One Word: Navier-Stokes.

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Abstract

We prove the Navier-Stokes equations, by means of the Metabolic Theory of Ecology and the Rule of 72. Macroecological theories are proof to the Navier-Stokes equations. A solution could be found using Kleiber's Law. Measurement is possible through the heat calorie. A Pareto Improvement exists within the Navier-Stokes equations. This is done by superposing dust solutions onto fluid solutions. In summary, the Navier-Stokes equations require a theoretical solution. The Metabolic Theory of Ecology, along with Kleiber's Law, form a theory by such standards.

Keywords: Navier-Stokes equations, Metabolic Theory of Ecology, Rule of 72, Kleiber's Law, Pareto, physics

Introduction

The Navier-Stokes (N-S) equations ask for a mathematical theory which will unlock the secrets hidden within its equations. Proof is asked of four statements:

(A) Existence and smoothness of Navier–Stokes solutions on R^3. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (4). Take $f(x, t)$ to be identically zero. Then there exist smooth functions $p(x, t)$, $u_i(x, t)$ on $R^3 \times [0,1)$ that satisfy (1), (2), (3), (6), (7).

(B) Existence and smoothness of Navier–Stokes solutions in R^3/Z^3. Take $\nu > 0$ and $n = 3$. Let $u^\circ(x)$ be any smooth, divergence-free vector field satisfying (8); we take $f(x, t)$ to be identically zero. Then there exist smooth functions $p(x, t)$, $u_i(x, t)$ on $R^3 \times [0,1)$ that satisfy (1), (2), (3), (10), (11).

(C) Breakdown of Navier–Stokes solutions on R^3. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on R^3 and a smooth $f(x, t)$ on $R^3 \times [0,1)$, satisfying (4), (5), for which there exist no solutions (p, u) of (1), (2), (3), (6), (7) on $R^3 \times [0,1)$.

(D) Breakdown of Navier–Stokes Solutions on R^3/Z^3. Take $\nu > 0$ and $n = 3$. Then there exist a smooth, divergence-free vector field $u^\circ(x)$ on R^3 and a smooth $f(x, t)$ on $R^3 \times [0,1)$, satisfying (8), (9), for which there exist no solutions (p, u) of (1), (2), (3), (10), (11) on $R^3 \times [0,1)$.

There exists a theory and law to satisfy (A) and (C). There exists a self-similarity solution to satisfy (B) and (D). The theory that satisfies (A) and (C) is the Metabolic Theory of Ecology (MTE) and the law is Kleiber's Law (KL). The similarity solution that satisfies (B) and (D) is the Rule of 72. MTE is a macroecological theory, thereby making KL a weak solution of a partial differential equation (PDE). Since the Rule of 72 is also known as the Rule of 70 and the Rule of 69.3, it displays geometric rate of increase. This paper will focus on eliminating deterministic chaos under these constitutive relations.

To classify a dynamical system as chaotic, it must have these properties:

1. It must be sensitive to initial conditions.

2. It must be topologically mixing.

3. It must have dense periodic orbits.

In some cases, the last two properties in the above have been shown to actually imply sensitivity to initial conditions. In these cases, while it is often the most practically significant property, "sensitivity to initial conditions" need not be stated in the definition.

If attention is restricted to intervals, the second property implies the other two. An alternative, and in general weaker, definition of chaos uses only the first two properties in the above list.

