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Introduction to Neutrosophic Soft Groups

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Abstract. The notion of neutrosophic soft group is introduced, together with several related properties. Its structural characteristics are investigated with suitable examples. The Cartesian product on neutrosophic soft groups and on neutrosophic soft subgroup is defined and illustrated by examples.
Related theorems are established.

Keywords: neutrosophic soft set, neutrosophic soft groups, neutrosophic soft subgroup, Cartesian product.

1 Introduction

The concept of Neutrosophic Set (NS), firstly introduced by Smarandache [1], is a generalisation of classical sets, fuzzy set [2], intuitionistic fuzzy set [3] etc. Researchers in economics, sociology, medical science and many other several fields deal daily with the complexities of modelling uncertain data. Classical methods are not always successful because the uncertainty appearing in these domains may be of various types. While probability theory, theory of fuzzy set, intuitionistic fuzzy set and other mathematical tools are well known and often useful approaches to describe uncertainty, each of these theories has its different difficulties, as pointed out by Molodtsov [4].

In 1999, Molodtsov [4] introduced a new concept of soft set theory, which is free from the parameterization inadequacy syndrome of different theories dealing with uncertainty. This makes the theory very convenient and easy to apply in practice. The classical group theory was extended over fuzzy set, intuitionistic fuzzy set and soft set by Rosenfeld [5], Sharma [6], Aktas et.al. [7], and many others. Consequently, several authors applied the theory of fuzzy soft sets, intuitionistic fuzzy soft sets to different algebraic structures, e.g. Maji et. al. [8, 9, 10], Dinda and Samanta [11], Ghosh et. al. [12], Mondal [13], Chetia and Das [14], Basu et. al. [15], Augunoglu and Aygun [16], Yaqoob et. al [17], Varol et. al. [18], Zhang [19].

Later, Maji [20] has introduced a combined concept, the Neutrosophic Soft Set (NSS). Using this concept, several mathematicians have produced their research works in different mathematical structures, e.g. Sahin et. al [21], Broumi [22], Bera and Mahapatra [23], Maji [24], Broumi and Smarandache [25]. Later, the concept has been redefined by Deli and Broumi [26].

This paper presents the notion of neutrosophic soft groups along with an investigation of some related properties and theorems. Section 2 gives some useful definitions. In Section 3, neutrosophic soft group is defined, along with some properties. Section 4 deals with the Cartesian product of neutrosophic soft groups. Finally, the concept of neutrosophic soft subgroup is studied, with suitable examples, in Section 5.

2 Preliminaries

We recall basic definitions related to fuzzy set, soft set, and neutrosophic soft.

2.1 Definition: [27]

A binary operation $* : [0,1]\times[0,1] \rightarrow [0,1]$ is continuous t - norm if it satisfies the following conditions:

(i) $*$ is commutative and associative.

(ii) $*$ is continuous,

(iii) $a * 1 = 1 * a = a,\ \forall a \in [0,1]$,

(iv) $a * b \leq c * d$ if $a \leq c, b \leq d$,

with $a, b, c, d \in [0,1]$.

A few examples of continuous t-norm are $a * b = ab$, $a * b = min\,(a, b)$, $a * b = max\,(a + b - 1, 0)$.

2.2 Definition: [27]

A binary operation $\diamond : [0,1]\times[0,1] \rightarrow [0,1]$ is continuous t - conorm (s - norm) if it satisfies the following conditions:

(i) $\diamond$ is commutative and associative,

(ii) $\diamond$ is continuous,

(iii) $a \diamond 0 = 0 \diamond a = a,\ \forall a \in [0,1]$,

(iv) $a \diamond b \leq c \diamond d$ if $a \leq c,\ b \leq d$,

with $a, b, c, d \in [0,1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab$, $a \diamond b = \max(a, b)$, $a \diamond b = \min(a + b, 1)$. $\forall\, a \in [0,1]$, if $a * a = a$ and $a \diamond a = a$, then $*$ is called an idempotent t-norm and $\diamond$ is called an idempotent s-norm.

2.3 Definition: [1]

A neutrosophic set (NS) on the universe of discourse U is defined as : $A = \{x, T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T, I, F : U \longrightarrow]^-0,1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

From a philosophical point of view, the neutrosophic set (NS) takes its values from real standard or nonstandard subsets of $]^-0,1^+[$. But in real life application, in scientific and engineering problems, it is difficult to use NS with values from real standard or nonstandard subset of $]^-0,1^+[$. Hence, we consider the NS which takes the values from the subset of [0,1].

2.4 Definition: [4]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F: A \rightarrow P(U)$ is a mapping.

2.5 Definition: [20]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F: A \rightarrow P(U)$ is a mapping.

This concept has been modified by Deli and Broumi [26] as given below.

2.6 Definition: [26]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \rightarrow P(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(U)$ and therefore it can be written as a set of ordered pairs, $N = \{(e, \{< x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) > : x \in U\}) : e \in E\}$ where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0,1]$, respectively, called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

2.6.1 Example

Let, $U = \{h_1, h_2, h_3\}$ be a set of houses and $E = \{e_1(beautiful), e_2(wooden), e_3(costly)\}$ be a set of parameters with respect to which the nature of houses is described. Let

$f_N(e_1) = \{<h_1, (0.5, 0.6, 0.3)>, <h_2, (0.4, 0.7, 0.6)>, <h_3, (0.6, 0.2, 0.3)>\}$;

$f_N(e_2) = \{<h_1, (0.6, 0.3, 0.5)>, <h_2, (0.7, 0.4, 0.3)>, <h_3, (0.8, 0.1, 0.2)>\}$;

$f_N(e_3) = \{<h_1, (0.7, 0.4, 0.3)>, <h_2, (0.6, 0.7, 0.2)>, <h_3, (0.7, 0.2, 0.5)>\}$;

Then, $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)]\}$ is an NSS over (U, E).

The tabular representation of the NSS N is as:

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)

Table 1: Tabular form of NSS *N*.

