# Isomorphism of Interval Valued Neutrosophic Hypergraphs

Muhammad Aslam Malik[1], Ali Hassan[2], Said Broumi[3], Assia Bakali[4], Mohamed Talea[5], Florentin Smarandache[6]

[1] Department of Mathematics, University of Punjab, Lahore, Pakistan
aslam@math.pu.edu.pk
[2] Department of Mathematics, University of Punjab, Lahore, Pakistan
alihassan.iiui.math@gmail.com
[3, 5] University Hassan II, Sidi Othman, Casablanca, Morocco
broumisaid78@gmail.com
[4] Ecole Royale Navale, Casablanca, Morocco
assiabakali@yahoo.fr
[6] University of New Mexico, Gallup, NM, USA
smarand@unm.edu

## Abstract

In this paper, we introduce the homomorphism, weak isomorphism, co-weak isomorphism and isomorphism of interval valued neutrosophic hypergraphs. The properties of order, size and degree of vertices, along with isomorphism, are included. The isomorphism of interval valued neutrosophic hypergraphs equivalence relation and weak isomorphism of interval valued neutrosophic hypergraphs partial order relation are also verified.

## Keywords

homomorphism, weak-isomorphism, co-weak-isomorphism, isomorphism of interval valued neutrosophic hypergraphs.

## 1 Introduction

The neutrosophic sets are characterized by a truth-membership function ($t$), an indeterminacy-membership function ($i$) and a falsity membership function ($f$) independently, which are within the real standard or non-standard unit interval $]^-0 , 1^+[$.

Smarandache [8] proposed the notion of neutrosophic set (NS) as a generalization of the fuzzy set [14], intuitionistic fuzzy set [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy set [13] theories.

For convenient use of NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1].

More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyze architecture structures and to represent system partitions. Mordesen and Nasir gave the definitions for fuzzy hyper graphs. Parvathy R. and M. G. Karunambigai's paper introduced the concepts of intuitionistic fuzzy hypergraphs and analyze its components. Radhamani and Radhika introduced the concept of Isomorphism on Fuzzy Hypergraphs.

In this paper, we extend the concept to isomorphism of interval valued neutrosophic hypergraphs, and some of their important properties are introduced.

## 2 Preliminaries

### Definition 2.1

A hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ is a family of subsets of $X$.

(3) $E_j$ are not-empty for $j= 1, 2, 3, \dots, m$ and $\cup_j(E_j) = X$.

The set $X$ is called set of vertices and $E$ is the set of edges (or hyper-edges).

### Definition 2.2

A fuzzy hypergraph $H = (X, E)$ is a pair, where $X$ is a finite set and $E$ is a finite family of non-trivial fuzzy subsets of $X$, such that $X = \cup_j Supp(E_j)$, $j = 1, 2, 3, \dots, m$.

### Remark 2.3

$E = \{E_1, E_2, E_3, \dots, E_m\}$ is the collection of edge set of $H$.

Definition 2.4

A fuzzy hypergraph with underlying set $X$ is of the form $H = (X, E, R)$, where $E = \{E_1, E_2, E_3, \ldots, E_m\}$ is the collection of fuzzy subsets of X, i.e. $E_j : X \to [0, 1]$, $j = 1, 2, 3, \ldots, m$ and $R : E \to [0, 1]$ is a fuzzy relation on fuzzy subsets $E_j$, such that:

$$R(x_1, x_2, \ldots, x_r) \leq \min(E_j(x_1), \ldots, E_j(x_r)), \quad (1)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of $X$.

Definition 2.5

Let $X$ be a space of points (objects) with generic elements in $X$, which is denoted by $x$. A single valued neutrosophic set $A$ (SVNS $A$) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x $\in$X; $T_A(x)$, $I_A(x)$,$F_A(x)$ $\in [0, 1]$.

Definition 2.6

A single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \ldots, x_n\}$ is a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ is a family of SVNSs of $X$.

(3)$E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \ldots, m$ and $\bigcup_j Supp(E_j) = X$.

The set $X$ is called set of vertices and $E$ is the set of SVN-edges (or SVN-hyper-edges).

Proposition 2.7

The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

Note that a given a SVNHG$H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \ldots, E_m\}$ is the collection of non-empty family of SVN subsets of $X$, and $R$ is SVN relation on SVN subsets $E_j$, such that:

$$R_T(x_1, x_2, \ldots, x_r) \leq \min([T_{E_j}(x_1)], \ldots, [T_{E_j}(x_r)]), \quad (2)$$

$$R_I(x_1, x_2, \ldots, x_r) \geq \max([I_{E_j}(x_1)], \ldots, [I_{E_j}(x_r)]), \quad (3)$$

$$R_F(x_1, x_2, \ldots, x_r) \geq \max([F_{E_j}(x_1)], \ldots, [F_{E_j}(x_r)]), \quad (4)$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of $X$.

Definition 2.8

Let $X$ be a space of points (objects) with generic elements in $X$ denoted by $x$. An interval valued neutrosophic set $A$ (IVNS $A$) is characterized by lower truth membership function $TL_A(x)$, lower indeterminacy membership function $IL_A(x)$, lower falsity membership function $FL_A(x)$, upper truth membership function $TU_A(x)$, upper indeterminacy membership function $IU_A(x)$, upper falsity membership function $FU_A(x)$, for each point x ∈ X; $[TL_A(x), TU_A]$, $[IL_A(x), IU_A(x)]$, $[FL_A(x), FU_A(x)]$ subsets of [0, 1].

Definition 2.9

An interval valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where:

(1) $X = \{x_1, x_2, \dots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ be a family of IVNSs of $X$.

