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Multi-Objective Structural Design Optimization using Neutrosophic Goal Programming Technique

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Abstract: This paper develops a multi-objective Neutrosophic Goal Optimization (NSGO) technique for optimizing the design of three bar truss structure with multiple objectives subject to a specified set of constraints. In this optimum design formulation, the objective functions are weight and deflection; the design variables are the cross-sections of the bar; the constraints are the stress in member. The classical three bar truss structure is presented here in to demonstrate the efficiency of the neutrosophic goal programming approach. The model is numerically illustrated by generalized NSGO technique with different aggregation method. The result shows that the Neutrosophic Goal Optimization technique is very efficient in finding the best optimal solutions.

Keywords: Neutrosophic Set, Single Valued Neutrosophic Set, Generalized Neutrosophic Goal Programming, Arithmetic Aggregation, Geometric Aggregation, Structural Optimization.

1 Introduction

The research area of optimal structural design has been receiving increasing attention from both academia and industry over the past four decades in order to improve structural performance and to reduce design costs. In the real world, uncertainty or vagueness is prevalent in the Engineering Computations. In the context of structural design the uncertainty is connected with lack of accurate data of design factors. This tendency has been changing due to the increase in the use of fuzzy mathematical algorithm for dealing with such kind of problems.

Fuzzy set (FS) theory has long been introduced to deal with inexact and imprecise data by Zadeh [1], Later on the fuzzy set theory was used by Bellman and Zadeh [2] to the decision making problem. A few work has been done as an application of fuzzy set theory on structural design. Several researchers like Wang et al. [3] first applied α-cut method to structural designs where various design levels α were used to solve the non-linear problems. In this regard ,a generalized fuzzy number has been used Dey et al. [4] in context of a non-linear structural design optimization. Dey et al. [5] used basic t-norm based fuzzy optimization technique for optimization of structure and Dey et al. [6] developed parameterized t-norm based fuzzy optimization method for optimum structural design.

In such extension, Intuitionistic fuzzy set which is one of the generalizations of fuzzy set theory and was characterized by a membership, a non- membership and a hesitancy function was first introduced by Atanassov [21] (IFS). In fuzzy set theory the degree of acceptance is only considered but in case of IFS it is characterized by degree of membership and non-membership in such a way that their sum is less or equal to one. Dey et al. [7] solved two bar truss non-linear problem by using intuitionistic fuzzy optimization problem.Again Dey et al. [8] used intuitionistic fuzzy optimization technique to solve multi objective structural design. R-x Liang et al. [9] applied interdependent inputs of single valued trapezoidal neutrosophic information on Multi-criteria group decision making problem. P Ji et al. [10], S Yu et al. [11] did so many research study on application based neutosophic sets and intuitionistic linguistic number. Z-p Tian et al. [12] Simplified neutrosophic linguistic multi-criteria group decision-making approach to green product development. Again J-j Peng et al. [13] introduced multi-valued neutrosophic qualitative flexible approach based on likelihood for multi-criteria decision-making problems. Also, H Zhang et. al. [22] investigates a case study on a novel decision support model for satisfactory restaurants utilizing social information. P Ji et al. [14] developed a projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection.Intuitionistic fuzzy sets consider both truth and falsity membership and can only handle incomplete information but not the information which is connected with indeterminacy or inconsistency.

In neutrosophic sets indeterminacy or inconsistency is quantified explicitly by indeterminacy membership function. Neutrosophic Set (NS), introduced by Smarandache [15] was characterized by truth, falsity and indeterminacy membership so that in case of single valued NS set their sum is less or equal to three. In early [17] Charnes and Cooper first introduced Goal programming problem for a linear model. Usually conflicting goal are presented in a multi-objective goal programming problem. Dey et al. [16] used intuitionistic goal programming on nonlinear structural model. This is the first time NSGO technique is in application to multi-objective structural design. Usually objective goals of existing structural model are considered to be deterministic and a fixed quantity. In a situation, the decision maker can be doubtful with regard to accomplishment of the goal. The DM may include the idea of truth, indeterminacy and falsity bound on objectives goal.The goal may have a target value with degree of truth,indeterminacy as well as degree of falsity.Precisely ,we can say a human being that express degree of truth membership of a given element in a fuzzy set,truth and falsity membership in a intuitionistic fuzzy set,very often does not express the corresponding degree of falsity membership as complement to 3. This fact seems to take the objective goal as a neutrosophic set. The present study investigates computational algorithm for solving multi-objective structural problem by single valued generalized NSGO technique. The results are compared numerically for different aggregation method of NSGO technique. From our numerical result, it has been seen the best result obtained for geometric aggregation method for NSGO technique in the perspective of structural optimization technique.

2 Multi-objective structural model

In the design problem of the structure i.e. lightest weight of the structure and minimum deflection of the loaded joint that satisfies all stress constraints in members of the structure. In truss structure system, the basic parameters (including allowable stress,etc.) are known and the optimization's target is that identify the optimal bar truss cross-section area so that the structure is of the smallest total weight with minimum nodes displacement in a given load conditions .

