NSS

University of New Mexico

Neutrosophic Graphs of Finite Groups

T.Chalapathi [1] and R. V M S S Kiran Kumar [2]

[1]Assistant Professor, Department of Mathematics, Sree Vidyanikethan Eng.College Tirupati,-517502, Andhra Pradesh, India. Email: chalapathi.tekuri@gmail.com
[2] Research Scholar, Department of Mathematics, S.V.University, Tirupati,-517502, Andhra Pradesh, India. Email: kksaisiva@gmail.com

Abstract: Let G be a finite multiplicative group with identity e and $N(G)$ be the Neutrosophic group with indeterminate I. We denote by $Ne(G, I)$, the Neutrosophic graph of G, $N(G)$ and I. In this paper, we study the graph $Ne(G, I)$ and its properties. Among the results, it is shown that for any finite multiplicative group G, $Ne(G, I)$ is a connected graph of diameter less than or equal to 2. Moreover, for finite group G, we obtain a formula for enumerating basic Neutrosophic triangles in $Ne(G, I)$. Furthermore, for every finite groups G and G', we show that $G \cong G'$ if and only if $N(G) \cong N(G')$, and if $N(G) \cong N(G')$, then $Ne(G, I) \cong Ne(G', I')$.

Keywords: Indeterminacy; Finite Multiplicative group; Neutrosophic Group; Basic Neutrosophic triangle; Neutrosophic group and graph isomorphism.

1 Introduction

Most of the real world problems in the fields of philosophy, physics, statistics, finance, robotics, design theory, coding theory, knot theory, engineering, and information science contain subtle uncertainty and inconsistent, which causes complexity and difficulty in solving these problems. Conventional methods failed to handle and estimate uncertainty in the real world problems with near tendency of the exact value. The determinacy of uncertainty in the real world problems have been great challenge for the scientific community, technological people, and quality control of products in the industry for several years. However, different models or methods were presented systematically to estimate the uncertainty of the problems by various incorporated computational systems and algebraic systems. To estimate the uncertainty in any system of the real world problems, first attempt was made by the Lotfi A Zadesh [1] with help of Fuzzy set theory in 1965. Fuzzy set theory is very powerful technique to deal and describe the behavior of the systems but it is very difficult to define exactly. Fuzzy set theory helps us to reduce the errors of failures in modeling and different fields of life. In order to define system exactly, by using Fuzzy set theory many authors were modified, developed and generalized the basic theories of classical algebra and modern algebra. Along with Fuzzy set theory there are other different theories have been study the properties of uncertainties in the real world problems, such as probability theory, intuitionistic Fuzzy set theory, rough set theory, paradoxist set theory [2-5]. Finally, all above theories contributed to explained uncertainty and inconsistency up to certain extent in real world problems. None of the above theories were not studied the properties of indeterminacy of the real world problems in our daily life. To analyze and determine the existence of indeterminacy in various real world problems, the author Smarandache [6] introduced philosophical theory such as Neutrosophic theory in 1990.

Neutrosophic theory is a specific branch of philosophy, which investigates percentage of Truthfulness, falsehood and neutrality of the real world problem. It is a generalization of Fuzzy set theory and intuitionistic Fuzzy set theory. This theory is considered as complete representation of a mathematical model of a real world problem. Consequently, if uncertainty is involved in a problem we use Fuzzy set theory, and if indertminancy is involved in a problem we essential Neutrosophic theory.

Kandasamy and Smarandache [7] introduced the philosophical algebraic structures, in particular, Neutrosophic algebraic structures with illustrations and examples in 2006 and initiated the new way for the emergence of a new class of structures, namely, Neutrosophic groupoids, Neutrosophic groups, Neutrosophic rings etc. According to these authors, the Neutrosophic algebraic structures N(I) was a nice composition of indeterminate I and the elements of a given algebraic structure $(N, *)$. In particular, the new algebraic structure $(N(I), *)$ is called Neutrosophic algebraic structure which is generated by N and I.

In [8], Agboola and others have studied some properties of Neutrosophic group and subgroup. Neutrosophic group denoted by $(N(G), \cdot)$ and defined by $N(G) = \langle G \cup I \rangle$, where G is a group with respect to multiplication. These authors also shown that all Neutrosophic groups generated by the Neutrosophic element I and any group isomorphic to Klein 4-group are Lagrange Neutrosophic groups.

Recent research in Neutrosophic algebra has concerned developing a graphical representation of the elements of a given finite Neutrosophic set, and then graph theoretically developing and analyzing the depiction to research Neutrosophic algebraic conclusions about the finite Neutrosophic set. The most well-known of these models is the Neutrosophic graph of Neutrosophic set, first it was introduced by Kandasamy and Smarandache [9].