(3) $E_j \neq O = ([0, 0], [0, 0], [0, 0])$ for $j = 1, 2, 3, \dots, m$ and $\bigcup_j Supp(E_j) = X$.

The set $X$ is called set of vertices and $E$ is the set of IVN-edges (or IVN-hyper-edges).

Note that a given IVNHG$H = (X, E, R)$ with underlying set X, where $E = \{E_1, E_2, \dots, E_m\}$ is the collection of non-empty family of IVN subsets of $X$, and $R$ is IVN relation on IVN subsets $E_j$ such that:

$$R_{TL}(x_1, x_2, \dots, x_r) \leq \min([TL_{E_j}(x_1)], \dots, [TL_{E_j}(x_r)]), \quad (5)$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq \max([IL_{E_j}(x_1)], \dots, [IL_{E_j}(x_r)]), \quad (6)$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq \max([FL_{E_j}(x_1)], \dots, [FL_{E_j}(x_r)]), \quad (7)$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq \min([TU_{E_j}(x_1)], \dots, [TU_{E_j}(x_r)]), \quad (8)$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq \max([IU_{E_j}(x_1)], \dots, [IU_{E_j}(x_r)]), \quad (9)$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq \max([FU_{E_j}(x_1)], \dots, [FU_{E_j}(x_r)]), \quad (10)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Proposition 2.10

The interval valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs, intuitionistic fuzzy hypergraphs, interval valued fuzzy hypergraphs and interval valued intuitionistic fuzzy hypergraphs.

Example 2.11

Consider the IVNHG *H = (X, E, R)* with underlying set *X = {a, b, c},* where *E = {A, B}* and *R,* which are defined in the *Tables* given below:

| H | A | B |
|---|---|---|
| a | ([0.5,0.7], [0.2, 0.9], [0.5,0.8]) | ([0.3,0.5],[0.5,0.6], [0.0,0.1]) |
| b | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.1,0.4],[0.3,0.9],[0.9,1.0]) |
| c | ([0.2,0.3], [0.1,0.5], [0.4,0.7]) | ([0.5,0.9],[0.2,0.3],[0.5,0.8]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.1, 0.2] | [0.6, 1.0] | [0.5, 0.9] |
| B | [0.1, 0.3] | [0.9, 0.9] | [0.9, 1.0] |

By routine calculations, *H = (X, E, R)* is IVNHG.

## 2 Isomorphism of SVNHGs

Definition 3.1

A homomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a mapping *f: X → Y* which satisfies the conditions:

$$\min[TL_{E_j}(x)] \leq \min[TL_{F_j}(f(x))], \tag{11}$$

$$\max[IL_{E_j}(x)] \geq \max[IL_{F_j}(f(x))], \tag{12}$$

$$\max[FL_{E_j}(x)] \geq \max[FL_{F_j}(f(x))], \tag{13}$$

$$\min[TU_{E_j}(x)] \leq \min[TU_{F_j}(f(x))], \tag{14}$$

$$\max[IU_{E_j}(x)] \geq \max[IU_{F_j}(f(x))], \tag{15}$$

$$\max[FU_{E_j}(x)] \geq \max[FU_{F_j}(f(x))], \text{ for all } x \in X. \tag{16}$$

$$R_{TL}(x_1, x_2, \ldots, x_r) \leq S_{TL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{17}$$

$$R_{IL}(x_1, x_2, \ldots, x_r) \geq S_{IL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{18}$$

$$R_{FL}(x_1, x_2, \ldots, x_r) \geq S_{FL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{19}$$

$$R_{TU}(x_1, x_2, \ldots, x_r) \leq S_{TU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{20}$$

$$R_{IU}(x_1, x_2, \ldots, x_r) \geq S_{IU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{21}$$

$$R_{FU}(x_1, x_2, \ldots, x_r) \geq S_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{22}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of *X.*

Example 3.2

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below:

| H | A | B |
|---|---|---|
| a | ([0.2,0.3], [0.3,0.4], [0.9,1.0]) | ([0.5,0.6], [0.2,0.3], [0.7,0.8]) |
| b | ([0.5,0.6], [0.5,0.6], [0.5,0.6]) | ([0.1,0.2], [0.6,0.7], [0.4,0.5]) |
| c | ([0.8,0.9], [0.8,0.9], [0.3, 0.4]) | ([0.5,0.6], [0.9,1.0], [0.8,0.9]) |
| K | C | D |
| x | ([0.3,0.4], [0.2,0.3], [0.2,0.3]) | ([0.2,0.3], [0.1,0.2], [0.3,0.4]) |
| y | ([0.2,0.4], [0.4,0.5], [0.2,0.3]) | ([0.3,0.4], [0.2,0.3], [0.1,0.2]) |
| z | ([0.5,0.6], [0.8,0.9], [0.2, 0.3]) | ([0.9,0.1], [0.7,0.8], [0.1,0.2]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| B | [0.1,0.2] | [0.9,1.0] | [0.8,0.9] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.2,0.3] | [0.8,0.9] | [0.3,0.4] |
| D | [0.1,0.2] | [0.7,0.8] | [0.3,0.4] |

and *f: X → Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a homomorphism between *H* and *K*.

Definition 3.3

A weak isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping $f : X \to Y$ which satisfies the condition $f$ is homomorphism, such that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{23}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{24}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{25}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{26}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{27}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{28}$$

for all $x \in X$.

Note

The weak isomorphism between two IVNHGs preserves the weights of vertices.