The multi-objective structural model can be expressed as

$$\textit{Minimize } WT(A)$$

(1)

$$\textit{minimize } \delta(A)$$

$$\textit{subject to } \sigma(A) \le [\sigma]$$

$$A^{\min} \le A \le A^{\max}$$

where $A=[A_1, A_2, ..., A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar , $WT(A)=\sum_{i=1}^{n}\rho_i A_i L_i$ is the total weight of the structure , $\delta(A)$ is the deflection of the loaded joint ,where L_i, A_i and ρ_i are the bar length, cross section area and density of the i^{th} group bars respectively. $\sigma(A)$ is the stress constraint and $[\sigma]$ is allowable stress of the group bars under various conditions, $A^{\min}$ and $A^{\max}$ are the lower and upper bounds of cross section area A respectively.

3 Mathematical preliminaries

3.1 Fuzzy set

Let X be a fixed set. A fuzzy set A set of X is an object having the form $\tilde{A}=\{(x, T_A(x)) : x \in X\}$ where the function $T_A : X \to [0,1]$ defined the truth membership of the element $x \in X$ to the set A .

3.2 Intuitionistic fuzzy set

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form

$$\tilde{A}^i = \{< X, T_A(x), F_A(x) > | x \in X\} \text{ where}$$

$$T_A : X \to [0,1] \text{ and } F_A : X \to [0,1]$$

define the truth membership and falsity membership respectively, for every element of $x \in X$ $0 \le T_A + F_A \le 1$.

3.3 Neutrosophic set

Let a set X be a space of points (objects) and $x \in X$ .A neutrosophic set $\tilde{A}^n$ in X is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$,and denoted by $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$.

$T_A(x)$ $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ .That is $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$, . There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $0^- \le \sup T_A(x) + I_A(x) + \sup F_A(x) \le 3^+$.

3.4 Single valued neutrosophic set

Let a set X be the universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ where $T_A : X \to [0,1]$, $I_A : X \to [0,1]$, and $F_A : X \to [0,1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

3.5 Complement of neutrosophic Set

Complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - F_A(x)$, $F_{c(A)}(x) = T_A(x)$

3.6 Union of neutrosophic sets

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cup B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \max(T_A(x), T_B(x))$,

$I_{c(A)}(x) = \max(I_A(x), I_B(x))$,

$F_{c(A)}(x) = \min(F_A(x), F_B(x))$ for all $x \in X$.

3.7 Intersection of neutrosophic sets

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cap B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \min(T_A(x), T_B(x))$,

$I_{c(A)}(x) = \min(I_A(x), I_B(x))$,

$F_{c(A)}(x) = \max(F_A(x), F_B(x))$ for all $x \in X$.

4 Mathematical analysis

4.1 Neutrosophic Goal Programming

Neutrosophic Goal Programming problem is an extension of intuitionistic fuzzy as well as fuzzy goal programming problem in which the degree of indeterminacy of objective(s) and constraints are considered with degree of truth and falsity membership degree.

Goal programming can be written as

Find

$$x = (x_1, x_2, ..., x_n)^T \tag{1}$$

to achieve:

$z_i = t_i \quad i = 1, 2, ..., k$

Subject to $x \in X$ where t_i are scalars and represent the target achievement levels of the objective functions that the decision maker wishes to attain provided, X is feasible set of constraints.

The nonlinear goal programming problem can be written as

Find

$$x = (x_1, x_2, ..., x_n)^T \tag{2}$$

So as to

Minimize z_i with target value t_i ,acceptance tolerance a_i ,indeterminacy tolerance d_i rejection tolerance c_i

$x \in X$

$g_j(x) \le b_j$, $j = 1, 2, ..., m$

$x_i \ge 0$, $i = 1, 2, ..., n$

This neutrosophic goal programming can be transformed into crisp programming and can be transformed into crisp programming problem model by maximizing the degree of truth and indeterminacy and minimizing the degree of falsity of neutrosophic objectives and constraints. In the above problem (2), multiple objectives are considered as neutrosophic with some relaxed target. This representation demonstrates that decision maker (DM) is not sure about minimum value of $z_i, i = 1, 2, .., k$. DM has some illusive ideas of some optimum values of $z_i, i = 1, 2, .., k$. Hence it is quite natural to have desirable values violating the set target. Then question arises that how much bigger the optimum values may be .DM has also specified it with the use of tolerances. The tolerances are set in such a manner that the sum of truth, indeterminacy and falsity membership of objectives $z_i, i = 1, 2, .., k$ will lie between 0 and 3 . Let us consider the following theorem on membership function:

Theorem 1.