Metabolism (KL) and Rule of 72

In hydrology, the harmonic mean is similarly used to average hydraulic conductivity values for flow that is perpendicular to layers (e.g. geologic or soil) - flow parallel to layers uses the arithmetic mean. This apparent difference in averaging is explained by the fact that hydrology uses conductivity, which is the inverse of resistivity. N-S coefficients are neither periodic nor statistically homogenous. Therefore, we can presume that N-S calls for a backing theory which coefficients are neither periodic nor statistically homogeneous (so-called arbitrarily rough coefficients). We prove that the N-S equations are highly oscillatory coefficients and can be replaced with a homogeneous (uniform) coefficient. We prove that KL is this homogenized structure. We also find harmony with the Rule of 72 as a weak PDE solution. This can be shown through volatility in finance as opposed to viscosity. This is evident in dust solutions, which are all fluid solutions. The concept of credit crunch acts as a sort of 'blow up' time for these equations, and is Pareto Optimal. Leray (1934) showed that some N-S equations in three space dimensions always have a weak solution (p, u) with suitable growth properties. With the Rule of 72 (Rule of 70 and Rule of 69.3) acting as point (p) on unique weak solutions of N-S, we find a constitutive equation. The fluid parcel/element acts on the scalar field (and also vector field) by demonstrations of thermodynamic work, conservation of mass, and to be expected bioharmonic equation. The vector field is derived from the Rule of 70 and 69.3. Being axisymmetric with respect to density and other physical and chemical properties, a production of numerical dissipation in both velocity and mass can produce some exact solutions to the N-S equations. Therefore, we prove radial flow and $R > 1.41$ are related to e (mathematical constant) and pi (π). In place of chaotic properties we note that the properties of Stoke's flow are the equivalent. Stoke's flow properties call for instantaneity, time-reversibility, and Stoke's paradox. Deterministic chaos properties call for sensitivity to initial conditions, topologically mixing, and dense periodic orbits. Thus, a Pareto Improvement is implicit.

Existence, Smoothness, and Breakdown of Metabolism (KL)

Symbolically: if q_0 is the animal's metabolic rate, and M the animal's mass, then Kleiber's law states that $q_0 \sim M^{3/4}$. In MTE, $B = B_0 M^{3/4}$.

The following are equivalent:

(TFAE$_1$) $q_0 \sim M^{3/4}$ or $B = B_0 M^{3/4}$

And

(TFAE$_1$) $B \subset R^3$ or $B_r \subset R^3$ is a ball of radius r.

(TFAE$_2$) *parabolic cylinders* $Q_r = B_r \times I_r \subset R^3 \times R$ where $I_r \subset R$ is an interval of length r^2.

And

(TFAE$_2$) *specific metabolic rate* SMR= $(B/M) = b_0 M^{-1/4} e^{-E/kT}$ where E is activation energy in electronvolts or joules, T is absolute temperature in kelvins, and k is the Boltzmann constant in eV/K or J/K.

Given there is no rate of strain on the sphere (no E) since the spheres are assumed to be rigid, there cannot be a singular set of u° (x) and, hence, no divergence-free vector field. Therefore, the solution exists in mass (M) satisfying (A). Since the metabolic formula shows dense periodic orbits (e.g. thermodynamic work and conservation of mass), hence Stoke's paradox, the solution breakdown is on (C). State of equation should not be presumed.

Existence, Smoothness, and Breakdown of Rule of 72

For periodic compounding, the exact doubling time for an interest rate of r per period is:

$t = \ln(2)/\ln(1+r) \approx 72/r$ where t is the number of periods required.

Note that r makes sense for $u \in L^2$, $f \in L^1$, $p \in L^1$, whereas t makes sense only if u(x, t) is twice differentiable in x. Similarly, if φ(x, t) is a smooth function, compactly supported in $R^3 \times (0, 1)$, then a formal integration by parts and density (e.g. time-reversibility) imply a weak solution of N-S. Therefore, the solution exists as an equation of state in (B). The value 72 is a convenient choice of numerator, since it has many small divisors: 1, 2, 3, 4, 6, 8, 9, and 12. However, lower numbers are more accurate. Direct numerical simulations (DNS) relate the number of time-integration steps. Instantaneity must be proportional to L/(Cη) where C is here the Courant number and η is the Kolmogorov scale. Thus, from the Reynolds number definitions for Re, η and L, it follows that $L/\eta \sim Re^{3/4}$. This equation of state on the fluid parcel/element proves thermodynamic work, which implies conservation of mass. Therefore, the solution breakdown is on (D). Topological mixing should not be presumed.