Example 3.4

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y={x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below:

| H | A | B |
|---|---|---|
| a | ([0.2,0.3], [0.3,0.4], [0.9,1.0]) | ([0.5,0.6], [0.2,0.3], [0.7,0.8]) |
| b | ([0.5,0.6], [0.5,0.6], [0.5,0.6]) | ([0.1,0.2], [0.6,0.7], [0.4,0.5]) |
| c | ([0.8,0.9], [0.8,0.9], [0.3, 0.4]) | ([0.5,0.6], [0.9,1.0], [0.8,0.9]) |
| K | C | D |
| x | ([0.2,0.3], [0.3,0.4], [0.2,0.3]) | ([0.2,0.3], [0.1,0.2], [0.8,0.9]) |
| y | ([0.2,0.3], [0.4,0.5], [0.2,0.3]) | ([0.1,0.2], [0.6,0.7], [0.5,0.6]) |
| z | ([0.5,0.6], [0.8,0.9], [0.9, 1.0]) | ([0.9,1.0], [0.9,1.0], [0.1,0.2]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| B | [0.1,0.2] | [0.9,1.0] | [0.9,1.0] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| D | [0.1,0.2] | [0.9,1.0] | [0.8,0.9] |

and *f : X → Y* defined by, *f(a)=x*, *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H → K* is a weak isomorphism between *H* and *K*.

Definition 3.5

A co-weak isomorphism *f: H → K* between two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* is a bijective mapping *f : X → Y* which satisfies the condition *f* is homomorphism, such that:

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{29}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{30}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{31}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{32}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{33}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{34}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Note

The co-weak isomorphism between two IVNHGs preserves the weights of edges.

Example 3.6

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c\}$ and $Y = \{x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, $R$ and $S$, which are defined in the *Tables* given below:

| H | A | B |
|---|---|---|
| a | ([0.2,0.3], [0.3,0.4], [0.9,1.0]) | ([0.5,0.6], [0.2,0.3], [0.7,0.8]) |
| b | ([0.5,0.6], [0.5,0.6], [0.5,0.6]) | ([0.1,0.2], [0.6,0.7], [0.4,0.5]) |
| c | ([0.8,0.9], [0.8,0.9], [0.3, 0.4]) | ([0.5,0.6], [0.9,1.0], [0.8,0.9]) |
| K | C | D |
| x | ([0.3,0.4], [0.2,0.3], [0.2,0.3]) | ([0.2,0.3], [0.1,0.2], [0.3,0.4]) |
| y | ([0.2,0.3], [0.4,0.5], [0.2,0.3]) | ([0.3,0.4], [0.2,0.3], [0.1,0.2]) |
| z | ([0.5,0.6], [0.8,0.9], [0.2, 0.3]) | ([0.9,1.0], [0.7,0.8], [0.1,0.2]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| B | [0.1,0.2] | [0.9,1.0] | [0.8,0.9] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| D | [0.1,0.2] | [0.9,1.0] | [0.8,0.9] |

and $f: X \to Y$ defined by, $f(a)=x$, $f(b)=y$ and $f(c)=z$. Then, by routine calculations, $f: H \to K$ is a co-weak isomorphism between $H$ and $K$.

Definition 3.7

An isomorphism $f: H \to K$ between two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ is a bijective mapping $f: X \to Y$ which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{35}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{36}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{37}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{38}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{39}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{40}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{41}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{42}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{43}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{44}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{45}$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{46}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Note

The isomorphism between two IVNHGs preserves the both weights of vertices and weights of edges.

Example 3.8

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c}* and *Y = {x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below**,**

| H | A | B |
|---|---|---|
| a | ([0.2,0.3], [0.3,0.4], [0.7,0.8]) | ([0.5,0.6], [0.2,0.3], [0.7,0.8]) |
| b | ([0.5,0.6], [0.5,0.6], [0.5,0.6]) | ([0.1,0.2], [0.6,0.7], [0.4,0.5]) |
| c | ([0.8,0.9], [0.8,0.9], [0.3, 0.4]) | ([0.5,0.6], [0.9,1.0], [0.8,0.9]) |
| K | C | D |
| x | ([0.2,0.3], [0.3,0.4], [0.2,0.3]) | ([0.2,0.3], [0.1,0.2], [0.8,0.9]) |
| y | ([0.2,0.3], [0.4,0.5], [0.2,0.3]) | ([0.1,0.2], [0.6,0.7], [0.5,0.6]) |
| z | ([0.5,0.6], [0.8,0.9], [0.7, 0.8]) | ([0.9,1.0], [0.9,1.0], [0.1,0.2]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| B | [0.0,0.1] | [0.9,1.0] | [0.8,0.9] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.2,0.3] | [0.8,0.9] | [0.9,1.0] |
| D | [0.0,0.1] | [0.9,1.0] | [0.8,0.9] |

and *f: X* → *Y* defined by, *f(a)=x* , *f(b)=y* and *f(c)=z*. Then, by routine calculations, *f: H* → *K* is a isomorphism between *H* and *K*.

Definition 3.9

Let *H = (X, E, R)* be a IVNHG; then, the order of *H,* which is denoted and defined by:

$$O(H)= ([\sum \min TL_{E_j}(x) , \sum \min TU_{E_j}(x)] ,\ [\sum \max IL_{E_j}(x) , \sum \max IU_{E_j}(x)], [\sum \max FL_{E_j}(x) , \sum \max FU_{E_j}(x)] ) \quad (47)$$

and the size of *H,* which is denoted and defined by:

$$S(H) = ([\sum R_{TL}(E_j), \sum R_{TU}(E_j)] , [\sum R_{IL}(E_j), \sum R_{IL}(E_j)], [\sum R_{FL}(E_j), \sum R_{FU}(E_j)]\ ) \quad (48)$$

Theorem 3.10

Let *H = (X, E, R)* and *K = (Y, F, S)* be two IVNHGs such that *H* is isomorphic to *K*; then:

*(1) O(H) = O(K),*

*(2) S(H) = S(K).*

Proof.