For a generalized neutrosophic goal programming problem (2)

The sum of truth, indeterminacy and falsity membership function will lie between 0 and $w_1 + w_2 + w_3$

Proof:

Let the truth, indeterminacy and falsity membership functions be defined as membership functions

$$T_i^{w_1}(z_i) = \begin{cases} w_1 & \text{if } z_i \le t_i \\ w_1\left(\dfrac{t_i + a_i - z_i}{a_i}\right) & \text{if } t_i \le z_i \le t_i + a_i \\ 0 & \text{if } z_i \ge t_i + a_i \end{cases}$$

$$I_i^{w_2}(z_i) = \begin{cases} 0 & \text{if } z_i \le t_i \\ w_2\left(\dfrac{z_i - t_i}{d_i}\right) & \text{if } t_i \le z_i \le t_i + a_i \\ w_2\left(\dfrac{t_i + a_i - z_i}{a_i - d_i}\right) & \text{if } t_i + d_i \le z_i \le t_i + a_i \\ 0 & \text{if } z_i \ge t_i + a_i \end{cases}$$

$$F_i^{w_3}(z_i)=\begin{cases} 0 & if\ z_i \le t_i \\ w_3\left(\dfrac{z_i-t_i}{c_i}\right) & if\ t_i \le z_i \le t_i+c_i \\ w_3 & if\ z_i \ge t_i+c_i \end{cases}$$

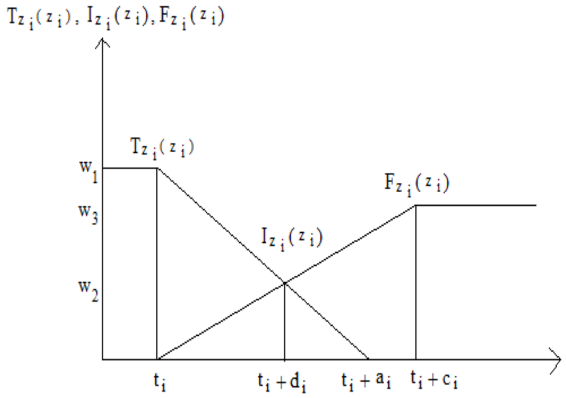

Fig. 1. Truth membership, Indeterminacy membership and Falsity membership function of z_i

From Fig. (1) and definition of generalized single valued neutrosophic set, it is clear that:

$0 \le T_{z_i}(z_i) \le w_1, 0 \le I_{z_i}(z_i) \le w_2$ and $0 \le F_{z_i}(z_i) \le w_1$

when $(z_i) \le t_i$

$T_{z_i}(z_i) = w_1$ and $I_{z_i}(z_i) = 0$ and $F_{z_i}(z_i) = 0$

Therefore $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) = w_1 \le w_1+w_2+w_3$

and $w_1 \ge 0$ implies that $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \ge 0$

when $z_i \in (t_i, t_i+a_i)$ from fig (A) we see that $T_{z_i}(z_i)$ and $F_{z_i}(z_i)$ intersects each other and the point whose coordinate is $(t_i+d_i, d_i c_i)$,

where $d_i = \dfrac{w_1}{\dfrac{w_1}{a_i}+\dfrac{w_2}{c_i}}$.

Now in the interval $z_i \in (t_i, t_i+d_i)$ we see that

$$T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) = w_2\left(\frac{z_i-t_i}{d_i}\right) \le w_2 \le w_1+w_2+w_3$$

Again, in the interval $z_i \in (t_i+d_i, t_i+a_i)$ we see that

$$T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) = w_2\left(\frac{t_i+a_i-z_i}{a_i-d_i}\right) \le w_2 \le w_1+w_2+w_3.$$

Also, for $t_i \le z_i \le t_i+a_i$

when $z_i \ge t_i$, $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) > w_2 \ge 0$ and

$T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) > w_1 \ge 0$ and when

$z_i \le t_i+a_i$, $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \le w_1 \dfrac{a_i}{c_i} < w_1 \le w_1+w_2$

(as $\dfrac{a_i}{c_i} \le 1$).

In the interval $z_i \in (t_i+a_i, t_i+c_i]$

when $z_i > t_i+a_i$, $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) > w_2 \dfrac{a_i}{c_i} > w_2 \ge 0$

(as $\dfrac{a_i}{c_i} \le 1$)

and when

$z_i \le t_i+c_i$, $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \le w_1 \le w_1+w_2+w_3$

for $z_i > t_i+c_i$,

$T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) = w_3 \le w_1+w_2+w_3$

and as $w_2 \ge 0$, $T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \ge 0$.

Therefore, combining all the cases we get

$0 \le T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \le w_1+w_2+w_3$

Hence the proof.

4.2. Solution Procedure of Neutrosophic Goal Programming Technique

In fuzzy goal programming, Zimmermann [18] has given a concept of considering all membership functions greater than a single value α which is to be maximized. Previously many researcher like Bharti and Singh [20], Parvathi and Malathi [19] have followed him in intutionistic fuzzy optimization. Along with the variable α and β, γ is optimized in neutrosophic goal programming problem.