2.6.2 Definition: [26]

The complement of a neutrosophic soft set N is denoted by N^c and is defined as:

$$N^c = \{(e, \{< x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

2.6.3 Definition: [26]

Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x), F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x);\ \forall e \in E \text{ and } x \in U.$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

2.6.4 Definition: [26]

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as :

$$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\},$$

where

$$T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x),$$
$$I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their intersection is denoted by $N_1 \cap N_2 = N_4$ and it is defined as:

$$N_4 = \{(e, \{< x, T_{f_{N_4}(e)}(x), I_{f_{N_4}(e)}(x), F_{f_{N_4}(e)}(x) >: x \in U\}) : e \in E\}$$

where

$$T_{f_{N_4}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x),$$
$$I_{f_{N_4}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_4}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x);$$

2.7 Definition: [8]

Let (F, A) be a soft set over the group G. Then (F, A) is called a soft group over G if $F(a)$ is a subgroup of G, $\forall a \in A$.

3 Neutrosophic soft groups

In this section, we define the neutrosophic soft group and some basic properties related to it. Unless otherwise stated, E is treated as the parametric set throughout this paper and $e \in E$, an arbitrary parameter.

3.1 Definition:

A neutrosophic set $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in G\}$ over a group $(G,\circ)$ is called a neutrosophic subgroup of $(G,\circ)$ if

(i) $T_A(x \circ y) \geq T_A(x) * T_A(y)$,
$I_A(x \circ y) \leq I_A(x) \diamond I_A(y)$,
$F_A(x \circ y) \leq F_A(x) \diamond F_A(y)$;
for $x, y \in G$.

(ii) $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \leq I_A(x)$, $F_A(x^{-1}) \leq F_A(x)$, for $x \in G$.

An NSS N over a group $(G,\circ)$ is called a neutrosophic soft group if $f_N(e)$ is a neutrosophic subgroup of $(G,\circ)$ for each $e \in E$.

3.1.1 Example:

1. Let us consider the Klein's -4 group $V = \{e, a, b, c\}$ and $E = \{\alpha, \beta, \gamma, \delta\}$ be the set of parameters. We define $f_N(\alpha), f_N(\beta), f_N(\gamma), f_N(\delta)$ as given by the following table:

	$f_N(\alpha)$	$f_N(\beta)$	$f_N(\gamma)$	$f_N(\delta)$
e	(0.65, 0.34, 0.14)	(0.88, 0.12,0.72)	(0.72, 0.21, 0.16)	(0.69, 0.31, 0.32)
a	(0.71, 0.22, 0.78)	(0.71, 0.19, 0.44)	(0.84, 0.16, 0.25)	(0.62, 0.32, 0.42)
b	(0.75, 0.25, 0.52)	(0.83, 0.11, 0.28)	(0.69, 0.31, 0.39)	(0.58, 0.41, 0.66)
c	(0.67, 0.32, 0.29)	(0.75, 0.21, 0.19)	(0.79, 0.19, 0.41)	(0.71, 0.27, 0.53)

Table 2: Tabular form of neutrosophic soft group *N*.

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = max\,(a + b - 1, \; 0)$, $a \diamond b = min\,(a + b, 1)$. Then, N forms a neutrosophic soft group over (V, E).

2. Let $E = \mathbb{N}$ (the set of natural no.), be the parametric set and $G = (\mathbb{Z}, +)$ be the group of all integers. Define a mapping $f_M : \mathbb{N} \to NS(\mathbb{Z})$ where, for any $n \in \mathbb{N}$ and $x \in \mathbb{Z}$,

$$T_{f_{M(n)}}(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ \frac{1}{n} & \text{if } x \text{ is even} \end{cases}$$

$$I_{f_{M(n)}}(x) = \begin{cases} \frac{1}{2n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

$$F_{f_{M(n)}}(x) = \begin{cases} 1 - \frac{1}{n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = min\,(a, b), a \diamond b = max\,(a, b)$.

Then, M forms a neutrosophic soft set as well as neutrosophic soft group over $[(\mathbb{Z}, +), \mathbb{N}]$.

3.2 Proposition:

An NSS N over the group $(G,\circ)$ is called a neutrosophic soft group iff followings hold on the assumption that truth membership (T), indeterministic membership (I) and falsity membership (F) functions of an NSS obey the idempotent t-norm and idempotent s-norm disciplines.

$$T_{f_{N(e)}}(x \circ y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$$
$$I_{f_{N(e)}}(x \circ y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$$
$$F_{f_{N(e)}}(x \circ y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y);$$
$$\forall\, x, y \in G; \quad \forall e \in E;$$

Proof:

Firstly, suppose N is an NSS group over $(G,\circ)$.
Then,

$$T_{f_{N(e)}}(x \circ y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$$
$$I_{f_{N(e)}}(x \circ y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$$
$$F_{f_{N(e)}}(x \circ y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y);$$

Conversely, for the identity element e_G in G;

$$T_{f_{N(e)}}(e_G) = T_{f_{N(e)}}(x \circ x^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x),$$
$$I_{f_{N(e)}}(e_G) = I_{f_{N(e)}}(x \circ x^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x),$$
$$F_{f_{N(e)}}(e_G) = F_{f_{N(e)}}(x \circ x^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x);$$

Now,

$$T_{f_{N(e)}}(x^{-1}) = T_{f_{N(e)}}(e_G \circ x^{-1}) \geq T_{f_{N(e)}}(e_G) * T_{f_{N(e)}}(x^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x),$$
$$I_{f_{N(e)}}(x^{-1}) = I_{f_{N(e)}}(e_G \circ x^{-1}) \leq I_{f_{N(e)}}(e_G) \diamond I_{f_{N(e)}}(x^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x),$$

$$F_{f_{N(e)}}(x^{-1}) = F_{f_{N(e)}}(e_G \circ x^{-1}) \leq F_{f_{N(e)}}(e_G) \diamond I_{f_{N(e)}}(x^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x);$$

Next,

$$T_{f_{N(e)}}(x \circ y) = T_{f_{N(e)}}(x \circ (y^{-1})^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$$

$$I_{f_{N(e)}}(x \circ y) = I_{f_{N(e)}}(x \circ (y^{-1})^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$$

$$F_{f_{N(e)}}(x \circ y) = F_{f_{N(e)}}(x \circ (y^{-1})^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y).$$

This completes the proof.

3.2.1 Proposition:

Let N be a neutrosophic soft group over the group $(G,\circ)$. Then for $x \in G$, followings hold.

