))$ is different zero. Still, to increase the strength of evidence of this proof we increase the number of experiments from t=1 to t=10000000000…

At every of these experiments we obtain the result that 1=0, *a logical contradiction*. Thus far, in order not to waste time any more, we perform a last experiment t+1 and do obtain the values $f(y_{t+1})=4$ and $f(x_{t+1})=3$. Rearranging equation above we obtain

$$\frac{4-3}{4-3}=\frac{1}{1}=1=0 \tag{50}$$

a logical contradiction. At this moment, we just don't know, what is the result of the division $((f(y_t)-f(x_t))/(f(y_t)-f(x_t)))$, i. e. if something equivalent to itself is divided by it-self. Still, according to this *reductio ad absurdum* it is generally valid that

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)} \neq 0 \tag{51}$$

QUOD ERAT DEMONSTRANDUM.

3.5. Theorem (Zero Divided By Zero Is Not Equal To Infinity)

Let a set of integers consists of zero (0), the positive natural numbers and the negative natural numbers. Let X denote a set of integers thus that X={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Further, let Y denote a set of integers thus that Y={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Let the set X be independent of the set Y and vice versa. Let $f(y_t)$ denote a function which returns one single value out of Y at an single experiment t. Let $f(x_t)$ denote a function which returns one single value out of X at an single experiment t. Let $f(y_t)$ be independent of $f(x_t)$.

CLAIM/THEOREM.
In general, it is

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)} \neq \infty \tag{52}$$

PROOF BY CONTRADICTION.
A proof by contradiction is based on the law of non-contradiction. According to the rules of a proof by contradiction, we are starting this proof by assuming that *the opposite of our above claim is true*. A proof by assuming the opposite is not allowed to lead to a logical contradiction. Thus far, we assume that the claim above is not true; the opposite of our theorem above is true. It is generally valid that

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)} = \infty \tag{53}$$

Now we perform a single thought experiment t=1. At this single experiment, we obtain the value $f(y_1)=7$ and $f(x_1)= 6$. Rearranging the equation above, we obtain

$$\frac{7-6}{7-6}=\frac{1}{1}=1=\infty \tag{54}$$

a logical contradiction. This single experiment is enough to provide strict evidence that the term $((f(y_t)-f(x_t))/(f(y_t)-f(x_t)))$ is different zero. Still, to increase the strength of evidence of this proof we increase the number of experiments from t=1 to t=10000000000… At every of these experiments we obtain the result that 1=∞, *a logical contradiction*. Thus far, in order not to waste time any more, we perform a last experiment t+1 and do obtain the values $f(y_{t+1})=8$ and $f(x_{t+1})=7$. Rearranging equation above we obtain

$$\frac{8-7}{8-7}=\frac{1}{1}=1=\infty \tag{55}$$

a logical contradiction. At this moment, we just don't know, what is the result of the division $((f(y_t)-f(x_t))/(f(y_t)-f(x_t)))$, i. e. if something equivalent to itself is divided by itself. Still, according to this *reductio ad absurdum* it is generally valid that

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)} \neq \infty \tag{56}$$

QUOD ERAT DEMONSTRANDUM.

Scholium.

At this stage, we don't know what is the result is zero is divided by zero. Still, we are looking for a generally valid solution of this problem. In point of fact, according to the theorems before we are authorized to accept that following. 1. ***It is generally valid*** that 0/0 is not equal to zero. 2. ***It is generally valid*** that 0/0 is not equal to infinity. The experiments above can be repeated "without" and end. After a very long and "endless" series of proofs by contradiction, we will find out that 0/0 is not equal to 5, that 0/0 is not equal to 6, that 0/0 is not equal to 7, that 0/0 is not equal to … How many proof by contradiction are necessary to recognize [10], [11], [12] the evident?