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming problem (2) can be formulated as:

$$Maximize\ T_{z_i}(z_i),\ i=1,2,....,k \qquad (3)$$

$$Maximize\ I_{z_i}(z_i),\ i=1,2,....,k$$

$$Minimize\ F_{z_i}(z_i),\ i=1,2,....,k$$

Subject to

$$0 \le T_{z_i}(z_i)+I_{z_i}(z_i)+F_{z_i}(z_i) \le w_1+w_2+w_3,\ i=1,2,....,k$$

$$T_{z_i}(z_i) \ge 0, I_{z_i}(z_i) \ge 0, F_{z_i}(z_i)\ I=1,2,...,k$$

$$T_{z_i}(z_i) \ge I_{z_i}(z_i),\ I=1,2,....,k$$

$$T_{z_i}(z_i) \ge F_{z_i}(z_i),\ i=1,2,...,k$$

$$0 \le w_1+w_2+w_3 \le 3$$

$$w_1, w_2, w_3 \in [0,1]$$

$$g_j(x) \le b_j,\ j=1,2,.....,m$$

$$x_i \ge 0,\ i=1,2,.....,n$$

Now the decision set $\tilde{D}^n$, a conjunction of Neutrosophic objectives and constraints is defined:

$$\tilde{D}^n = \left(\bigcap_{i=1}^{k} \tilde{z}_i^n\right) \bigcap \left(\bigcap_{j=1}^{q} \tilde{g}_j^n\right) = \left\{\left(x, T_{\tilde{D}^n}(x)\right), I_{\tilde{D}^n}(x), F_{\tilde{D}^n}(x)\right\}$$

$$\text{Here } \alpha = T_{\tilde{D}^n}(x) = \min\left\{\begin{array}{l} T_{\tilde{z}_1^n}(x), T_{\tilde{z}_2^n}(x), T_{\tilde{z}_3^n}(x), \ldots\ldots\ldots\ldots T_{\tilde{z}_p^n}(x); \\ T_{\tilde{g}_1^n}(x), T_{\tilde{g}_2^n}(x), T_{\tilde{g}_3^n}(x), \ldots\ldots\ldots\ldots T_{\tilde{g}_q^n}(x) \end{array}\right\} \text{ for all } x \in X$$

$$\gamma = I_{\tilde{D}^n}(x) = \min\left\{\begin{array}{l} I_{\tilde{z}_1^n}(x), I_{\tilde{z}_2^n}(x), I_{\tilde{z}_3^n}(x), \ldots\ldots\ldots\ldots I_{\tilde{z}_p^n}(x); \\ I_{\tilde{g}_1^n}(x), I_{\tilde{g}_2^n}(x), I_{\tilde{g}_3^n}(x), \ldots\ldots\ldots\ldots I_{\tilde{g}_q^n}(x) \end{array}\right\} \text{ for all } x \in X$$

$$\beta = F_{\tilde{D}^n}(x) = \min\left\{\begin{array}{l} F_{\tilde{z}_1^n}(x), F_{\tilde{z}_2^n}(x), F_{\tilde{z}_3^n}(x), \ldots\ldots\ldots\ldots F_{\tilde{z}_p^n}(x); \\ F_{\tilde{g}_1^n}(x), F_{\tilde{g}_2^n}(x), F_{\tilde{g}_3^n}(x), \ldots\ldots\ldots\ldots F_{\tilde{g}_q^n}(x) \end{array}\right\} \text{ for all } x \in X$$

where $T_{\tilde{D}^n}(x), I_{\tilde{D}^n}(x), F_{\tilde{D}^n}(x)$ are truth-membership function, indeterminacy membership function,falsity membership function of neutrosophic decision set respectively .Now using the neutrosophic optimization, problem (2) is transformed to the non-linear programming problem as

$$\text{Maximize } \alpha, \text{Maximize } \gamma, \text{Minimize } \beta \tag{4}$$

$$z_i \le t_i + a_i\left(1 - \frac{\alpha}{w_1}\right), i = 1, 2, \ldots, k$$

$$z_i \ge t_i + \frac{d_i}{w_2}\gamma, i = 1, 2, \ldots, k$$

$$z_i \le t_i + a_i - \frac{\gamma}{w_2}\left(a_i - d_i\right), i = 1, 2, \ldots, k$$

$$z_i \le t_i + \frac{c_i}{w_3}\beta, i = 1, 2, \ldots, k$$

$$z_i \le t_i, i = 1, 2, \ldots.., k$$

$$0 \le \alpha + \beta + \gamma \le w_1 + w_2 + w_3;$$

$$\alpha \in [0, w_1], \gamma \in [0, w_2], \beta \in [0, w_3];$$

$$w_1 \in [0,1], w_2 \in [0,1], w_3 \in [0,1];$$

$$0 \le w_1 + w_2 + w_3 \le 3.$$

Now, based on arithmetic aggregation operator above problem can be formulated as

$$\text{Minimize } \left\{\frac{(1-\alpha) + \beta + (1-\gamma)}{3}\right\} \tag{5}$$

Subjected to the same constraint as (4).