(i) $T_{f_{N(e)}}(x^{-1}) = T_{f_{N(e)}}(x)$, $I_{f_{N(e)}}(x^{-1}) = I_{f_{N(e)}}(x)$, $F_{f_{N(e)}}(x^{-1}) = F_{f_{N(e)}}(x)$;

(ii) $T_{f_{N(e)}}(e_G) \geq T_{f_{N(e)}}(x)$, $I_{f_{N(e)}}(e_G) \leq I_{f_{N(e)}}(x)$, $F_{f_{N(e)}}(e_G) \leq F_{f_{N(e)}}(x)$;

if T follows the idempotent t-norm and I, F follow the idempotent s-norm disciplines, respectively. (e_G being the identity element of G.)

Proof:

(i) $T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x^{-1})^{-1} \geq T_{f_{N(e)}}(x^{-1})$

$I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x^{-1})^{-1} \leq I_{f_{N(e)}}(x^{-1})$

$F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x^{-1})^{-1} \leq F_{f_{N(e)}}(x^{-1})$

Now, from definition of neutrosophic soft group, the result follows.

(ii) For the identity element e_G in G,

$$T_{f_{N(e)}}(e_G) = T_{f_{N(e)}}(x \circ x^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x),$$

$$I_{f_{N(e)}}(e_G) = I_{f_{N(e)}}(x \circ x^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x),$$

$$F_{f_{N(e)}}(e_G) = F_{f_{N(e)}}(x \circ x^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x);$$

Hence, the proposition is proved.

3.3 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group $(G,\circ)$. Then, $N_1 \cap N_2$ is also a neutrosophic soft group over $(G,\circ)$.

Proof:

Let $N_1 \cap N_2 = N_3$; Now for $x, y \in G$;

$$T_{f_{N_3(e)}}(x \circ y) = T_{f_{N_1(e)}}(x \circ y) * T_{f_{N_2(e)}}(x \circ y)$$
$$\geq \left[T_{f_{N_1(e)}}(x) * T_{f_{N_1(e)}}(y)\right] * \left[T_{f_{N_2(e)}}(x) * T_{f_{N_2(e)}}(y)\right]$$
$$= \left[T_{f_{N_1(e)}}(x) * T_{f_{N_1(e)}}(y)\right] * \left[T_{f_{N_2(e)}}(y) * T_{f_{N_2(e)}}(x)\right] \text{ (as } * \text{ is commutative)}$$
$$= T_{f_{N_1(e)}}(x) * \left[T_{f_{N_1(e)}}(y) * T_{f_{N_2(e)}}(y)\right] * T_{f_{N_2(e)}}(x) \text{ (as } * \text{ is associative)}$$
$$= T_{f_{N_1(e)}}(x) * T_{f_{N_3(e)}}(y) * T_{f_{N_2(e)}}(x)$$
$$= T_{f_{N_1(e)}}(x) * T_{f_{N_2(e)}}(x) * T_{f_{N_3(e)}}(y) \text{ (as } * \text{ is commutative)}$$
$$= T_{f_{N_3(e)}}(x) * T_{f_{N_3(e)}}(y)$$

Also,

$$T_{f_{N_3(e)}}(x^{-1}) = T_{f_{N_1(e)}}(x^{-1}) * T_{f_{N_2(e)}}(x^{-1}) \geq T_{f_{N_1(e)}}(x) * T_{f_{N_2(e)}}(x) = T_{f_{N_3(e)}}(x);$$

Next,

$$I_{f_{N_3(e)}}(x \circ y) = I_{f_{N_1(e)}}(x \circ y) \diamond I_{f_{N_2(e)}}(x \circ y)$$
$$\leq \left[I_{f_{N_1(e)}}(x) \diamond I_{f_{N_1(e)}}(y)\right] \diamond \left[I_{f_{N_2(e)}}(x) \diamond I_{f_{N_2(e)}}(y)\right]$$
$$= \left[I_{f_{N_1(e)}}(x) \diamond I_{f_{N_1(e)}}(y)\right] \diamond \left[I_{f_{N_2(e)}}(y) \diamond I_{f_{N_2(e)}}(x)\right] \text{ (as } \diamond \text{ is commutative)}$$
$$= I_{f_{N_1(e)}}(x) \diamond \left[I_{f_{N_1(e)}}(y) \diamond I_{f_{N_2(e)}}(y)\right] \diamond I_{f_{N_2(e)}}(x) \text{ (as } \diamond \text{ is associative)}$$
$$= I_{f_{N_1(e)}}(x) \diamond I_{f_{N_3(e)}}(y) \diamond I_{f_{N_2(e)}}(x)$$
$$= I_{f_{N_1(e)}}(x) \diamond I_{f_{N_2(e)}}(x) \diamond I_{f_{N_3(e)}}(y) \text{ (as } \diamond \text{ is commutative)}$$
$$= I_{f_{N_3(e)}}(x) \diamond I_{f_{N_3(e)}}(y)$$

Also,

$$I_{f_{N_3(e)}}(x^{-1}) = I_{f_{N_1(e)}}(x^{-1}) \diamond I_{f_{N_2(e)}}(x^{-1}) \leq I_{f_{N_1(e)}}(x) \diamond I_{f_{N_2(e)}}(x) = I_{f_{N_3(e)}}(x);$$

Similarly,

$$F_{f_{N_3(e)}}(x \circ y) \leq F_{f_{N_3(e)}}(x) \diamond F_{f_{N_3(e)}}(y),$$
$$F_{f_{N_3(e)}}(x^{-1}) \leq F_{f_{N_3(e)}}(x);$$

This ends the theorem. The theorem is also true for a family of neutrosophic soft groups over a group.

3.3.1 Remark:

For two neutrosophic soft groups N_1 and N_2 over the group G, $N_1 \cup N_2$ is not generally a neutrosophic soft group over G. It is possible if anyone is contained in other.