Let *f : H* → *K* be an isomorphism between two IVNHGs *H* and *K* with underlying sets *X* and *Y* respectively; then, by definition, we have that:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (49)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (50)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (51)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (52)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (53)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (54)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (55)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (56)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (57)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (58)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (59)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (60)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Consider:

$$O_{TL}(H) = \sum \min TL_{E_j}(x) = \sum \min TL_{F_j}(f(x)) = O_{TL}(K) \quad (61)$$

$$O_{TU}(H) = \sum \min TU_{E_j}(x) = \sum \min TU_{F_j}(f(x)) = O_{TU}(K) \quad (62)$$

Similarly:

$$O_{IL}(H) = O_{IL}(K) and O_{FL}(H) = O_{FL}(K), \quad (63)$$

$$O_{IU}(H) = O_{IU}(K) and O_{FU}(H) = O_{FU}(K). \quad (64)$$

Hence, $O(H) = O(K)$.

Next,

$$S_{TL}(H) = \sum R_{TL}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TL}(K), \quad (65)$$

and similarly:

$$S_{TU}(H) = \sum R_{TU}(x_1, x_2, \dots, x_r)$$

$$= \sum S_{TU}(f(x_1), f(x_2), \dots, f(x_r)) = S_{TU}(K) \quad (66)$$

Similarly,

$$S_{IL}(H) = S_{IL}(K), S_{FL}(H) = S_{FL}(K), \quad (67)$$

$$S_{IU}(H) = S_{IU}(K), S_{FU}(H) = S_{FU}(K), \quad (68)$$

hence $S(H) = S(K)$.

Remark 3.11

The converse of the above theorem needs not to be true in general.

Example 3.12

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b, c, d\}$ and $Y = \{w, x, y, z\}$, where $E = \{A, B\}$, $F = \{C, D\}$, $R$ and $S$, which are defined in the *Tables* given below:

| H | A | B |
|---|---|---|
| a | ([0.2, 0.3],[0.5,0.6],[0.33,0.43]) | ([0.16,0.26],[0.5,0.6],[0.33,0.43]) |
| b | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.2,0.3],[0.5,0.6],[0.33,0.43]) |
| c | ([0.33,0.43],[0.5,0.6],[0.33,0.43]) | ([0.2,0.3], [0.5,0.6],[0.33,0.43]) |
| d | ([0.5,0.6],[0.5,0.6],[0.33,0.43]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |
| K | C | D |
| w | ([0.2,0.3],[0.5,0.6],[0.33,0.43]) | ([0.16,0.26],[0.5,0.6],[0.33,0.43]) |
| x | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.2,0.3],[0.5,0.6],[0.33,0.43]) |
| y | ([0.33,0.43],[0.5,0.6],[0.33,0.43]) | ([0.2,0.3], [0.5,0.6],[0.33,0.43]) |
| z | ([0.5,0.6],[0.5,0.6],[0.33,0.43]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.5,0.6] | [0.33,0.43] |
| B | [0.16,0.26] | [0.5,0.6] | [0.33,0.43] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.16,0.26] | [0.5,0.6] | [0.33,0.43] |
| D | [0.2,0.3] | [0.5,0.6] | [0.33,0.43] |

where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z*.

Here, *O(H) = ([1.06, 1.46], [2.0, 2.4], [1.32, 1.72]) = O(K)* and *S(H) = ([0.36, 0.56], [1.0, 1.2], [0.66, 0.86]) = S(K)*.

By routine calculations, *H* is not isomorphism to *K*.

Corollary 3.13

The weak isomorphism between any two IVNHGs *H* and *K* preserves the orders.

Remark 3.14

The converse of the above corollary need not to be true in general.

Example 3.15

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S,* which are defined in the *Tables* given below, where *f* is defined by *f(a)=w, f(b)=x, f(c)=y, f(d)=z.*
Here *O(H)= ([1.0,1.4], [2.0,2.4], [1.2,1.6]) = O(K).*
By routine calculations, *H* is not weak isomorphism to *K*.

| H | A | B |
|---|---|---|
| a | ([0.2,0.3],[0.5,0.6],[0.3,0.4]) | ([0.14,0.24],[0.5,0.6],[0.3,0.4]) |
| b | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.2,0.3],[0.5,0.6],[0.3,0.4]) |
| c | ([0.33,0.43],[0.5,0.6],[0.3,0.4]) | ([0.16,0.26], [0.5,0.6], [0.3,0.4]) |
| d | ([0.5,0.6], [0.5,0.6], [0.3,0.4]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |
| K | C | D |
| w | ([0.14,0.24],[0.5,0.6],[0.3,0.4]) | ([0.16,0.26],[0.5,0.6],[0.3,0.4]) |
| x | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.16,0.26],[0.5,0.6],[0.3,0.4]) |
| y | ([0.33,0.43],[0.5,0.6],[0.33,0.43]) | ([0.2,0.3], [0.5,0.6],[0.3,0.4]) |
| z | ([0.5,0.6],[0.5,0.6],[0.3,0.4]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |

Corollary 3.16

The co-weak isomorphism between any two IVNHGs *H* and *K* preserves the sizes.

Remark 3.17

The converse of the above corollary need not to be true in general.