3.6. Theorem (Zero Divided By Zero Is One)

Let a set of integers consists of zero (0), the positive natural numbers and the negative natural numbers. Let X denote a set of integers thus that X={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Further, let Y denote a set of integers thus that Y={-∞, …, -2, -1- 0, +1, +2, …,+∞}. Let the set X be independent of the set Y and vice versa. Let $f(y_t)$ denote a function which returns one single value out of Y at an single experiment t. Let $f(x_t)$ denote a function which returns one single value out of X at an single experiment t. Let $f(y_t)$ be independent of $f(x_t)$.

CLAIM/THEOREM.

In general, it is

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)}=1 \tag{57}$$

PROOF BY INDUCTION.

A proof by induction is a form of *direct proof* and usually done in several steps. The first step, known as *the base case*, is to prove the given equation, a statement for the first run of a certain thought experiment t=1. One than assumes the *induction hypothesis* that an equation, a statement holds for the run of t=n thought experiments. Finally, the inductive step proves an equation, a statement holds for the run of t= n + 1 thought experiments.

Thus far, we perform a single thought experiment t=1. At this single experiment, we obtain the value $f(y_1)=5$ and $f(x_1)= 4$. Rearranging the equation above, we obtain

$$\frac{5-4}{5-4}=\frac{1}{1}=1=1 \tag{58}$$

a contradiction free and correct result. This single experiment is enough to provide strict evidence that the term $((f(y_t)-f(x_t))/(f(y_t)-f(x_t)))$ is different zero. Often a real experiment as an analogue of a thought experiment is impossible for physical, technological, ethical, or other reasons. Thus far, to increase the strength of evidence of this proof too we increase the number of experiments from t=1 to t=10000000000… At every of these experiments we obtain the result that 1=1 (the induction hypothesis), *a contradiction free and correct result*. Thus far, convince ourselves definitely, we perform a last experiment t+1 and do obtain the values $f(y_{t+1})=6$ and $f(x_{t+1})=5$. Rearranging equation above we obtain

$$\frac{6-5}{6-5}=\frac{1}{1}=1=1 \tag{59}$$

again a contradiction free and correct result. In general, it is valid that

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)}=1 \tag{60}$$

QUOD ERAT DEMONSTRANDUM.

Scholium.

This proof is based on the fact that $f(y_t)$ is independent of $f(x_t)$ and vice versa. This includes the possibility that $f(y_t) = f(x_t)$ with the consequence that

$$\frac{f(y_t)-f(x_t)}{f(y_t)-f(x_t)}=\frac{f(y_t)-f(y_t)}{f(y_t)-f(y_t)}=\frac{0}{0}=1 \tag{61}$$

Thus far the theoretical challenge of the thought experiment before is the simple dilemma that *either* we must accept that 0/0=1 *or* we cannot accept the result of the theorem before that $((f(y_t)-f(x_t))/(f(y_t)-f(x_t))) = 1$. Clearly, the thought experiment before is characterized by an intriguing plasticity and clearness and grounded on an imaginary number of experiments and confirms the theorem that $((f(y_t)-f(x_t))/(f(y_t)-f(x_t))) = 1$. In the following we will highlight whether classical logic may contribute anything to the problem of the division of 0 by 0. Assumed that *classical logic is generally valid*, then the same logic should be valid for the division of 0 by 0 too with the consequence that the problem of the division of 0 by 0 could be solved by while relying only on classical logic.

3.7. Theorem (The Relationship between ${}_0X_t$ and ${}_0\underline{X}_t$ normalized)

Let ${}_RC_t$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_0x_t$ denote an event, a subset of the sample space ${}_RC_t$. Let ${}_0\underline{x}_t$ denote the negation of an event ${}_0x_t$, another, complementary subset of the sample space ${}_RC_t$.

CLAIM.