For example, let, $G = (\mathbb{Z}, +)$, $E = 2\mathbb{Z}$. Consider two neutrosophic soft groups N_1 and N_2 over G as following. For $x, n \in \mathbb{Z}$,

$$T_{f_{N_1}(2n)}(x) = \begin{cases} \frac{1}{2} & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_1}(2n)}(x) = \begin{cases} 0 & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{4} & \text{others} \end{cases}$$

$$F_{f_{N_1}(2n)}(x) = \begin{cases} 0 & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{10} & \text{others} \end{cases}$$

and

$$T_{f_2(2n)}(x) = \begin{cases} \frac{2}{3} & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_2}(2n)}(x) = \begin{cases} 0 & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{5} & \text{others} \end{cases}$$

$$F_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{6} & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 1 & \text{others} \end{cases}$$

Corresponding t-norm ($*$) and s-norm ($\diamond$) are defined as $a * b = min\,(a, b)$, $a \diamond b = max\,(a, b)$. Let $N_1 \cup N_2 = N_3$; Then for $n = 3$, $x = 12$, $y = 18$ we have,

$$T_{f_{N_3}(6)}(12 - 18) = T_{f_{N_1}(6)}(-6) \diamond T_{f_{N_2}(6)}(-6)$$
$$= \max\,(0, 0) = 0 \quad \text{and}$$
$$T_{f_{N_3}(6)}(12) * T_{f_{N_3}(6)}(18)$$
$$= \left\{ T_{f_{N_1}(6)}(12) \diamond T_{f_{N_2}(6)}(12) \right\} * \left\{ T_{f_{N_1}(6)}(18) \diamond T_{f_{N_2}(6)}(18) \right\}$$
$$= min\, \left\{ max\left(\frac{1}{2}, 0\right), max\left(0, \frac{2}{3}\right) \right\}$$
$$= min\, \left(\frac{1}{2}, \frac{2}{3}\right) = \frac{1}{2}$$

Hence,

$$T_{f_{N_3}(6)}(12 - 18) < T_{f_{N_3}(6)}(12) * T_{f_{N_3}(6)}(18);$$

i,e $N_1 \cup N_2$ is not a neutrosophic soft group, here.

Now, if we define N_2 over G as following:

$$T_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{8} & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_2}(2n)}(x) = \begin{cases} 0 & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{2}{5} & \text{others} \end{cases}$$

$$F_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{6} & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{2} & \text{others} \end{cases}$$

Then, it can be easily verified that $N_2 \subseteq N_1$ and $N_1 \cup N_2$ is a neutrosophic soft group over G.

3.4 Definition:

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'AND' operation is denoted by $N_1 \wedge N_2 = N_3$ and is defined as:

$$N_3 = \{[(a,b), \{< x, T_{f_{N_3}(a,b)}(x),\ I_{f_{N_3}(a,b)}(x),\ F_{f_{N_3}(a,b)}(x) >: x \in U\}]: (a,b) \in E \times E\} \text{ where}$$
$$T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x),$$
$$I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x),$$
$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'OR' operation is denoted by $N_1 \vee N_2 = N_4$ and is defined as:

$$N_4 = \{[(a,b), \{< x, T_{f_{N_4}(a,b)}(x),\ I_{f_{N_4}(a,b)}(x),\ F_{f_{N_4}(a,b)}(x) >: x \in U\}]: (a,b) \in E \times E\}$$

where

$$T_{f_{N_4}(a,b)}(x) = T_{f_{N_1}(a)}(x) \diamond T_{f_{N_2}(b)}(x),$$
$$I_{f_{N_4}(a,b)}(x) = I_{f_{N_1}(a)}(x) * I_{f_{N_2}(b)}(x),$$
$$F_{f_{N_4}(a,b)}(x) = F_{f_{N_1}(a)}(x) * F_{f_{N_2}(b)}(x);$$

3.5 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group $(G, \circ)$. Then, $N_1 \wedge N_2$ is also a neutrosophic soft group over $(G, \circ)$.

Proof:

Let $N_1 \wedge N_2 = N_3$. Then for $x, y \in G$ and $(a, b) \in E \times E$,

$$T_{f_{N_3}(a,b)}(x \circ y)$$
$$= T_{f_{N_1}(a)}(x \circ y) * T_{f_{N_2}(b)}(x \circ y)$$
$$\geq \left[T_{f_{N_1}(a)}(x) * T_{f_{N_1}(a)}(y) \right] * \left[T_{f_{N_2}(b)}(x) * T_{f_{N_2}(b)}(y) \right]$$
$$= \left[T_{f_{N_1}(a)}(x) * T_{f_{N_1}(a)}(y) \right] * \left[T_{f_{N_2}(b)}(y) * T_{f_{N_2}(b)}(x) \right] \quad \text{(as } * \text{ is commutative)}$$
$$= T_{f_{N_1}(a)}(x) * \left[T_{f_{N_1}(a)}(y) * T_{f_{N_2}(b)}(y) \right] * T_{f_{N_2}(b)}(x) \quad \text{(as } * \text{ is associative)}$$
$$= T_{f_{N_1}(a)}(x) * T_{f_{N_3}(a,b)}(y) * T_{f_{N_2}(b)}(x)$$
$$= T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x) * T_{f_{N_3}(a,b)}(y) \quad \text{(as } * \text{ is commutative)}$$
$$= T_{f_{N_3}(a,b)}(x) * T_{f_{N_3}(a,b)}(y)$$
$$T_{f_{N_3}(a,b)}(x^{-1}) = T_{f_{N_1}(a)}(x^{-1}) * T_{f_{N_2}(b)}(x^{-1})$$
$$\geq T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x)$$
$$= T_{f_{N_3}(a,b)}(x)$$

Similarly,

$I_{f_{N_3}(a,b)}(x \circ y) \leq I_{f_{N_3}(a,b)}(x) \diamond I_{f_{N_3}(a,b)}(y)$,
$I_{f_{N_3}(a,b)}(x^{-1}) \leq I_{f_{N_3}(a,b)}(x)$;

$F_{f_{N_3}(a,b)}(x \circ y) \leq F_{f_{N_3}(a,b)}(x) \diamond F_{f_{N_3}(a,b)}(y)$,
$F_{f_{N_3}(a,b)}(x^{-1}) \leq F_{f_{N_3}(a,b)}(x)$;

This completes the proof.

The theorem is true for a family of neutrosophic soft groups over a group.

3.6 Definition:

Let g be a mapping from a set X to a set Y. If M and N are two neutrosophic soft sets over X and Y, respectively, then the image of M under g is defined as a neutrosophic soft set $g(M) = \{[e, f_{g(M)}(e)]: e \in E\}$ over Y, where $T_{f_{g(M)}(e)}(y) = T_{f_M(e)}[g^{-1}(y)]$, $I_{f_{g(M)}(e)}(y) = I_{f_M(e)}[g^{-1}(y)]$, $F_{f_{g(M)}(e)}(y) = F_{f_M(e)}[g^{-1}(y)]$, $\forall y \in Y$.