Example 3.18

Consider the two IVNHGs *H = (X, E, R)* and *K = (Y, F, S)* with underlying sets *X = {a, b, c, d}* and *Y = {w, x, y, z}*, where *E = {A, B}*, *F = {C, D}*, *R* and *S*, which are defined in the *Tables* given below**,** where *f* is defined by, *f(a)=w, f(b)=x, f(c)=y, f(d)=z.* Here *S(H)= ([0.34,0.54], [1.0,1.2], [0.6,0.8]) = S(K)*, but, by routine calculations, *H* is not co-weak isomorphism to *K*.

| H | A | B |
|---|---|---|
| a | ([0.2,0.3],[0.5,0.6],[0.3,0.4]) | ([0.14,0.24],[0.5,0.6],[0.3,0.4]) |
| b | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) | ([0.16,0.26],[0.5,0.6],[0.3,0.4]) |
| c | ([0.3,0.4],[0.5,0.6],[0.3,0.4]) | ([0.2,0.3], [0.5,0.6], [0.3,0.4]) |
| d | ([0.5,0.6], [0.5,0.6], [0.3,0.4]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |
| K | C | D |
| w | ([0.0,0.0],[0.0,0.0],[0.0,0.0]) | ([0.2,0.3],[0.5,0.6],[0.3,0.4]) |
| x | ([0.14,0.24], [0.5,0.6], [0.3,0.4]) | ([0.25,0.35],[0.5,0.6],[0.3,0.4]) |
| y | ([0.5,0.6], [0.5,0.6], [0.3,0.4]) | ([0.2,0.3], [0.5,0.6],[0.3,0.4]) |
| z | ([0.3,0.4], [0.5,0.6], [0.3,0.4]) | ([0.0,0.0], [0.0,0.0], [0.0,0.0]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.2,0.3] | [0.5,0.6] | [0.3,0.4] |
| B | [0.14,0.24] | [0.5,0.6] | [0.3,0.4] |
| S | $S_T$ | $S_I$ | $S_F$ |
| C | [0.14,0.24] | [0.5,0.6] | [0.3,0.4] |
| D | [0.2,0.3] | [0.5,0.6] | [0.3,0.4] |

Definition 3.19

Let *H = (X, E, R)* be a IVNHG; then, the degree of vertex $x_i$ is denoted and defined by:

$$\deg(x_i) = ([deg_{TL}(x_i), deg_{TU}(x_i)], [deg_{IL}(x_i), deg_{IU}(x_i)],$$

$$[deg_{FL}(x_i), deg_{FU}(x_i)]), \quad (69)$$

where

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r)\,, \quad (70)$$

$$deg_{IL}(x_i) = \sum R_{IL}(x_1, x_2, \dots, x_r), \quad (71)$$

$$deg_{FL}(x_i) = \sum R_{FL}(x_1, x_2, \dots, x_r), \quad (72)$$

$$deg_{TU}(x_i) = \sum R_{TU}(x_1, x_2, \dots, x_r)\,, \quad (73)$$

$$deg_{IU}(x_i) = \sum R_{IU}(x_1, x_2, \dots, x_r), \quad (74)$$

$$deg_{FU}(x_i) = \sum R_{FU}(x_1, x_2, \dots, x_r), \quad (75)$$

for $x_i \neq x_r$.

Theorem 3.20

If $H$ and $K$ are two isomorphic IVNHGs, then the degree of their vertices are preserved.

Proof.

Let *f: H → K* be an isomorphism between two IVNHGs $H$ and $K$ with underlying sets $X$ and $Y$, respectively. Then, by definition, we have:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (75)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (77)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (78)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (79)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (80)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (81)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_1), \dots, f(x_r)), \quad (82)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (83)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (84)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (85)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (86)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (87)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Consider,

$$deg_{TL}(x_i) = \sum R_{TL}(x_1, x_2, \dots, x_r) = \sum S_{TL}(f(x_1), f(x_2), \dots, f(x_r)) = deg_{TL}(f(x_i)) \quad (88)$$

and similarly:

$$deg_{TU}(x_i) = deg_{TU}(f(x_i)), \quad (89)$$

$$deg_{IL}(x_i) = deg_{IL}(f(x_i)), deg_{FL}(x_i) = deg_{FL}(f(x_i)), \quad (90)$$

$$deg_{IU}(x_i) = deg_{IU}(f(x_i)), deg_{FU}(x_i) = deg_{FU}(f(x_i)). \quad (91)$$

Hence,

$$deg(x_i) = deg(f(x_i)). \quad (92)$$

Remark 3.21

The converse of the above theorem may not be true in general.

Example 3.22

Consider the two IVNHGs $H = (X, E, R)$ and $K = (Y, F, S)$ with underlying sets $X = \{a, b\}$ and $Y = \{x, y\}$, where $E = \{A, B\}$, $F = \{C, D\}$, $R$ and $S$, which are defined in the *Tables* given below, where $f$ is defined by $f(a)=x$, $f(b)=y$, where $deg(a) = ([0.8,1.0], [1.0,1.2], [0.6,0.8]) = deg(x)$ and $deg(b) = ([0.45,0.65], [1.0,1.2], [0.6,0.8]) = deg(y)$. But $H$ is not isomorphic to $K$, i.e. $H$ is neither weak isomorphic nor co-weak isomorphic $K$.

