The Phenomena Violating Gauss' Law

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Abstract: By analyzing the charge and electric field distribution for some charge systems that containing neutral conductor cavity, we found phenomena that violate Gauss' Law. In some cases, the net electric flux through a closed surface is equal to zero, although there is net electric charge within it. In the other case, there is net electric flux through a closed surface, but the net electric charge within that surface is not zero.
Key words: Violate Gauss' law; Gauss' law; Electric flux.

1. Introduction

In classical electromagnetic physics, Gauss' law is a law describing the distribution of electric charge and the resulting electric field. The law states that the net electric flux through any hypothetical closed surface is equal to 4π times the net electric charge within that closed surface. It is expressed as, [1,2,3]

$$\int \boldsymbol{E} \cdot d\boldsymbol{a} = 4\pi \mathrm{Q}$$

Where Q is the total charge enclosed within the hypothetical closed surface.

The law was proposed and formulated by Carl Friedrich Gauss in 1813.[4] As a basic law of classical electromagnetism, it is one of Maxwell's four equations which form the basis of classical electrodynamics. Gauss' law can be used to derive Coulomb's law, and vice versa. [4] The law is widely used for electric charge and field distribution analysis and has very general applicability. The law is appropriate for not only static electric charge system, but also for moving electric charge system.[1,2]

Are there any cases that are against Gauss' law? Up to now, there is no any relative report on that. In this article, the phenomena violating Gauss' law are found in some charge systems containing neutral conductor cavity. Here, it is so obvious that Gauss' law is invalid under these cases. The fact that we overlooked these phenomena is simply because we believe Gauss' law too much on its universality.

2. Systems description and analysis

There is a neutral conductor cavity in which a electric charge Q is enclosed. The cross section view of this system is shown in figure1. This is a typical and familiar charge system. We know that the electric field is zero everywhere in the region outside the conductor and in the area within the conductor, as seen in figure1. For a closed surface S enclosing the whole conductor (see figure2), the net electric flux through this surface is equal to zero because there is no electric field in this area. However, there is net electric charge Q within this closed surface. That is:

$$\int \boldsymbol{E} \cdot d\boldsymbol{a} = 0 \neq 4\pi Q$$

This is a clear violation of Gauss' law.

For the same system, let's see another closed surface that consists of two parts, as shown in figure3. One part is within the body area of the conductor and the other is outside the conductor. Because the electric field is zero everywhere in the two regions, the net electric flux through this surface must be zero, but there is really net electric charge q ($q < Q$) within the closed surface as seen in figure3. That is:

$$\int \boldsymbol{E} \cdot d\boldsymbol{a} = 0 \neq 4\pi q$$

Gauss' law is violated in this case, too.

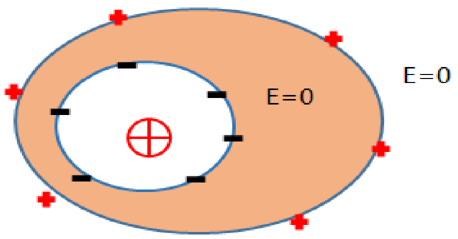

Figure1. A nuetral conductor cavity containing a positive electric charge. Under electrostatic equilibrium state, the electric field is zero inside the body area of the conductor and in the area outside the conductor.

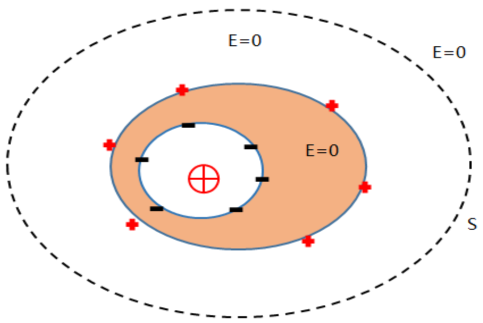

Figure2. A closed surface S enclosing the whole conductor. The net electric flux through the closed surface is zero, due to the electric field is zero everywhere outside the conductor.

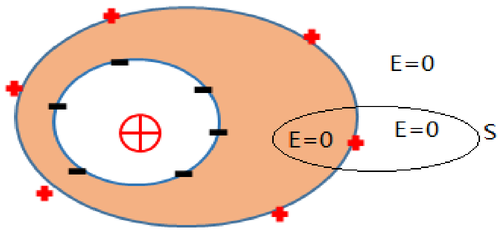

Figure3. A closed surface S consists of two parts. One part is within the body of the conductor and the other outside the conductor.

Now, let's look the other two systems. The system in figure4 is a neutral conductor cavity enclosing a positive electric charge Q and there is another negative electric charge -Q in the vicinity of the conductor. S is a closed surface enclosing the whole sytem. Because the charge within the conductor cavity has not effect on the outside electric field created by the –Q, so there is net electric flux through the closed surface S. However, the net electric charge within the closed surface is zero (Q – Q = 0). This fact is against Gauss' law.

In figure5, there is spherical electric charge layer which encloses a conductor cavity. The electric field is zero everywhere in the cavity and inside the body of the conductor. A closed surface S contains two parts, as shown in figure5. One part is in the cavity and the other inside the body of the conductor. Because the electric field is zero in the cavity and the body of the conductor, the net electric flux through the closed surface is zero. But, it is apparent that the net electric charge within the closed surface is not zero. So, Gauss' law is violated.

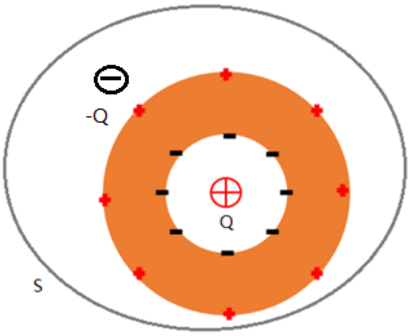

Figure4. A neutral conductor cavity enclosing a positive electric charge Q and there is another negative electric charge -Q in the vicinity of the conductor. S is a closed surface enclosing the whole sytem.

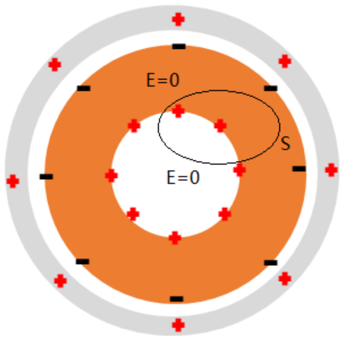

Figure5. A spherical electric charge layer encloses a conductor cavity. A closed surface S contains two parts. One part is in the cavity and the other inside the body of the conductor.

3. Conclusions

As our analysis above for three charge systems containing conductor cavities, the phenomena of violation of Gauss' law are found in four cases. Under these situations, the net electric flux for a hypothetical closed surface is not equal to 4π times the net electric charge within that closed surface.

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