The pre-image of N under g is defined as a neutrosophic soft set given by:

$g^{-1}(N) = \{[e, f_{g^{-1}(N)}(e)]: e \in E\}$ over X, where
$T_{f_{g^{-1}(N)}(e)}(x) = T_{f_N(e)}[g(x)]$, $I_{f_{g^{-1}(N)}(e)}(x) = I_{f_N(e)}[g(x)]$, $F_{f_{g^{-1}(N)}(e)}(x) = F_{f_N(e)}[g(x)]$,
$\forall x \in X$.

3.7 Theorem:

Let $g : X \to Y$ be an isomorphism in classical sense. If M is a neutrosophic soft group over X then $g(M)$ is a neutrosophic soft group over Y.

Proof:

Let $x_1, x_2 \in X$; $y_1, y_2 \in Y$; such that $y_1 = g(x_1)$, $y_2 = g(x_2)$. Now,

$T_{f_{g(M)}(e)}(y_1 \circ y_2)$
$= T_{f_M(e)}[g^{-1}(y_1 \circ y_2)]$
$= T_{f_M(e)}[g^{-1}(y_1) \circ g^{-1}(y_2)]$
(as g^{-1} is homomorphism)
$= T_{f_M(e)}(x_1 \circ x_2)$
$\geq T_{f_M(e)}(x_1) * T_{f_M(e)}(x_2)$
$= T_{f_M(e)}[g^{-1}(y_1)] * T_{f_M(e)}[g^{-1}(y_2)]$
$= T_{f_{g(M)}(e)}(y_1) * T_{f_{g(M)}(e)}(y_2)$

Next, $I_{f_{g(M)}(e)}(y_1 \circ y_2)$
$= I_{f_M(e)}[g^{-1}(y_1 \circ y_2)]$
$= I_{f_M(e)}[g^{-1}(y_1) \circ g^{-1}(y_2)]$
(as g^{-1} is homomorphism)
$= I_{f_M(e)}(x_1 \circ x_2)$
$\leq I_{f_M(e)}(x_1) \diamond I_{f_M(e)}(x_2)$
$= I_{f_M(e)}[g^{-1}(y_1)] \diamond I_{f_M(e)}[g^{-1}(y_2)]$
$= I_{f_{g(M)}(e)}(y_1) \diamond I_{f_{g(M)}(e)}(y_2)$

Similarly, $F_{f_{g(M)}(e)}(y_1 \circ y_2)$
$\leq F_{f_{g(M)}(e)}(y_1) \diamond F_{f_{g(M)}(e)}(y_2)$

Next, $T_{f_{g(M)}(e)}(y_1^{-1}) = T_{f_M(e)}[g^{-1}(y_1^{-1})]$
$= T_{f_M(e)}[(g^{-1}(y_1))^{-1}] = T_{f_M(e)}(x_1^{-1})$
$\geq T_{f_M(e)}(x_1) = T_{f_M(e)}[g^{-1}(y_1)]$
$= T_{f_{g(M)}(e)}(y_1)$ i,e
$T_{f_{g(M)}(e)}(y_1^{-1}) \geq T_{f_{g(M)}(e)}(y_1)$;

$I_{f_{g(M)}(e)}(y_1^{-1}) = I_{f_M(e)}[g^{-1}(y_1^{-1})]$
$= I_{f_M(e)}[(g^{-1}(y_1))^{-1}] = I_{f_M(e)}(x_1^{-1})$
$\leq I_{f_M(e)}(x_1) = I_{f_M(e)}[g^{-1}(y_1)]$
$= I_{f_{g(M)}(e)}(y_1)$ i,e
$I_{f_{g(M)}(e)}(y_1^{-1}) \leq I_{f_{g(M)}(e)}(y_1)$;

Similarly, $F_{f_{g(M)}(e)}(y_1^{-1}) \leq F_{f_{g(M)}(e)}(y_1)$;

This proves the theorem.

3.8 Theorem:

Let $g : X \to Y$ be an homomorphism in classical sense. If N is a neutrosophic soft group over Y, then $g^{-1}(N)$ is a neutrosophic soft group over X. [Note that $g^{-1}(N)$ is the inverse image of N under the mapping g. Here g^{-1} may not be a mapping.]

Proof:

Let $x_1, x_2 \in X$; $y_1, y_2 \in Y$; such that $y_1 = g(x_1)$, $y_2 = g(x_2)$. Now,

$T_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$= T_{f_N(e)}[g(x_1 \circ x_2)]$
$= T_{f_N(e)}[g(x_1) \circ g(x_2)]$
(as g is homomorphism)
$= T_{f_N(e)}(y_1 \circ y_2)$
$\geq T_{f_N(e)}(y_1) * T_{f_N(e)}(y_2)$
$= T_{f_N(e)}[g(x_1)] * T_{f_N(e)}[g(x_2)]$
$= T_{f_{g^{-1}(N)}(e)}(x_1) * T_{f_{g^{-1}(N)}(e)}(x_2)$

Next, $I_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$= I_{f_N(e)}[g(x_1 \circ x_2)]$
$= I_{f_N(e)}[g(x_1) \circ g(x_2)]$
(as g is homomorphism)
$= I_{f_N(e)}(y_1 \circ y_2)$
$\leq I_{f_N(e)}(y_1) \diamond I_{f_N(e)}(y_2)$
$= I_{f_N(e)}[g(x_1)] \diamond I_{f_N(e)}[g(x_2)]$
$= I_{f_{g^{-1}(N)}(e)}(x_1) \diamond I_{f_{g^{-1}(N)}(e)}(x_2)$

Similarly, $F_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$\leq F_{f_{g^{-1}(N)}(e)}(x_1) \diamond F_{f_{g^{-1}(N)}(e)}(x_2)$

Next, $T_{f_{g^{-1}(N)}(e)}(x_1^{-1}) = T_{f_N(e)}[g(x_1^{-1})]$

$= T_{f_N(e)}\left[\left(g(x_1)\right)^{-1}\right] = T_{f_N(e)}(y_1^{-1}) \geq T_{f_N(e)}(y_1) = T_{f_N(e)}[g(x_1)]$

$= T_{f_{g^{-1}(N)}(e)}(x_1)$ i,e

$T_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \geq T_{f_{g^{-1}(N)}(e)}(x_1)$;

Similarly, $I_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \leq I_{f_{g^{-1}(N)}(e)}(x_1)$,

$F_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \leq F_{f_{g^{-1}(N)}(e)}(x_1)$;

Hence, the theorem is proved.