| H | A | B |
|---|---|---|
| a | ([0.5,0.6], [0.5,0.6], [0.3,0.4]) | ([0.3,0.4],[0.5,0.6],[0.3,0.4]) |
| b | ([0.25,0.35], [0.5,0.6], [0.3,0.4]) | ([0.2,0.3], [0.5,0.6], [0.3, 0.4]) |

| K | C | D |
|---|---|---|
| x | ([0.3,0.4], [0.5,0.6], [0.3,0.4]) | ([0.5,0.6],[0.5,0.6],[0.3,0.4]) |
| y | ([0.2,0.3], [0.5,0.6], [0.3,0.4]) | ([0.25,0.34], [0.5,0.6], [0.3,0.4]) |

| R | $R_T$ | $R_I$ | $R_F$ |
|---|---|---|---|
| A | [0.25,0.35] | [0.5,0.6] | [0.3,0.4] |
| B | [0.2,0.3] | [0.5,0.6] | [0.3,0.4] |

| S | $S_T$ | $S_I$ | $S_F$ |
|---|---|---|---|
| C | [0.2,0.3] | [0.5,0.6] | [0.3,0.4] |
| D | [0.25,0.35] | [0.5,0.6] | [0.3,0.4] |

Theorem 3.23

The isomorphism between IVNHGs is an equivalence relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be IVNHGs with underlying sets X, Y and *Z*, respectively:

Reflexive.

Consider the map (identity map) $f: X \to X$, defined as follows: $f(x) = x$ for all $x \in X$, since the identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \tag{93}$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \tag{94}$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \tag{95}$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \tag{96}$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \tag{97}$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \tag{98}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{99}$$

$$R_{IL}(x_1, x_2, \dots, x_r) = R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{100}$$

$$R_{FL}(x_1, x_2, \dots, x_r) = R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{101}$$

$$R_{TU}(x_1, x_2, \dots, x_r) = R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{102}$$

$$R_{IU}(x_1, x_2, \dots, x_r) = R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{103}$$

$$R_{FU}(x_1, x_2, \ldots, x_r) \;=\; R_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{104}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of $X$.

Hence $f$ is an isomorphism of IVNHG $H$ to itself.

Symmetric.

Let $f: X \rightarrow Y$ be an isomorphism of $H$ and $K$, then $f$ is bijective mapping defined as: $f(x) = y$ for all $x \in X$. Then, by definition:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{105}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{106}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{107}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{108}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{109}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{110}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \ldots, x_r) = S_{TL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{111}$$

$$R_{IL}(x_1, x_2, \ldots, x_r) = S_{IL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{112}$$

$$R_{FL}(x_1, x_2, \ldots, x_r) = S_{FL}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{113}$$

$$R_{TU}(x_1, x_2, \ldots, x_r) = S_{TU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{114}$$

$$R_{IU}(x_1, x_2, \ldots, x_r) = S_{IU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{115}$$

$$R_{FU}(x_1, x_2, \ldots, x_r) = S_{FU}(f(x_1), f(x_2), \ldots, f(x_r)), \tag{116}$$

for all $\{x_1, x_2, \ldots, x_r\}$ subsets of $X$. Since $f$ is bijective, then we have $f^{-1}(y) = x$ for all $y \in Y$. Thus, we get:

$$\min[TL_{E_j}\left(f^{-1}(y)\right)] = \min[TL_{F_j}(y)], \tag{117}$$

$$\max[IL_{E_j}\left(f^{-1}(y)\right)] = \max[IL_{F_j}(y)], \tag{118}$$

$$\max[FL_{E_j}\left(f^{-1}(y)\right)] = \max[FL_{F_j}(y)], \tag{119}$$

$$\min[TU_{E_j}\left(f^{-1}(y)\right)] = \min[TU_{F_j}(y)], \tag{120}$$

$$\max[IU_{E_j}\left(f^{-1}(y)\right)] = \max[IU_{F_j}(y)], \tag{121}$$

$$\max[FU_{E_j}\left(f^{-1}(y)\right)] = \max[FU_{F_j}(y)], \tag{122}$$

for all $x \in X$.

$$R_{TL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TL}(y_1, y_2, \dots, y_r), \quad (123)$$

$$R_{IL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IL}(y_1, y_2, \dots, y_r), \quad (124)$$

$$R_{FL}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FL}(y_1, y_2, \dots, y_r), \quad (125)$$

$$R_{TU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{TU}(y_1, y_2, \dots, y_r), \quad (126)$$

$$R_{IU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{IU}(y_1, y_2, \dots, y_r), \quad (127)$$

$$R_{FU}\left(f^{-1}(y_1), f^{-1}(y_2), \dots, f^{-1}(y_r)\right) = S_{FU}(y_1, y_2, \dots, y_r), \quad (128)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of $Y$.

Hence we have a bijective map $f^{-1} : Y \to X$, which is an isomorphism from $K$ to $H$.

Transitive.

Let $f : X \to Y$ and $g : Y \to Z$ be two isomorphism of IVNHGs of $H$ onto $K$ and $K$ onto $M$ respectively. Then $gof$ is bijective mapping from $X$ to $Z$, where $gof$ is defined as $(gof)(x) = g\big(f(x)\big)$ for all $x \in X$.