In general, the relationship between ${}_0x_t$ and ${}_0\underline{x}_t$ can be normalized as

$$\frac{{}_0x_t}{{}_RC_t}+\frac{{}_0\underline{x}_t}{{}_RC_t}=+1 \tag{62}$$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1=+1 \tag{63}$$

Multiplying this equation by ${}_0x_t$, we obtain

$$1\times {}_0x_t = 1\times {}_0x_t \tag{64}$$

or

$$ {}_0x_t = {}_0x_t \tag{65}$$

x

Adding ${}_0\underline{x}_t$, we obtain

$$ {}_0x_t + {}_0\underline{x}_t = {}_0x_t + {}_0\underline{x}_t \tag{66}$$

According to our definition, this is equivalent with

$$ {}_{0}x_{t} + {}_{0}\underline{x}_{t} \equiv {}_{R}C_{t} \tag{67} $$

Dividing by ${}_{R}C_{t}$, it is

$$ \frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} \equiv \frac{{}_{R}C_{t}}{{}_{R}C_{t}} = +1 \tag{68} $$

QUOD ERAT DEMONSTRANDUM.

Scholium.

Especially under conditions of Euclidean geometry and Einstein's special theory of relativity, ${}_{R}C_{t}$ denotes the hypotenuse, the side opposite to the right angle of a right angled triangle while ${}_{0}a_{t}$ and ${}_{0}b_{t}$ denote the other two sides of a right angled triangle. According to Euclid's theorem, the Pythagorean Theorem follows as

$$ \frac{{}_{0}x_{t} \times {}_{R}C_{t}}{{}_{R}C_{t} \times {}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t} \times {}_{R}C_{t}}{{}_{R}C_{t} \times {}_{R}C_{t}} \equiv \frac{{}_{0}a_{t}^{2}}{{}_{R}C_{t}^{2}} + \frac{{}_{0}b_{t}^{2}}{{}_{R}C_{t}^{2}} = +1 \tag{69} $$

One consequence of such an approach to the Pythagorean Theorem and the logical negation is that the Lorenz factor of Einstein's special theory of relativity is nothing else but a mathematical reformulation of logical negation under conditions of Einstein's special theory of relativity.

3.8. Theorem (The Number One According To Classical Logic)

Let ${}_{R}C_{t}$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_{0}x_{t}$ denote an event, a subset of the sample space ${}_{R}C_{t}$. Let ${}_{0}\underline{x}_{t}$ denote the negation of an event ${}_{0}x_{t}$, another, complementary subset of the sample space ${}_{R}C_{t}$.

CLAIM.

In general, it is

$$ \frac{{}_{0}x_{t}}{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)} = +1 \tag{70} $$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$ +1 = +1 \tag{71} $$

Due to the theorem before, this equation can be rearranged as

$$ \frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} = +1 \tag{72} $$

Rearranging equation, it is

$$\frac{{}_0x_t}{{}_RC_t} = 1 - \frac{{}_0\underline{x}_t}{{}_RC_t} \tag{73}$$

or

$${}_0x_t = {}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right) \tag{74}$$

and at the end

$$\frac{{}_0x_t}{{}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right)} = \frac{{}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right)}{{}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right)} = +1 \tag{75}$$

or as

$$\frac{{}_0x_t}{{}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right)} = +1 \tag{76}$$

QUOD ERAT DEMONSTRANDUM.

SUB-PROOF.
It is

$$\frac{{}_0x_t}{{}_RC_t \times \left(1 - \frac{{}_0\underline{x}_t}{{}_RC_t}\right)} = +1 \tag{77}$$

An experiment provided the values ${}_0x_t = 1$. Thus far, ${}_0\underline{x}_t = 0$ while ${}_RC_t = 1$. We obtain

$$\frac{1}{1 \times \left(1 - \frac{0}{1}\right)} = \frac{1}{1 \times (1-0)} = \frac{1}{1 \times 1} = \frac{1}{1} = +1 \tag{78}$$

QUOD ERAT DEMONSTRANDUM.