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming, based on geometric aggregation operator can be formulated as:

$$\text{Minimize } \sqrt[3]{(1-\alpha)\beta(1-\gamma)} \tag{6}$$

Subjected to the same constraint as (4).

Now this non-linear programming problem (4 or 5 or 6) can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (1) by generalized neutrosophic goal optimization approach.

5. Solution of Multi-Objective Structural Optimization Problem (MOSOP) by Generalized Neutrosophic Goal Programming Technique

The multi-objective neutrosophic fuzzy structural model can be expressed as :

Minimize $WT(A)$ with target value WT_0 ,truth tolerance a_{WT} ,indeterminacy tolerance d_{WT} and rejection tolerance c_{WT} (7)

minimize $\delta(A)$ with target value δ_0 ,truth tolerance a_{δ_0} ,indeterminacy tolerance d_{δ_0} and rejection tolerance c_{δ_0}

$$\text{subject to } \sigma(A) \le [\sigma]$$

$$A^{\min} \le A \le A^{\max}$$

where $A = [A_1, A_2, \ldots., A_n]^T$ are the design variables for the cross section, n is the group number of design variables for the cross section bar.

To solve this problem we first calculate truth, indeterminacy and falsity membership function of objective as follows:

$$T_{WT}^{w_1}\left(WT(A)\right) = \begin{cases} w_1 & \text{if } WT(A) \le WT_0 \\ w_1\left(\dfrac{WT_0 + a_{WT} - WT(A)}{a_{WT}}\right) & \text{if } WT_0 \le WT(A) \le WT_0 + a_{WT} \\ 0 & \text{if } WT(A) \ge WT_0 + a_{WT} \end{cases}$$

$$I_{WT(A)}^{w_2}\left(WT(A)\right) = \begin{cases} 0 & \text{if } WT(A) \le WT_0 \\ w_2\left(\dfrac{WT(A) - WT_0}{d_{WT}}\right) & \text{if } WT_0 \le WT(A) \le WT_0 + a_{WT} \\ w_2\left(\dfrac{WT_0 + a_{WT} - WT(A)}{a_{WT} - d_{WT}}\right) & \text{if } WT_0 + d_{WT} \le WT(A) \le WT_0 + a_{WT} \\ 0 & \text{if } WT(A) \ge WT_0 + a_{WT} \end{cases}$$

where $d_{WT} = \dfrac{w_1}{\dfrac{w_1}{a_{WT}} + \dfrac{w_2}{c_{WT}}}$

$$F_{WT(A)}^{w_3}\left(WT(A)\right)=\begin{cases}0 & if\ WT(A)\le WT_0\\ w_3\left(\dfrac{WT(A)-WT_0}{c_{WT}}\right) & if\ WT_0\le WT(A)\le WT_0+c_{WT}\\ w_3 & if\ WT(A)\ge WT_0+c_{WT}\end{cases}$$

and

$$T_{\delta(A)}^{w_1}\left(\delta(A)\right)=\begin{cases}w_1 & if\ \delta(A)\le \delta_0\\ w_1\left(\dfrac{\delta_0+a_{\delta_0}-\delta(A)}{a_{\delta_0}}\right) & if\ \delta_0\le \delta(A)\le \delta_0+a_{\delta_0}\\ 0 & if\ \delta(A)\ge \delta_0+a_{\delta_0}\end{cases}$$

$$I_{\delta(A)}^{w_2}\left(\delta(A)\right)=\begin{cases}0 & if\ \delta(A)\le \delta_0\\ w_2\left(\dfrac{\delta(A)-\delta_0}{d_\delta}\right) & if\ \delta_0\le \delta(A)\le \delta_0+a_\delta\\ w_2\left(\dfrac{\delta_0+a_\delta-WT(A)}{a_\delta-d_\delta}\right) & if\ \delta_0+d_\delta\le \delta(A)\le \delta_0+a_\delta\\ 0 & if\ \delta(A)\ge \delta_0+a_\delta\end{cases}$$

$$d_\delta=\frac{w_1}{\dfrac{w_1}{a_\delta}+\dfrac{w_2}{c_\delta}}$$

$$F_{\delta(A)}^{w_3}\left(\delta(A)\right)=\begin{cases}0 & if\ \delta(A)\le \delta_0\\ w_3\left(\dfrac{\delta(A)-\delta_0}{c_\delta}\right) & if\ \delta_0\le \delta(A)\le \delta_0+c_\delta\\ w_3 & if\ \delta(A)\ge \delta_0+c_\delta\end{cases}$$

According to generalized neutrosophic goal optimization technique using truth, indeterminacy and falsity membership function, MOSOP (7) can be formulated as:

Model I

$$Maximize\ \alpha, Maximize\ \gamma, Minimize\ \beta \tag{8}$$

$$WT(A)\le WT_0+a_{WT}\left(1-\frac{\alpha}{w_1}\right),$$

$$WT(A)\ge WT_0+\frac{d_{WT}}{w_2}\gamma,$$

$$WT(A)\le WT_0+a_{WT}-\frac{\gamma}{w_2}\left(a_{WT}-d_{WT}\right),$$

$$WT(A)\le WT_0+\frac{c_{WT}}{w_3}\beta,$$

$$WT(A)\le WT_0,$$

$$\delta(A)\le \delta_0+a_\delta\left(1-\frac{\alpha}{w_1}\right),$$

$$\delta(A)\ge \delta_0+\frac{d_\delta}{w_2}\gamma,$$

$$\delta(A)\le \delta_0+a_\delta-\frac{\gamma}{w_2}\left(a_\delta-d_\delta\right),$$

$$\delta(A)\le \delta_0+\frac{c_\delta}{w_3}\beta,\ \delta(A)\le \delta_0,$$

$$0\le \alpha+\beta+\gamma\le w_1+w_2+w_3;$$

$$\alpha\in[0,w_1],\gamma\in[0,w_2],\beta\in[0,w_3];$$

$$w_1\in[0,1],w_2\in[0,1],w_3\in[0,1];$$

$$0\le w_1+w_2+w_3\le 3;$$

$$g_j(x)\le b_j,\ j=1,2,.....,m$$

$$x_j\ge 0,\ \ j=1,2,....,n$$

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming based on arithmetic aggregation operator can be formulated as:

Model II

$$Minimize\left\{\frac{(1-\alpha)+\beta+(1-\gamma)}{3}\right\} \tag{9}$$

Subjected to the same constraint as (8)

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming based on geometric aggregation operator can be formulated as:

Model -III

$$Minimize\ \sqrt[3]{(1-\alpha)\beta(1-\gamma)} \tag{10}$$

Subjected to the same constraint as (8)

Now these non-linear programming Model-I, II, III can be easily solved through an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (7) by generalized neutrosophic goal optimization approach.

6 Numerical illustration

A well-known three bar planer truss is considered in Fig.2 to minimize weight of the structure $WT(A_1,A_2)$ and minimize the deflection $\delta(A_1,A_2)$ at a loading point of a statistically loaded three bar planer truss subject to stress constraints on each of the truss members.

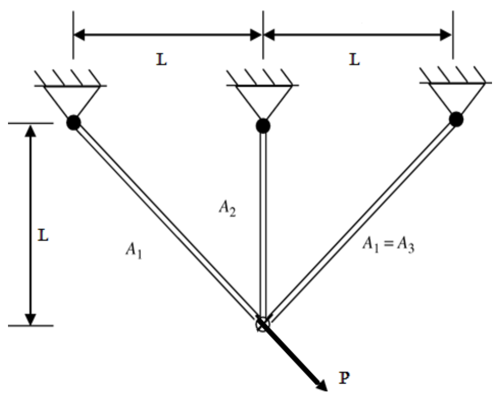

Fig. 2 Design of three bar planar truss

The multi-objective optimization problem can be stated as follows:

$$Minimize \quad WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right) \quad (11)$$

$$Minimize \quad \delta(A_1, A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}$$

Subject to

$$\sigma_1(A_1, A_2) = \frac{P\left(\sqrt{2}A_1 + A_2\right)}{\left(\sqrt{2}A_1^2 + 2A_1A_2\right)} \le \left[\sigma_1^T\right];$$

$$\sigma_2(A_1, A_2) = \frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right];$$

$$\sigma_3(A_1, A_2) = \frac{PA_2}{\left(\sqrt{2}A_1^2 + 2A_1A_2\right)} \le \left[\sigma_3^C\right];$$

$$A_i^{\min} \le A_i \le A_i^{\max} \quad i = 1, 2$$

where $P =$ applied load ; $\rho =$ material density ; $L =$ length ; $E =$ Young's modulus ; $A_1 =$ Cross section of bar-1 and bar-3; $A_2 =$ Cross section of bar-2; δ is deflection of loaded joint. $\left\lfloor \sigma_1^T \right\rfloor$ and $\left\lfloor \sigma_2^T \right\rfloor$ are maximum allowable tensile stress for bar 1 and bar 2 respectively, σ_3^C is maximum allowable compressive stress for bar 3.The input data is given in table1.

