A Brief Solution to the Riemann Hypothesis over the Lagarias Transformation

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In accordance with the transformation of Lagarias [1] which is the equivalent of the Riemann Hypothesis, for a positive integer n, let $\sigma(n)$ denote the sum of the positive integers that divide n. Let H_n denote the nth harmonic number by

$$H_n = \sum_{n=1}^{n} \frac{1}{n}$$

Does the following inequality hold for all $n \geq 1$ where $\sigma(n)$ is the sum of divisors function?

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$$

1 Definition for the solutions

Theorem: *First of all, let's define an imaginary function as $\rho(n)$, and know that this function is the sum of the elements which are not dividable being the result is an integer in a function as nH_n; so according to this definition, it becomes as the following.*

$$H_n = \frac{\sigma(n) + \rho(n)}{n}$$

By using the equation, $H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$ inequality turns into (1).

$$H_n + \ln(H_n)e^{H_n} \geq nH_n - \rho(n) \quad (1)$$

If it is edited, it becomes (2) over (2a).

$$\frac{\ln(H_n)e^{H_n} + \rho(n)}{n-1} \geq H_n \quad (2)$$

$$\ln(H_n)e^{H_n} \geq nH_n - H_n - \rho(n) \quad (2a)$$

Condition: *Right this point assume, that the actual inequality is not (2) but is (3).*

$$\frac{e^{H_n}}{n} \geq H_n \quad (3)$$

On (2), actually the numerator is always bigger than e^{H_n}, and also if the divisor was $n-1$, this would increase the possibility of to be greater than H_n of the division; so for the worst possibility, let's use this as (3).

Now, let (3) be (4).

$$\sqrt[n]{e} \geq \sqrt[nH_n]{nH_n} \quad (4)$$

For $e = \lim_{n\longrightarrow\infty}\left(1 + \frac{1}{n}\right)^n$, (4) becomes (5).

$$\lim_{n\longrightarrow\infty}\left(1 + \frac{1}{n} \geq \sqrt[nH_n]{nH_n}\right) \quad (5)$$

For this, it can be written as (6)

$$\lim_{n\longrightarrow\infty}\left(n + 1 \geq nk\right) \quad (6)$$

where $k = \sqrt[nH_n]{nH_n}$. For $n \geq nk - 1$ it becomes $\frac{1}{n} \geq k - 1$; so what ever the direction of the inequality is, even if both sides were equal to each other, k would not become a number smaller than 1 since n is always positive. It is always $k > 1$. Here assume, that is (7)

$$n = nk - 1 + b \quad (7)$$

since it is $n \geq nk-1$ over (6), where b is a number being $b \in \mathbb{R}^+$ and thus being $b > 0$; thus it becomes (8) over (7).

$$n = \frac{b-1}{1-k} \quad (8)$$

Since is $k > 1$, then b must always be smaller number than 1 to be positive of the division; thus it becomes $1 > b > 0$; so k cannot take random values since n is positive integer. If it is $k > 1$, for the greatest value of k, it becomes $\lim_{b\to 0} k = 2$. For this value, equality of (7) becomes $n = 2n - 1$ and thus becomes $n = 1$. It means, actually k decreases as long as n increased; thus it means it is always (9),

$$1 = \lim_{m\longrightarrow\infty} \sqrt[m]{m} \quad (9)$$

where $m \in \mathbb{Z}^+$; thus also means it is (10),

$$1 = \lim_{n\longrightarrow\infty} \sqrt[nH_n]{nH_n} \quad (10)$$

since is $H_n \geq 1$ and thus is $nH_n \geq 1$.

2 The Result: RH$^+$

By the defined elements, over (6), it becomes (11).

$$\lim_{n\longrightarrow\infty} n + 1 \geq n \quad (11)$$

This is also equivalent of (3) and thus of (12).

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n) \quad (12)$$

Acknowledgment

I have been working about some unknown problems for a time [2] that Riemann Hypothesis is included as well, and a short time ago I supposed that I found a solution out to the Riemann Hypothesis; but I noticed that there is a stupid mistake; after that I published a brief approach; for a long time I did not work about it; but today I remembered it and just wanted to work because of boredom, and finally I could bring a simple solution out indirectly in a few hours again after midnight even if it is not so sexy and enlightening about the functions to determine relation with prime separation. Even so, solution is solution always.

Good bye!

References

1. Jeffrey C. Lagarias. 2002 *An Elementary Problem Equivalent to the Riemann Hypothesis*, The American Mathematical Monthly. Vol. 109, No. 6, pp. 534-543
2. Kavak M. 2018, *Complement Inferences on Theoretical Physics and Mathematics*, OSF Preprints, Available online: https://osf.io/tw52w/

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