3.9. Theorem (The Number One According To Classical Logic)

Let ${}_{R}C_{t}$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_{0}x_{t}$ denote an event, a subset of the sample space ${}_{R}C_{t}$. Let ${}_{0}\underline{x}_{t}$ denote the negation of an event ${}_{0}x_{t}$, another, complementary subset of the sample space ${}_{R}C_{t}$. In general, under conditions of classical logic, it is ${}_{0}x_{t} + {}_{0}\underline{x}_{t} = {}_{R}C_{t} = 1$.

CLAIM.

In general, it is

$$\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)} = +1 \tag{79}$$

PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1 = +1 \tag{80}$$

Due to the theorem before, this equation can be rearranged as

$$\frac{{}_{0}x_{t}}{{}_{R}C_{t}} + \frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} = +1 \tag{81}$$

Rearranging equation, yields

$$\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}} = 1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}} \tag{82}$$

or

$${}_{0}\underline{x}_{t} = {}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right) \tag{83}$$

and at the end

$$\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)} = \frac{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)}{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)} = +1 \tag{84}$$

or as

$$\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t} \times \left(1 - \frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)} = +1 \tag{85}$$

QUOD ERAT DEMONSTRANDUM.

SUB-PROOF.
It is

$$\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}\times\left(1-\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)}=+1 \tag{86}$$

An experiment provided the values ${}_{0}x_{t}=0$. Thus far, ${}_{0}\underline{x}_{t}=1$ while ${}_{R}C_{t}=1$. We obtain

$$\frac{1}{1\times\left(1-\frac{0}{1}\right)}=\frac{1}{1\times(1-0)}=\frac{1}{1\times1}=\frac{1}{1}=+1 \tag{87}$$

QUOD ERAT DEMONSTRANDUM.

3.10. Theorem (According To Classical Logic it is (+0/+0)=+1)

Let ${}_{R}C_{t}$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_{0}x_{t}$ denote an event, a subset of the sample space ${}_{R}C_{t}$. Let ${}_{0}\underline{x}_{t}$ denote the negation of an event ${}_{0}x_{t}$, another, complementary subset of the sample space ${}_{R}C_{t}$. In general, under conditions of classical logic, it is ${}_{0}x_{t}+{}_{0}\underline{x}_{t}={}_{R}C_{t}=1$.

CLAIM.
In general, it is

$$\frac{+0}{+0}=+1 \tag{88}$$

PROOF.
Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1=+1 \tag{89}$$

Classical logic define the number 1 as

$$\frac{{}_{0}x_{t}}{{}_{R}C_{t}\times\left(1-\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)}=+1 \tag{90}$$

and equally as

$$\frac{{}_{0}x_{t}}{{}_{R}C_{t}\times\left(1-\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}}\right)}=\frac{{}_{0}\underline{x}_{t}}{{}_{R}C_{t}\times\left(1-\frac{{}_{0}x_{t}}{{}_{R}C_{t}}\right)}=+1 \tag{91}$$

It is the same number 1 which is defined or determined by classical logic in two different ways. Now we perform a thought experiment. The experiment performed provided the values ${}_{0}x_{t}=1$. Thus far, ${}_{0}\underline{x}_{t}=0$ while ${}_{R}C_{t}=1$. We obtain

$$\frac{{}_0x_t}{{}_RC_t\times\left(1-\frac{{}_0\underline{x}_t}{{}_RC_t}\right)}=\frac{1}{1\times\left(1-\frac{0}{1}\right)}=\frac{{}_0\underline{x}_t}{{}_RC_t\times\left(1-\frac{{}_0x_t}{{}_RC_t}\right)}=\frac{0}{1\times\left(1-\frac{1}{1}\right)}=+1 \tag{92}$$

or

$$\frac{1}{1}=\frac{0}{0}=+1 \tag{93}$$

In other words, according to classical logic, it is

$$\frac{+0}{+0}=+1 \tag{94}$$

QUOD ERAT DEMONSTRANDUM.