3.9 Definition:

Let N be a neutrosophic soft group over the group G and $\lambda, \mu, \eta \in (0,1]$ with $\lambda + \mu + \eta \leq 3$. Then,

1. N is called (λ, μ, η) -identity neutrosophic soft group over G if $\forall e \in E$,

$T_{f_{N(e)}}(x) = \lambda,\ I_{f_{N(e)}}(x) = \mu,\ F_{f_{N(e)}}(x) = \eta$;

for $x = e_G$, the identity element of G.

$T_{f_{N(e)}}(x) = 0,\ I_{f_{N(e)}}(x) = F_{f_{N(e)}}(x) = 1$;

otherwise.

2. N is called (λ, μ, η) -absolute neutrosophic soft group over G if $\forall x \in G,\ e \in E,\quad T_{f_{N(e)}}(x) = \lambda,\ I_{f_{N(e)}}(x) = \mu,\ F_{f_{N(e)}}(x) = \eta$.

3.10 Theorem:

Let $\phi : X \to Y$ be an isomorphism in classical sense.

1. If N is a neutrosophic soft group over X, then $\phi(N)$ is a (λ, μ, η) -identity neutrosophic soft group over Y if $T_{f_{N(e)}}(x) = \lambda,\ I_{f_{N(e)}}(x) = \mu,\ F_{f_{N(e)}}(x) = \eta$; when $x \in Ker\phi$.

$T_{f_{N(e)}}(x) = 0,\ I_{f_{N(e)}}(x) = F_{f_{N(e)}}(x) = 1$;

otherwise, $\forall x \in X,\ e \in E$.

2. If N is a (λ, μ, η) -absolute neutrosophic soft group over X, then $\phi(N)$ is also so over Y.

Proof:

1. Clearly, $\phi(N)$ is a neutrosophic soft group over Y by theorem (3.7). Now, if $x \in ker\phi$ then $\phi(x) = e_Y$, the identity element of Y. Then,

$$T_{f_{\phi(N)(e)}}(e_Y) = T_{f_{N(e)}}[\phi^{-1}(e_Y)] = T_{f_{N(e)}}(x) = \lambda$$

$$I_{f_{\phi(N)(e)}}(e_Y) = I_{f_{N(e)}}[\phi^{-1}(e_Y)] = I_{f_{N(e)}}(x) = \mu$$

$$F_{f_{\phi(N)(e)}}(e_Y) = F_{f_{N(e)}}[\phi^{-1}(e_Y)] = F_{f_{N(e)}}(x) = \eta$$

Similarly, $T_{f_{N(e)}}(x) = 0,\ I_{f_{N(e)}}(x) = 1$,

$F_{f_{N(e)}}(x) = 1$; if x otherwise.

This ends the 1st part.

2. Let, $y = \phi(x)$ for $x \in X,\ y \in Y$. Then $\forall e \in E$,

$T_{f_{\phi(N)(e)}}(y) = T_{f_{N(e)}}[\phi^{-1}(y)] = T_{f_{N(e)}}(x) = \lambda$,

$I_{f_{\phi(N)(e)}}(y) = I_{f_{N(e)}}[\phi^{-1}(y)] = I_{f_{N(e)}}(x) = \mu$,

$F_{f_{\phi(N)(e)}}(y) = F_{f_{N(e)}}[\phi^{-1}(y)] = F_{f_{N(e)}}(x) = \eta$;

This completes the 2nd part.

4 Cartesian product of neutrosophic soft groups

4.1 Definition:

Let N_1 and N_2 be two neutrosophic soft groups over the groups X and Y, respectively. Then their cartesian product is $N_1 \times N_2 = N_3$ where $f_{N_3}(a, b) = f_{N_1}(a) \times f_{N_2}(b)$ for $(a, b) \in E \times E$. Analytically, $f_{N_3}(a, b) =$

$$\left\{ < (x, y),\ T_{f_{N_3(a,b)}}(x, y)\ I_{f_{N_3(a,b)}}(x, y),\ F_{f_{N_3(a,b)}}(x, y) >: (x, y) \in X \times Y \right\}$$

where

$T_{f_{N_3(a,b)}}(x, y) = T_{f_{N_1(a)}}(x) * T_{f_{N_2(b)}}(y)$,

$I_{f_{N_3(a,b)}}(x, y) = I_{f_{N_1(a)}}(x) \diamond I_{f_{N_2(b)}}(y)$,

$F_{f_{N_3(a,b)}}(x, y) = F_{f_{N_1(a)}}(x) \diamond F_{f_{N_2(b)}}(y)$;

This definition can be extended for more than two neutrosophic soft groups.

4.2 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the groups X and Y, respectively. Then their cartesian product $N_1 \times N_2$ is also a neutrosophic soft group over $X \times Y$.

Proof:

Let $N_1 \times N_2 = N_3$ where $f_{N_3}(a, b) = f_{N_1}(a) \times f_{N_2}(b)$ for $(a, b) \in E \times E$. Then for $(x_1, y_1),\ (x_2, y_2) \in X \times Y$,

$$\begin{aligned}
& T_{f_{N_3(a,b)}}[(x_1, y_1) \circ (x_2, y_2)] \\
&= T_{f_{N_3(a,b)}}(x_1 \circ x_2,\ y_1 \circ y_2) \\
&= T_{f_{N_1(a)}}(x_1 \circ x_2) * T_{f_{N_2(b)}}(y_1 \circ y_2) \\
&\geq [T_{f_{N_1(a)}}(x_1) * T_{f_{N_1(a)}}(x_2)] * [T_{f_{N_2(b)}}(y_1) * T_{f_{N_2(b)}}(y_2)] \\
&= [T_{f_{N_1(a)}}(x_1) * T_{f_{N_2(b)}}(y_1)] * [T_{f_{N_1(a)}}(x_2) * T_{f_{N_2(b)}}(y_2)] \\
&= T_{f_{N_3(a,b)}}(x_1, y_1) * T_{f_{N_3(a,b)}}(x_2, y_2)
\end{aligned}$$