This multi objective structural model can be expressed as neutrosophic fuzzy model as

$Minimize \quad WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right)$ with target value $4\times10^2 KN$ truth tolerance $2\times10^2 KN$ indeterminacy tolerance $\frac{w_1}{0.5w_1 + 0.22w_2}\times10^2 KN$ and rejection tolerance $4.5\times10^2 KN$ (12)

$Minimize \quad \delta(A_1, A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}$ with target value $2.5\times10^{-7} m$,truth tolerance $2.5\times10^{-7} m$,indeterminacy tolerance $\frac{w_1}{0.4w_1 + 0.22w_2}\times10^{-7} m$ and rejection tolerance $4.5\times10^{-7} m$

Subject to

$$\sigma_1(A_1, A_2) = \frac{P\left(\sqrt{2}A_1 + A_2\right)}{\left(\sqrt{2}A_1^2 + 2A_1A_2\right)} \le \left[\sigma_1^T\right];$$

$$\sigma_2(A_1, A_2) = \frac{P}{\left(A_1 + \sqrt{2}A_2\right)} \le \left[\sigma_2^T\right];$$

$$\sigma_3(A_1, A_2) = \frac{PA_2}{\left(\sqrt{2}A_1^2 + 2A_1A_2\right)} \le \left[\sigma_3^C\right];$$

$$A_i^{\min} \le A_i \le A_i^{\max} \quad i = 1, 2$$

According to generalized neutrosophic goal optimization technique using truth, indeterminacy and falsity membership function ,MOSOP (12) can be formulated as:

Model I

$$Maximize\ \alpha, Maximize\ \gamma, Minimize\ \beta \quad (13)$$

$$\left(2\sqrt{2}A_1 + A_2\right) \le 4 + 2\left(1 - \frac{\alpha}{w_1}\right),$$

$$\left(2\sqrt{2}A_1 + A_2\right) \ge 4 + \frac{w_1}{w_2\left(0.5w_1 + 0.22w_2\right)}\gamma,$$

$$\left(2\sqrt{2}A_1 + A_2\right) \le 4 + 2 - \frac{\gamma}{w_2}\left(2 - \frac{w_1}{\left(0.5w_1 + 0.22w_2\right)}\right),$$

$$\left(2\sqrt{2}A_1 + A_2\right) \le 4 + \frac{4.5}{w_3}\beta,$$

$$\left(2\sqrt{2}A_1 + A_2\right) \le 4,$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 2.5 + 2.5\left(1 - \frac{\alpha}{w_1}\right),$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \ge 2.5 + \frac{w_1}{w_2\left(0.4w_1 + 0.22w_2\right)}\gamma,$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 2.5 + 2.5 - \frac{\gamma}{w_2}\left(2.5 - \frac{w_1}{\left(0.4w_1 + 0.22w_2\right)}\right),$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 2.5 + \frac{4.5}{w_3}\beta,$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 2.5,$$

$$0 \le \alpha + \beta + \gamma \le w_1 + w_2 + w_3;$$

$$\alpha \in [0, w_1], \gamma \in [0, w_2], \beta \in [0, w_3];$$

$$w_1 \in [0,1], w_2 \in [0,1], w_3 \in [0,1];$$

$$0 \le w_1 + w_2 + w_3 \le 3;$$

$$\frac{20\left(\sqrt{2}A_1 + A_2\right)}{\left(\sqrt{2}A_1^2 + 2A_1A_2\right)} \le 20;$$

$$\frac{20}{\left(A_1 + \sqrt{2}A_2\right)} \le 20;$$

$$\frac{20A_2}{\left(\sqrt{2}A_1^2+2A_1A_2\right)}\le 15;$$

$$0.1\le A_i\le 5 \quad i=1,2$$

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming problem (12) based on arithmetic aggregation operator can be formulated as:

Model II

$$Minimize\left\{\frac{(1-\alpha)+\beta+(1-\gamma)}{3}\right\} \tag{14}$$

Subjected to the same constraint as (13)

With the help of generalized truth, indeterminacy, falsity membership function the generalized neutrosophic goal programming problem (12) based on geometric aggregation operator can be formulated as:

Model III

$$Minimize\ \sqrt[3]{(1-\alpha)\beta(1-\gamma)} \tag{15}$$

Subjected to the same constraint as (13)

The above problem can be formulated using Model I, II, III and can be easily solved by an appropriate mathematical programming to give solution of multi-objective non-linear programming problem (12) by generalized neutrosophic goal optimization approach and the results are shown in the table 2.

Again, value of membership function in GNGP technique for MOSOP (11) based on different Aggregation is given in Table 3.

Table 1: Input data for crisp model (11)

Applied load P (KN)	Volume density ρ (KN/m^3)	Length L (m)	Maximum allowable tensile stress $\lfloor\sigma^T\rfloor$ (KN/m^2)	Maximum allowable compressive stress $\lfloor\sigma^C\rfloor$ (KN/m^2)	Young's modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $(10^{-4}m^2)$
20	100	1	20	15	2×10^7	$A_1^{\min}=0.1$ $A_1^{\max}=5$ $A_2^{\min}=0.1$ $A_2^{\max}=5$

Table 2: Comparison of GNGP solution of MOSOP (11) based on different Aggregation