3.11. Theorem (According To Classical Logic it is (+0/+0)=+1)

Let ${}_RC_t$ denote the set of all the possible outcomes of a random experiment, a phenomenon in nature et cetera. Let ${}_0x_t$ denote an event, a subset of the sample space ${}_RC_t$. Let ${}_0\underline{x}_t$ denote the negation of an event ${}_0x_t$, another, complementary subset of the sample space ${}_RC_t$. In general, under conditions of classical logic, it is ${}_0x_t + {}_0\underline{x}_t = {}_RC_t =1$.

CLAIM.
In general, it is

$$\frac{+0}{+0}=+1 \tag{95}$$

PROOF.
Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1=+1 \tag{96}$$

Classical logic defines the number 1 in a mathematically correct way as

$$\frac{{}_0x_t}{{}_RC_t\times\left(1-\frac{{}_0\underline{x}_t}{{}_RC_t}\right)}=+1 \tag{97}$$

but at from another point of view equally as

$$+1=\frac{{}_0x_t}{{}_RC_t\times\left(1-\frac{{}_0\underline{x}_t}{{}_RC_t}\right)}=\frac{{}_0\underline{x}_t}{{}_RC_t\times\left(1-\frac{{}_0x_t}{{}_RC_t}\right)}=+1 \tag{98}$$

It is the same number 1 which is defined or determined by classical logic in two different but equally valid ways. Now we perform a second thought experiment. The experiment performed provided the values ${}_0x_t = 0$. Thus far, ${}_0\underline{x}_t = 1$ while ${}_RC_t = 1$. We obtain

$$+1=\frac{{}_0x_t}{{}_RC_t\times\left(1-\frac{{}_0x_t}{{}_RC_t}\right)}=\frac{0}{1\times\left(1-\frac{1}{1}\right)}=\frac{{}_0x_t}{{}_RC_t\times\left(1-\frac{{}_0x_t}{{}_RC_t}\right)}=\frac{1}{1\times\left(1-\frac{0}{1}\right)}=+1 \tag{99}$$

or

$$\frac{0}{0}=\frac{1}{1}=+1 \tag{100}$$

In other words, according to classical logic, it is

$$\frac{+0}{+0}=+1 \tag{101}$$

QUOD ERAT DEMONSTRANDUM.

3.12. Theorem (According To Probability Theory it is (+0/+0)=+1)

Let $p({}_0A_t)$ denote the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$. Le $p({}_RB_t)$ denote the probability $p({}_RB_t)$ as associated with an event ${}_RB_t$.

CLAIM.

In general, it is

$$\frac{+0}{+0}=+1 \tag{102}$$

DIRECT PROOF.

Given axiom I (principium identitatis, lex identitatis, the identity law) it is

$$+1=+1 \tag{103}$$

Classical logic defines the number 1 in a mathematically correct way as

$$1\times p({}_0A_t)=1\times p({}_0A_t) \tag{104}$$

or equally as

$$p({}_0A_t)=p({}_0A_t) \tag{105}$$

There are conditions where the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$, is independent from something third. This must not mean that the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$, must be constant. A probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ stay that what it is, a third has no influence on this fact. There is only one operation which assures the independence of the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$. We obtain the following equation

$$p({}_0A_t)\times 1=p({}_0A_t) \tag{106}$$

Only if the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ is multiplied by 1, it stays that what it is. Thus far, even if the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ is related to a probability $p({}_RB_t)$ as associated with an event ${}_RB_t$, under conditions, where it is given that $(p({}_RB_t)/\ p({}_RB_t)) = 1$, the probability $p({}_RB_t)$ as associated with an event ${}_RB_t$, has no influence on the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$. The equation before changes to

$$p({}_0A_t)\times\left(\frac{p({}_RB_t)}{p({}_RB_t)}\right)=p({}_0A_t) \tag{107}$$

In other words, especially under conditions of independence, it is

$$\frac{p({}_0A_t\cap{}_RB_t)}{p({}_RB_t)}=\frac{p({}_0A_t)\times p({}_RB_t)}{p({}_RB_t)}=p({}_0A_t) \tag{108}$$