Next,

$$\begin{aligned}
& I_{f_{N_3(a,b)}}[(x_1, y_1) \circ (x_2, y_2)] \\
&= I_{f_{N_3(a,b)}}(x_1 \circ x_2,\ y_1 \circ y_2) \\
&= I_{f_{N_1(a)}}(x_1 \circ x_2) \diamond I_{f_{N_2(b)}}(y_1 \circ y_2) \\
&\leq [I_{f_{N_1(a)}}(x_1) I_{f_{N_1(a)}}(x_2)] \diamond [I_{f_{N_2(b)}}(y_1) \diamond I_{f_{N_2(b)}}(y_2)] \\
&= [I_{f_{N_1(a)}}(x_1) \diamond I_{f_{N_2(b)}}(y_1)] \diamond [I_{f_{N_1(a)}}(x_2) \diamond I_{f_{N_2(b)}}(y_2)] \\
&= I_{f_{N_3(a,b)}}(x_1, y_1) \diamond I_{f_{N_3(a,b)}}(x_2, y_2)
\end{aligned}$$

Similarly, $F_{f_{N_3(a,b)}}[(x_1, y_1) \circ (x_2, y_2)] \leq F_{f_{N_3(a,b)}}(x_1, y_1) \diamond F_{f_{N_3(a,b)}}(x_2, y_2)$.

Next,

$$T_{f_{N_3}(a,b)}[(x_1,y_1)^{-1}] = T_{f_{N_3}(a,b)}(x_1^{-1},y_1^{-1})$$
$$= T_{f_{N_1}(a)}(x_1^{-1}) * T_{f_{N_2}(b)}(y_1^{-1})$$
$$\geq T_{f_{N_1}(a)}(x_1) * T_{f_{N_2}(b)}(y_1)$$
$$= T_{f_{N_3}(a,b)}(x_1,y_1)$$

Similarly,

$I_{f_{N_3}(a,b)}[(x_1,y_1)^{-1}] \leq I_{f_{N_3}(a,b)}(x_1,y_1)$,
$F_{f_{N_3}(a,b)}[(x_1,y_1)^{-1}] \leq F_{f_{N_3}(a,b)}(x_1,y_1)$.

Hence, the theorem is proved.

5 Neutrosophic soft subgroup

5.1 Definition:

Let N_1 and N_2 be two neutrosophic groups over the group G. Then N_1 is neutrosophic soft subgroup of N_2 if

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x);\ \forall x \in G,\ e \in E.$$

5.1.1 Example:

We consider the Klein's -4 group $V = \{ e,a,b,c \}$ and $E = \{ \alpha,\beta,\gamma,\delta \}$ be a set of parameters. The two neutrosophic soft groups M,N defined over (V,E) are given by the following tables when corresponding t-norm and s-norm are defined as

$$a * b = max\,(a + b - 1,0) \text{ and } a \diamond b = min\,(a+b,1).$$

	$f_M(\alpha)$ $f_M(\gamma)$	$f_M(\beta)$ $f_M(\delta)$
e	(0.65, 0.42, 0.54) (0.70, 0.31, 0.32)	(0.68, 0.21, 0.76) (0.59, 0.38, 0.62)
a	(0.61, 0.44, 0.78) (0.67, 0.41, 0.39)	(0.62, 0.31, 0.79) (0.41, 0.49, 0.64)
b	(0.55, 0.55, 0.59) (0.60, 0.36, 0.48)	(0.59, 0.42, 0.80) (0.56, 0.43, 0.68)
c	(0.47, 0.49, 0.69) (0.48, 0.52, 0.54)	(0.67, 0.43, 0.84) (0.49, 0.50, 0.70)

Table 3: Tabular form of neutrosophic soft group *M*.

	$f_N(\alpha)$ $f_N(\gamma)$	$f_N(\beta)$ $f_N(\delta)$
e	(0.65, 0.34, 0.14) (0.72, 0.21, 0.16)	(0.88, 0.12, 0.72) (0.69, 0.31, 0.32)
a	(0.71, 0.22, 0.78) (0.84, 0.16, 0.25)	(0.71, 0.19, 0.44) (0.62, 0.32, 0.42)
b	(0.75, 0.25, 0.52) (0.69, 0.31, 0.39)	(0.83, 0.11, 0.28) (0.58, 0.41, 0.66)
c	(0.67, 0.32, 0.29) (0.79, 0.19, 0.41)	(0.75, 0.21, 0.19) (0.71, 0.27, 0.53)

Table 4: Tabular form of neutrosophic soft group *N*.

Obviously, M is the neutrosophic soft subgroup of N over (V,E).

5.2 Theorem:

Let N be a neutrosophic soft group over the group G and N_1, N_2 be two neutrosophic soft subgroups of N. If T, I, F of neutrosophic soft group N obey the disciplines of idempotent t-norm and idempotent s-norm, then,

(i) $N_1 \cap N_2$ is a neutrosophic soft subgroup of N.
(ii) $N_1 \wedge N_2$ is a neutrosophic soft subgroup of $N \wedge N$.

Proof:

The intersection($\cap$), AND ($\wedge$) of two neutrosophic soft groups is also so by theorems (3.3) and (3.5). Now to complete this theorem, we only verify the criteria of neutrosophic soft subgroup in each case.

(i) Let $N_3 = N_1 \cap N_2$. For $x \in G$,

$$T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x)$$
$$\leq T_{f_N(e)}(x) * T_{f_N(e)}(x) = T_{f_N(e)}(x),$$

$$I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x)$$
$$\geq I_{f_N(e)}(x) \diamond I_{f_N(e)}(x) = I_{f_N(e)}(x),$$

$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x)$$
$$\geq F_{f_N(e)}(x) \diamond F_{f_N(e)}(x) = F_{f_N(e)}(x);$$

(ii) Let $N_3 = N_1 \wedge N_2$ and $x \in G$; Then,

$$T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x)$$
$$\leq T_{f_N(a)}(x) * T_{f_N(b)}(x) = T_{f_N(a,b)}(x),$$

$$I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x)$$
$$\geq I_{f_N(a)}(x) \diamond I_{f_N(b)}(x) = I_{f_N(a,b)}(x),$$

$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(x)$$
$$\geq F_{f_N(a)}(x) \diamond F_{f_N(b)}(x) = F_{f_N(a,b)}(x);$$

The theorems are also true for a family of neutrosophic soft subgroups of N.

5.3 Example:

We consider the group (S,.), cube root of unity where S= $\{1,\omega,\omega^2\}$ and let E= $\{ \alpha,\beta,\gamma \}$ be a set of parameters.

The t-norm and s-norm are defined as: $a * b = ab$ and $a \diamond b = a + b - ab$. The neutrosophic soft group N and it's two subgroups N_1, N_2 defined over (S,.) are given by the following tables.