Since $f$ is isomorphism, then, by definition, $f(x) = y$ for all $x \in X$, which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}\big(f(x)\big)], \quad (129)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}\big(f(x)\big)], \quad (130)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}\big(f(x)\big)], \quad (131)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}\big(f(x)\big)], \quad (132)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}\big(f(x)\big)], \quad (133)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}\big(f(x)\big)], \quad (134)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) = S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (135)$$

$$R_{IL}(x_1, x_2, \dots, x_r) = S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (136)$$

$$R_{FL}(x_1, x_2, \dots, x_r) = S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (137)$$

$$R_{TU}(x_1, x_2, \dots, x_r) = S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (138)$$

$$R_{IU}(x_1, x_2, \dots, x_r) = S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (139)$$

$$R_{FU}(x_1, x_2, \dots, x_r) = S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (140)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$. Since $g : Y \to Z$ is isomorphism, then by definition $g(y) = z$ for all $y \in Y$ satisfy the conditions:

$$\min[TL_{F_j}(y)] = \min\left[TL_{G_j}(g(y))\right], \quad (141)$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}(g(y))], \quad (142)$$

$$\max[FL_{F_j}(y)] = \max[FL_{G_j}(g(y))], \quad (143)$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}(g(y))], \quad (144)$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}(g(y))], \quad (145)$$

$$\max[FU_{F_j}(y)] = \max[FU_{G_j}(g(y))], \quad (146)$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) = W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (147)$$

$$S_{IL}(y_1, y_2, \dots, y_r) = W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (148)$$

$$S_{FL}(y_1, y_2, \dots, y_r) = W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (149)$$

$$S_{TU}(y_1, y_2, \dots, y_r) = W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (150)$$

$$S_{IU}(y_1, y_2, \dots, y_r) = W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (151)$$

$$S_{FU}(y_1, y_2, \dots, y_r) = W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (152)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of $Y$. Thus, from the above equations, we conclude that:

$$\min[TL_{E_j}(x)] = \min\left[TL_{G_j}\left(g(f(x))\right)\right], \quad (153)$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \quad (154)$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \quad (155)$$

$$\min[TU_{E_j}(x)] = \min\left[TU_{G_j}\left(g(f(x))\right)\right], \quad (156)$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \quad (157)$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \quad (158)$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) = W_{TL}(g(f(x_1)), \dots, g(f(x_r))), \quad (159)$$

$$R_{IL}(x_1, \dots, x_r) = W_{IL}(g(f(x_1)), \dots, g(f(x_r))), \quad (160)$$

$$R_{FL}(x_1, \dots, x_r) = W_{FL}(g(f(x_1)), \dots, g(f(x_r))), \quad (161)$$

$$R_{TU}(x_1, \dots, x_r) = W_{TU}(g(f(x_1)), \dots, g(f(x_r))), \quad (162)$$

$$R_{IU}(x_1, \dots, x_r) = W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \tag{163}$$

$$R_{FU}(x_1, \dots, x_r) = W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \tag{164}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Therefore, $gof$ is an isomorphism between $H$ and $M$. Hence, the isomorphism between IVNHGs is an equivalence relation.

Theorem 3.24

The weak isomorphism between IVNHGs satisfies the partial order relation.

Proof.

Let *H = (X, E, R), K = (Y, F, S)* and *M = (Z, G, W)* be IVNHGs with underlying sets X, Y and *Z* respectively,

Reflexive.

Consider the map (identity map) *f: X → X,* defined as follows: *f(x)=x* for all *x* ∈ *X,* since identity map is always bijective and satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{E_j}(f(x))], \tag{165}$$

$$\max[IL_{E_j}(x)] = \max[IL_{E_j}(f(x))], \tag{166}$$

$$\max[FL_{E_j}(x)] = \max[FL_{E_j}(f(x))], \tag{167}$$

$$\min[TU_{E_j}(x)] = \min[TU_{E_j}(f(x))], \tag{168}$$

$$\max[IU_{E_j}(x)] = \max[IU_{E_j}(f(x))], \tag{169}$$

$$\max[FU_{E_j}(x)] = \max[FU_{E_j}(f(x))], \tag{170}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \le R_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{171}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \ge R_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{172}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \ge R_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{173}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \le R_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{174}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \ge R_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{175}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \ge R_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{176}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of *X.*

Hence *f* is a weak isomorphism of IVNHG *H* to itself.

Anti-symmetric.

Let $f$ be a weak isomorphism between $H$ onto $K$, and $g$ be weak isomorphic between $K$ and $H$, i.e. $f\colon X \to Y$ is a bijective map defined by: $f(x) = y$ for all $x \in X$ satisfying the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \tag{177}$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \tag{178}$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \tag{179}$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \tag{180}$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \tag{181}$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \tag{182}$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \leq S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{183}$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{184}$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \tag{185}$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{186}$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{187}$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \tag{188}$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Since g is also bijective map $g(y) = x$ for all $y \in Y$ satisfying the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{E_j}(g(y))], \tag{189}$$

$$\max[IL_{F_j}(y)] = \max[IL_{E_j}(g(y))], \tag{190}$$

$$\max[FL_{F_j}(y)] = \max[FL_{E_j}(g(y))], \tag{191}$$

$$\min[TU_{F_j}(y)] = \min[TU_{E_j}(g(y))], \tag{192}$$

$$\max[IU_{F_j}(y)] = \max[IU_{E_j}(g(y))], \tag{193}$$

$$\max[FU_{F_j}(y)] = \max[FU_{E_j}(g(y))], \tag{194}$$

for all $y \in Y$.

$$R_{TL}(y_1, y_2, \dots, y_r) \leq S_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{195}$$

$$R_{IL}(y_1, y_2, \dots, y_r) \geq S_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{196}$$

$$R_{FL}(y_1, y_2, \dots, y_r) \geq S_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \tag{197}$$

$$R_{TU}(y, y_2, \dots, y_r) \leq S_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (198)$$

$$R_{IU}(y_1, y_2, \dots, y_r) \geq S_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (199)$$

$$R_{FU}(y_1, y_2, \dots, y_r) \geq S_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (200)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of $Y$.

The above inequalities hold for finite sets $X$ and $Y$ only whenever $H$ and $K$ have the same number of edges, and the corresponding edge have same weights, hence $H$ is identical to $K$.

Transitive.