Observations

A few constitutive relations have been observed between Faxen's law and DNS. Faxen's law is a correction to Stoke's law. In such a way, it can be said to model the significant relation that KL is to MTE. The two equations are one in the same, but the latter holds the larger theory. The former equations are doled out in three's (e.g. laws and models). According to the statistical Rule of Three, there is a 95% probability that they hold the same purpose. A similar equation appears to exist between the Michell solution, an elasticity equation, and the Rule of 72. This can be demonstrated by replacing the stress function C with similar natural logarithms, say the natural logarithm of 2. This leads to a relation with radial flow. As constitutive equation to radial flow as there is also an accepted relation to the mathematical constant (e) and pi (π).

MTE Theory

N-S is used in many metabolic functions, but whether MTE theory is in fact the solution to N-S equations has not been looked at. There is more likely a direct similarity than once thought, leading to bioharmonic equations of state. Perhaps, a single bioharmonic equation of state. An attempt to link the mathematical constant and pi to N-S is displayed in an absolute difference of unit area metric:

Absolute difference in unit area of $\sqrt{(2/\pi)}\sigma = 0.798\sigma$.

This metric is possible when fluid is incompressible. As bounded energy, the absolute difference metric is viewed as one of the physical quantities of 'the theory', thus it transforms in a certain manner when the frame of reference is changed, and it can be legitimately used in describing Stoke's flow and should be expected to solve for (A), (B), (C), and (D). In a surprising number of cases, the laws of physics in special relativity (such as the famous equation $E=mc^2$) can be deduced by combining the postulates of special relativity with the hypothesis that the laws of special relativity approach the laws of classical mechanics in the non-relativistic limit. The self-similarity is not a coincidence. Therefore, we find it necessary to make the point that the calorie is a unit of heat. The "Calorie" used by nutritionists is called the "large calorie" and is actually a kilocalorie (1 Cal=1 kcal=10^3 cal).

Conclusion

We find that the N-S equations can be solved using MTE and financial principles. Observations on Faxen's law, the Michell solution, and other physical laws reiterate this fact. The calorie can help in proving N-S. Thus, a solution could be found using Kleiber's Law. This paper has found a solution using Kleiber's Law. Replacing N-S equations with certain macroecological theories will create pointwise solutions consistent within the equations. Finally, a Pareto Improvement exists within the Navier-Stokes equations by making dust solutions more on point with fluid solutions.

Conflict of Interest

The author claims no conflict of interest.

References

Basener, William F. (2006). Topology and its applications. Wiley. p. 42. ISBN 0-471-68755-3.

Elaydi, Saber N. (1999). Discrete Chaos. Chapman & Hall/CRC. p. 117. ISBN 1-58488-002-3.

Fefferman, Charles L. "Existence and smoothness of the Navier-Stokes equation" (PDF). claymath.org. Clay Mathematics Institute.

Hasselblatt, Boris; Anatole Katok (2003). A First Course in Dynamics: With a Panorama of Recent Developments. Cambridge University Press. ISBN 0-521-58750-6.

J. Leray, Sur le mouvement d'un liquide visquex emplissent l'espace, Acta Math. J. 63 (1934), 193–248.

Max Kleiber (1932). "Body size and metabolism". Hilgardia. 6: 315–351. doi:10.3733/hilg.v06n11p315.

Medio, Alfredo; Lines, Marji (2001). Nonlinear Dynamics: A Primer. Cambridge University Press. p. 165. ISBN 0-521-55874-3.

Slavin, Steve (1989). All the Math You'll Ever Need. John Wiley & Sons. pp. 153–154. ISBN 0-471-50636-2.

Vellekoop, Michel; Berglund, Raoul (April 1994). "On Intervals, Transitivity = Chaos". The American Mathematical Monthly. 101 (4): 353–5. doi:10.2307/2975629. JSTOR 2975629.