All events must have a probability between 0.0 and 1.0, including 0.0 and 1.0. In other words, it is $0.0 \leq p({}_0A_t) \leq 1.0$. The equation before is and must be valid for any probability value and even in the case if $p({}_RB_t) =0$. Thus far, let $p({}_RB_t) =0$, we obtain

$$p({}_0A_t)\times\frac{0}{0}=p({}_0A_t) \tag{109}$$

Whatever the result of the operation (0/0) may be, under conditions of independence, the same operation must ensure that $p({}_0A_t)= p({}_0A_t)$. As we will see, there is only one value, as proofed before, which assures this result. Our assumption was that the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ is independent of a probability $p({}_RB_t)$ as associated with an event ${}_RB_t$. Thus far, the probability $p({}_0A_t)$ as associated with an event ${}_0A_t$ may take the value $p({}_0A_t)= 1$. Under conditions where $p({}_0A_t)= 1$ we obtain

$$1\times\frac{0}{0}=1 \tag{110}$$

Thus far, either the law of independence of the probability theory breaks down if $p({}_0A_t)=1$ and $p({}_RB_t) =0$ or we must accept according to probability theory that

$$\frac{+0}{+0}=+1 \tag{111}$$

QUOD ERAT DEMONSTRANDUM.

4. Discussion

Many-valued or dialectical logic as a non-classical logic does not restrict the number of truth values to only two, either true or false, usually denoted by "0" and "1". As usual in classical logic, the truth values of the dialectical logic may pass over into the truth values either true or false with the consequence that dialectical logic reduces to simple classical logic. A critical discussion of some main systems of many-valued logics (Lukasiewicz logics, Gödel logics, t-Norm based systems, Three-valued systems, Dunn/Belnap's 4-valued system, Product systems) can be found in literature. Fuzzy logic which belongs to the family of many-valued logics too, is already refuted [1].

The proof that 0/0=1 is based on classical logic and the Pythagorean theorem and generally valid. Still, some point are worth being mentioned.

PSEUDO-COUNTEREXAMPLE.

Let us claim something which is obviously incorrect. Let it be true that

$$+2 = +1 \tag{112}$$

In other words, multiplying by zero, we obtain

$$+2 \times 0 = +1 \times 0 \tag{113}$$

At the end, it is

$$+0 = +0 \tag{114}$$

Dividing by zero, it is

$$\frac{+0}{+0} = \frac{+0}{+0} \tag{115}$$

or, according to our proofs before,

$$+1 = +1 \tag{116}$$

which is a logical contradiction. We started with something incorrect (+2=+1) and obtained something correct (+1=+1). This is not possible. The arguments so far considered show that a superficial of the division of zero by zero can lead to invalid inferences with the result to treat the division of zero by zero as indeterminate and undefined. The problem above arises from a dilemma posed by the multiplication by zero and not by the division by zero. The multiplication by zero changes something not equivalent (2=1) to something equivalent (0=0). And yet, despite a long history of debate going back to Aristotle [10], the division of zero by zero is possible without any contradiction. Thus far, let as regard the following.

SUB-PROOF.

Again, let us claim something which is obviously incorrect. Let it be true that

$$+2 = +1 \tag{117}$$

In other words, multiplying by zero, we obtain

$$+2\times 0=+1\times 0 \tag{118}$$

In our understanding and following Euler, 2x0 is not equal to 1x0. Thus far, we change the notation [2] and do obtain

$$2\_0=1\_0 \tag{119}$$

Dividing by zero, it is

$$\frac{2\_0}{0}=\frac{1\_0}{0} \tag{120}$$

or, according to our proofs before,

$$+2=+1 \tag{121}$$

which is equivalent with the incorrect starting point, we started from.
QUOD ERAT DEMONSTRANDUM.

5. Conclusion

A division of zero by zero is possible and defined. Zero divided by zero is one. Still, some rules of precedence should be respected if this mathematical operation should be generally accepted.

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