Let $f: X \to Y$ and $g: Y \to Z$ be two weak isomorphism of IVNHGs of $H$ onto $K$ and $K$ onto $M$, respectively. Then $gof$ is bijective mapping from X to Z, where $gof$ is defined as $(gof)(x) = g(f(x))$ for all $x \in X$.

Since $f$ is a weak isomorphism, then by definition $f(x) = y$ for all $x \in X$ which satisfies the conditions:

$$\min[TL_{E_j}(x)] = \min[TL_{F_j}(f(x))], \quad (201)$$

$$\max[IL_{E_j}(x)] = \max[IL_{F_j}(f(x))], \quad (202)$$

$$\max[FL_{E_j}(x)] = \max[FL_{F_j}(f(x))], \quad (203)$$

$$\min[TU_{E_j}(x)] = \min[TU_{F_j}(f(x))], \quad (204)$$

$$\max[IU_{E_j}(x)] = \max[IU_{F_j}(f(x))], \quad (205)$$

$$\max[FU_{E_j}(x)] = \max[FU_{F_j}(f(x))], \quad (206)$$

for all $x \in X$.

$$R_{TL}(x_1, x_2, \dots, x_r) \leq S_{TL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (207)$$

$$R_{IL}(x_1, x_2, \dots, x_r) \geq S_{IL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (208)$$

$$R_{FL}(x_1, x_2, \dots, x_r) \geq S_{FL}(f(x_1), f(x_2), \dots, f(x_r)), \quad (209)$$

$$R_{TU}(x_1, x_2, \dots, x_r) \leq S_{TU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (210)$$

$$R_{IU}(x_1, x_2, \dots, x_r) \geq S_{IU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (211)$$

$$R_{FU}(x_1, x_2, \dots, x_r) \geq S_{FU}(f(x_1), f(x_2), \dots, f(x_r)), \quad (212)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Since $g: Y \to Z$ is a weak isomorphism, then by definition $g(y) = z$ for all $y \in Y$ which satisfies the conditions:

$$\min[TL_{F_j}(y)] = \min[TL_{G_j}(g(y))], \quad (213)$$

$$\max[IL_{F_j}(y)] = \max[IL_{G_j}(g(y))], \quad (214)$$

$$\max[FL_{F_j}(y)] = \max[FL_G(g(y))], \quad (215)$$

$$\min[TU_{F_j}(y)] = \min[TU_{G_j}(g(y))], \quad (216)$$

$$\max[IU_{F_j}(y)] = \max[IU_{G_j}(g(y))], \quad (217)$$

$$\max[FU_{F_j}(y)] = \max[FU_G(g(y))], \quad (218)$$

for all $x \in X$.

$$S_{TL}(y_1, y_2, \dots, y_r) \le W_{TL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (219)$$

$$S_{IL}(y_1, y_2, \dots, y_r) \ge W_{IL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (210)$$

$$S_{FL}(y_1, y_2, \dots, y_r) \ge W_{FL}(g(y_1), g(y_2), \dots, g(y_r)), \quad (211)$$

$$S_{TU}(y_1, y_2, \dots, y_r) \le W_{TU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (212)$$

$$S_{IU}(y_1, y_2, \dots, y_r) \ge W_{IU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (213)$$

$$S_{FU}(y_1, y_2, \dots, y_r) \ge W_{FU}(g(y_1), g(y_2), \dots, g(y_r)), \quad (214)$$

for all $\{y_1, y_2, \dots, y_r\}$ subsets of $Y$.

Thus, from the above equations, we conclude that,

$$\min[TL_{E_j}(x)] = \min[TL_{G_j}(g(f(x)))], \quad (215)$$

$$\max[IL_{E_j}(x)] = \max[IL_{G_j}(g(f(x)))], \quad (216)$$

$$\max[FL_{E_j}(x)] = \max[FL_{G_j}(g(f(x)))], \quad (217)$$

$$\min[TU_{E_j}(x)] = \min[TU_{G_j}(g(f(x)))], \quad (219)$$

$$\max[IU_{E_j}(x)] = \max[IU_{G_j}(g(f(x)))], \quad (220)$$

$$\max[FU_{E_j}(x)] = \max[FU_{G_j}(g(f(x)))], \quad (221)$$

for all $x \in X$.

$$R_{TL}(x_1, \dots, x_r) \le W_{TL}(g(f(x_1)), \dots, g(f(x_r))), \quad (222)$$

$$R_{IL}(x_1, \dots, x_r) \ge W_{IL}(g(f(x_1)), \dots, g(f(x_r))), \quad (223)$$

$$R_{FL}(x_1, \dots, x_r) \ge W_{FL}(g(f(x_1)), \dots, g(f(x_r))), \quad (224)$$

$$R_{TU}(x_1, \dots, x_r) \le W_{TU}(g(f(x_1)), \dots, g(f(x_r))), \quad (225)$$

$$R_{IU}(x_1, \dots, x_r) \ge W_{IU}(g(f(x_1)), \dots, g(f(x_r))), \quad (226)$$

$$R_{FU}(x_1, \dots, x_r) \ge W_{FU}(g(f(x_1)), \dots, g(f(x_r))), \quad (227)$$

for all $\{x_1, x_2, \dots, x_r\}$ subsets of $X$.

Therefore, $gof$ is a weak isomorphism between $H$ and $M$. Hence, the weak isomorphism between IVNHGs is a partial order relation.

## 4 Conclusion

The concepts of interval valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper, the isomorphism between IVNHGs is proved to be an equivalence relation and the weak isomorphism is proved to be a partial order relation. Similarly, it can be proved that the co-weak isomorphism in IVNHGs is a partial order relation.

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