A Brief Solution to the Riemann Hypothesis over the Lagarias Transformation

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Over the paper of Lagarias [1], for a positive integer n, let $\sigma(n)$ denote the sum of the positive integers that divide n. Let H_n denote the nth harmonic number by

$$H_n = \sum_{n=1}^{n} \frac{1}{n}$$

Does the following inequality hold for all $n \geq 1$ where $\sigma(n)$ is the sum of divisors function?

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$$

1 Definition for the solutions

Theorem: *First of all, let's define an imaginary function as $\rho(n)$, and know that this function is the sum of the elements which are not dividable being the result is an integer in a function as nH_n; so according to this definition, it becomes as the following.*

$$H_n = \frac{\sigma(n) + \rho(n)}{n}$$

By using the equation, $H_n + \ln(H_n)e^{H_n} \geq \sigma(n)$ inequality turns into (1).

$$H_n + \ln(H_n)e^{H_n} \geq nH_n - \rho(n) \tag{1}$$

If it is edited, it becomes (2) over (2a).

$$\frac{\ln(H_n)e^{H_n} + \rho(n)}{n-1} \geq H_n \tag{2}$$

$$\ln(H_n)e^{H_n} \geq nH_n - H_n - \rho(n) \tag{2a}$$

Condition: *Right this point assume, that the actual inequality is not (2) but is (3).*

$$\frac{e^{H_n}}{n} \geq H_n \tag{3}$$

On (2), actually the numerator is always bigger than e^{H_n}, and also if the divisor was $n-1$, this would increase the possibility of to be greater than H_n of the division; so for the worst possibility, let's use this as (3).

Now, let (3) be (4).

$$\sqrt[n]{e} \geq \sqrt[nH_n]{nH_n} \tag{4}$$

For $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$, (4) becomes (5).

$$\lim_{n \to \infty} \left(1 + \frac{1}{n} \geq \sqrt[nH_n]{nH_n}\right) \tag{5}$$

For this, it can be written as (6)

$$\lim_{n \to \infty} \left(n + 1 \geq nk\right) \tag{6}$$

where $k = \sqrt[nH_n]{nH_n}$. For $n \geq nk - n$ it becomes $\frac{1}{n} \geq k-1$; so what ever the direction of the inequality, even if both sides were equal to each other, k would not become a number smaller than 1 since n is always positive. It is always $k > 1$. Here assume, that is (7)

$$n = nk - 1 + b \tag{7}$$

since it is $n \geq nk-1$ over (6), where b is a number being $b \in \mathbb{R}^+$ and thus being $b > 0$; thus it becomes (8) over (7).

$$n = \frac{b-1}{1-k} \tag{8}$$

Since is $k > 1$, then b must always be smaller number than 1 to be positive of the division; thus it becomes $1 > b > 0$; so k cannot take random values since n is positive integer. If it is $k > 1$, for the greatest value of k, it becomes $\lim_{b \to 0} k = 2$. For this value, equality of (7) becomes $n = 2n - 1$ and thus becomes $n = 1$. It means, actually k decreases as long as n increased; thus it means it is always (9),

$$1 = \lim_{m \to \infty} \sqrt[m]{m} \tag{9}$$

where $m \in \mathbb{Z}^+$; thus also means it is (10),

$$1 = \lim_{n \to \infty} \sqrt[nH_n]{nH_n} \tag{10}$$

since is $H_n \geq 1$ and thus is $nH_n \geq 1$.

2 Conclusion

By the defined elements, over (6), it becomes (11).

$$\lim_{n \to \infty} n + 1 \geq n \tag{11}$$

This is also equivalent of (3) and thus of (12).

$$H_n + \ln(H_n)e^{H_n} \geq \sigma(n) \tag{12}$$

Acknowledgment

I have been working about some unknown problems for a time [2] that Riemann Hypothesis is included as well, and a short time ago I supposed that I found a solution out to the Riemann Hypothesis; but I noticed that there is a stupid mistake; after that I published a brief approach; for a long time I did not work about it; but today I remembered it and just wanted to work because of boredom, and finally I could bring a simple solution out indirectly in a few hours again after midnight even if it is not so sexy and enlightening about functions. Even so, solution is solution always.

Good bye!

References

1. Jeffrey C. Lagarias. 2002 *An Elementary Problem Equivalent to the Riemann Hypothesis*, The American Mathematical Monthly. Vol. 109, No. 6, pp. 534-543
2. Kavak M. 2018, *Complement Inferences on Theoretical Physics and Mathematics*, OSF Preprints, Available online: https://osf.io/tw52w/

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