Methods	A_1 $\times10^{-4}m^2$	A_2 $\times10^{-4}m^2$	$WT(A_1,A_2)$ $\times10^2KN$	$\delta(A_1,A_2)$ $\times10^{-7}m$
Generalized Fuzzy Goal programming(GFGP) $w_1=0.15$	0.5392616	4.474738	6	2.912270
Generalized Intuitionistic Fuzzy Goal programming(GIFGP) $w_1=0.15$ $w_3=0.8$	0.5392619	4.474737	6	2.912270
Generalized Neutrosophic Goal programming (GNGP) $w_1=0.4, w_2=0.3, w_3=0.7$	5	0.4321463	4.904282	3.564332
Generalized Intuitionistic Fuzzy optimization (GIFGP) based on Arithmetic Aggregation $w_1=0.15, w_3=0.8$	0.5392619	4.474737	6	2.912270

Generalized Neutosophic optimization (GNGP) based on Arithmetic Aggregation $w_1 = 0.4, w_2 = 0.3, w_3 = 0.7$	5	0.4321468	4.904282	3.564333
Generalized Intuitionistic Fuzzy optimization (GIFGP) based on Geometric Aggregation $w_1 = 0.15, w_3 = 0.8$	0.5727008	2.380158	4	5.077751
Generalized Neutosophic optimization (GNGP) based on Geometric Aggregation $w_1 = 0.4, w_2 = 0.3, w_3 = 0.7$	5	1.109954	4.462428	3.044273

Here we get best solutions for the different value of w_1, w_2, w_3 in geometric aggregation method for objective functions. From Table 2 it is clear that Neutrosophic Optimization technique is more fruitful in optimization of weight compare to fuzzy and intuitionistic fuzzy optimization technique.

Moreover it has been seen that more desired value is obtain in geometric aggregation method compare to arithmetic aggregation method in intuitionistic as well as neutrosophic environment in perspective of structural engineering.

Table 3: Value of membership function in GNGP technique for MOSOP (11) based on different Aggregation

Methods	$\alpha^*, \beta^*, \gamma^*$	Sum of Truth, Indeterminacy and Falsity Membership Function
Neutrosophic Goal programming (GNGP) $w_1 = 0.4, w_2 = 0.3, w_3 = 0.7$	$\alpha^* = .1814422$ $\beta^* = .2191435$ $\gamma^* = .6013477$	$T_{WT}\left(WT\left(A_1, A_2\right)\right) + I_{WT}\left(WT\left(A_1, A_2\right)\right) + F_{WT}\left(WT\left(A_1, A_2\right)\right)$ $= .2191435 + .1804043 + .1406661 = .5402139$ $T_\delta\left(\delta\left(A_1, A_2\right)\right) + I_\delta\left(\delta\left(A_1, A_2\right)\right) + F_\delta\left(\delta\left(A_1, A_2\right)\right)$ $= .2297068 + .1804043 + .1655628 = .5756739$
Generalized Neutosophic optimization (GNGP) based on Arithmetic Aggregation $w_1 = 0.4, w_2 = 0.3, w_3 = 0.7$	$\alpha^* = .2191435$ $\beta^* = .2191435$ $\gamma^* = .6013480$	$T_{WT}\left(WT\left(A_1, A_2\right)\right) + I_{WT}\left(WT\left(A_1, A_2\right)\right) + F_{WT}\left(WT\left(A_1, A_2\right)\right)$ $= .2191435 + .1804044 + .1406662 = .5402141$ $T_\delta\left(\delta\left(A_1, A_2\right)\right) + I_\delta\left(\delta\left(A_1, A_2\right)\right) + F_\delta\left(\delta\left(A_1, A_2\right)\right)$ $= .2297068 + .1804044 + .1655629 = .5756741$
Generalized Neutosophic optimization (GNGP) based on Geometric Aggregation $w_1 = 0.4, w_2 = 0.3, w_2 = 0.7$	$\alpha^* = .3075145$ $\beta^* = .3075145$ $\gamma^* = .3075145$	$T_{WT}\left(WT\left(A_1, A_2\right)\right) + I_{WT}\left(WT\left(A_1, A_2\right)\right) + F_{WT}\left(WT\left(A_1, A_2\right)\right)$ $= .3075145 + .0922543 + .07193320 = .471702$ $T_\delta\left(\delta\left(A_1, A_2\right)\right) + I_\delta\left(\delta\left(A_1, A_2\right)\right) + F_\delta\left(\delta\left(A_1, A_2\right)\right)$ $= .3129163 + .09225434 + .08466475 = .48983539$

From the above table it is clear that all the objective functions attained their goals as well as restriction of truth, indeterminacy and falsity membership function in neutrosophic goal programming problem based on different aggregation operator.

The sum of truth,indeterminacy and falsity membership function for each objective is less than sum of gradiation $\left(w_1 + w_2 + w_3\right)$. Hence the criteria of generalized neutrosophic set is satisfied.

7. Conclusions

The research study investigates that neutrosophic goal programming can be utilized to optimize a nonlinear structural problem. . The results obtained for different aggregation method of the undertaken problem show that the best result is achieved using geometric aggregation method. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; rather, they are independent with degree of indeterminacy. As we have considered a non-linear three bar truss design problem and find out minimum weight of the structure as well as minimum deflection of loaded joint, the results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in different field.

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