	$f_N(\alpha)$	$f_N(\beta)$	$f_N(\gamma)$
1	(0.7,0.3,0.2) (0.6,0.5,0.6)		(0.6,0.3,0.5)
ω	(0.7,0.2,0.4) (0.5,0.5,0.7)		(0.7,0.3,0.5)
ω^2	(0.6,0.3,0.3) (0.4,0.4,0.6)		(0.5,0.4,0.6)

Table 5: Tabular form of neutrosophic soft group *N*.

	$f_{N_1}(\alpha)$	$f_{N_1}(\beta)$	$f_{N_1}(\gamma)$
1	(0.4,0.4,0.9) (0.5,0.6,0.6)		(0.6,0.6,0.6)
ω	(0.6,0.4,0.7) (0.4,0.5,0.7)		(0.5,0.8,0.5)
ω^2	(0.3,0.5,0.8) (0.4,0.8,0.7)		(0.5,0.6,0.7)

Table 6: Tabular form of neutrosophic soft subgroup N_1.

	$f_{N_2}(\alpha)$	$f_{N_2}(\beta)$	$f_{N_2}(\gamma)$
1	(0.6,0.5,0.2) (0.5,0.5,0.7)		(0.6,0.4,0.6)
ω	(0.7,0.3,0.4) (0.4,0.5,0.8)		(0.6,0.4,0.5)
ω^2	(0.6,0.4,0.3) (0.3,0.6,0.7)		(0.5,0.5,0.7)

Table 7: Tabular form of neutrosophic soft subgroup N_2.

	$f_M(\alpha)$	$f_M(\beta)$	$f_M(\gamma)$
1	(0.24,0.70,0.92) (0.25,0.80,0.88)		(0.36,0.76,0.84)
ω	(0.42,0.58,0.82) (0.16,0.75,0.94)		(0.30,0.88,0.75)
ω^2	(0.18,0.70,0.86) (0.12,0.92,0.91)		(0.25,0.80,0.91)

Table 8: Tabular form of neutrosophic soft subgroup $M = N_1 \cap N_2$.

	$f_P(\alpha,\alpha)$ $f_P(\alpha,\beta)$ $f_P(\alpha,\gamma)$	$f_P(\beta,\alpha)$ $f_P(\beta,\beta)$ $f_P(\beta,\gamma)$	$f_P(\gamma,\alpha)$ $f_P(\gamma,\beta)$ $f_P(\gamma,\gamma)$
1	(0.24,0.70,0.92) (0.30,0.80,0.68) (0.24,0.64,0.96) (0.30,0.76,0.84) (0.20,0.70,0.97) (0.25,0.80,0.88)		(0.36,0.80,0.68) (0.36,0.76,0.84) (0.30,0.80,0.88)
ω	(0.42,0.58,0.82) (0.28,0.65,0.82) (0.36,0.64,0.85) (0.24,0.70,0.85) (0.24,0.70,0.94) (0.16,0.75,0.94)		(0.35,0.86,0.70) (0.30,0.88,0.75) (0.20,0.90,0.90)
ω^2	(0.18,0.70,0.86) (0.24,0.88,0.79) (0.15,0.75,0.94) (0.20,0.90,0.91) (0.09,0,80,0.94) (0.12,0.92,0.91)		(0.30,0.76,0.79) (0.25,0.80,0.91) (0.15,0.84,0.91)

Table 9: Tabular form of neutrosophic soft subgroup $P = N_1 \wedge N_2$.

	$f_P(\alpha,\alpha)$ $f_P(\alpha,\beta)$ $f_P(\alpha,\gamma)$	$f_P(\beta,\alpha)$ $f_P(\beta,\beta)$ $f_P(\beta,\gamma)$	$f_P(\gamma,\alpha)$ $f_P(\gamma,\beta)$ $f_P(\gamma,\gamma)$
1	(0.49,0.51,0.36) (0.42,0.65,0.68) (0.42,0.51,0.60) (0.36,0.65,0.80) (0.42,0.65,0.68) (0.36,0.75,0.84)		(0.42,0.51,0.60) (0.36,0.51,0.75) (0.36,0.65,0.80)
ω	(0.49,0.36,0.64) (0.35,0.60,0.82) (0.49,0.44,0.70) (0.35,0.65,0.85) (0.35,0.60,0.82) (0.25,0.75,0.91)		(0.49,0.44,0.70) (0.49,0.51,0.75) (0.35,0.65,0.85)
ω^2	(0.36,0.51,0.51) (0.24,0.58,0.72) (0.30,0.58,0.72) (0.20,0.64,0.84) (0.24,0.58,0.72) (0.16,0.64,0.84)		(0.30,0.58,0.72) (0.25,0.64,0.84) (0.20,0.64,0.84)

Table 10: Tabular form of neutrosophic soft subgroup $P = N \wedge N$.

Tables 5 & 8 show the 1st result and Tables 9 & 10 show the 2nd result in theorem (5.2).

5.4 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group X such that N_1 is the neutrosophic soft subgroup of N_2. Let $g : X \rightarrow Y$ be an isomorphism in classical sense. Then $g(N_1)$ and $g(N_2)$ are two neutrosophic soft groups over Y. Moreover $g(N_1)$ is the neutrosophic soft subgroup of $g(N_2)$.

Proof:

The 1st part is already proved in theorem (3.7).

Let $x \in X, y \in Y$ so that $y = g(x)$. Then,

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x)$$
$$\Rightarrow T_{f_{N_1}(e)}[g^{-1}(y)] \leq T_{f_{N_2}(e)}[g^{-1}(y)]$$
$$\Rightarrow T_{f_{g(N_1)}(e)}(y) \leq T_{f_{g(N_2)}(e)}(y)$$

Similarly, $I_{f_{g(N_1)}(e)}(y) \geq I_{f_{g(N_2)}(e)}(y)$, $F_{f_{g(N_1)}(e)}(y) \geq F_{f_{g(N_2)}(e)}(y)$;

Hence, the theorem is proved.

Conclusion

In the present paper, the theoretical point of view of neutrosophic soft group has been discussed with suitable examples. Here, we also have defined the Cartesian product on neutrosophic soft groups and neutrosophic soft subgroup. Some theorems have been established. We extended the concept of group in NSS theory context. This concept will bring a new opportunity in research and development of NSS theory.

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