Recently, the authors Kandasamy and Smarandache in [9-10] have introduced Neutrosophic graphs, Neutrosophic edge graphs and Neutrosophic vertex graphs, respectively. If the edge values are from the set $\langle G \cup I \rangle$ they will termed as Neutrosophic graphs, and a Neutrosophic graph is a graph in which at least one edge is indeterminacy. Let $V(G)$ be the set of all vertices of G. If the edge set $E(G)$, where at least one of the edges of G is an indeterminate one. Then we call such graphs as a Neutrosophic edge graphs. Further, a Neutrosophic vertex graph G_N is a graph G with finite non empty set $V_N = V_N(G)$ of $p-$points where at least one of the point in $V_N(G)$ is indeterminate vertex. Here $V_N(G) = V(G) + N$, where $V(G)$ are vertices of the graph G and N the non empty set of vertices which are indeterminate.

In the present paper, indeterminacy of the real world problems are expressed as mathematical model in the form of new algebraic structure $(GI, \cdot)$, and its properties are studied in second section, where G is finite group with respect to multiplication and I indeterminacy of the real world problems.

In the third section, to find the relation between G, I and N (G) we introduced Neutrosophic graph $Ne(G, I)$ of the Neutrosophic group $(N(G), \cdot)$, by studying its important concrete properties of these graphs.

In the fourth section, we introduced basic Neutrosophic triangles in the graph $Ne(G, I)$ and obtained a formula for enumerating basic Neutrosophic triangles in $Ne(G, I)$ to understand the internal mutual relations between the elements in G, I and N (G).

In the last section, all finite isomorphic groups G and G' such that $N(G) \cong N(G')$ and $Ne(G, I) \cong Ne(G', I')$ are characterized with examples.

Throughout this paper, all groups are assumed to be finite multiplicative groups with identity e. Let $N(G)$ be a Neutrosophic group generated by G and I. For classical theorems and notations in algebra and Neutrosophic algebra, the interest reader is refereed to [11] and [8].

Let X be a graph with vertex set $V(X)$ and edge set $E(X)$. The cardinality of $V(X)$ and $E(X)$ are denoted by $|V(X)|$ and $|E(X)|$, which are order and size of X, respectively. If X is connected, then there exist a path between any two vertices in X. We denote by K_n the complete graph of order n. Let $u \in V(X)$. Then degree of u, $\deg(u)$ in X is the number of edges incident at u. If $\deg(u) = 1$ then the vertex u is called pendent. The girth of X is the length of smallest cycle in X. The girth of X is infinite if X has no cycle. Let $d(x, y)$ be the length of the shortest path from two vertices x and y in X, and the diameter of X denoted by

$$Diam(X) = \max\{d(x, y) : x, y \in V(X)\}.$$

For further details about graph theory the reader should see [12].

2 Basic Properties of Neutrosophic set and GI

This section will present a few basic concepts of Neutrosophic set and Neutrosophic group that will then be used repeatedly in further sections, and it will introduce a convenient notations. A few illustrations and examples will appear in later sections.

Neutrosophic set is a mathematical tool for handling real world problems involving imprecise, inconsistent data and indeterminacy; also it generalizes the concept of the classic set, fuzzy set, rough set etc. According to authors Vasantha Kandasamy and Smarandache, the Neutrosophic set is a nice composition of an algebraic set and indeterminate element of the real world problem.

Let N be a non-empty set and I be an indeterminate. Then the set $N(I)=\langle N\cup I\rangle$ is called a Neutrosophic set generated by N and I .If '$\cdot$' is usual multiplication in N, then I has the following axioms.

1. $0\cdot I=0$
2. $1\cdot I=I=I\cdot 1$
3. $I^2=I$
4. $a\cdot I=I\cdot a$, for every $a\in N$.
5. I^{-1} does not exist.

For the definition, notation and basic properties of Neutrosophic group, we refer the reader to Agbool [8]. As treated in [8], we shall denote the finite Neutrosophic group by $N(G)$ for a group G.

Definition 2.1 Let G be any finite group with respect to multiplication. Then the set GI defined as $GI=\{gI: g\in G\}=\{Ig: g\in G\}$.

Definition 2.2 If a map f from a finite nonempty set S into a finite nonempty set S' is both one-one and onto then there exist a map g from S' into S that is also one-one and onto. In this case we say that the two sets are equivalent, and, abstractly speaking, these sets can be regarded as the same cardinality. We write $S\sim S'$ whenever there is a one-one map of a set S onto S'.

Two finite rings R and R' are equivalent if there is a one-one correspondence between R and R'. We write $R\sim R'$.

Definition 2.3 Let G be any finite group with respect to multiplication and let $N(G)=\langle G\cup I\rangle$ .Then $(N(G),\cdot)$ is called a Neutrosophic group generated by G and I under the binary operation '$\cdot$'on G. The Neutrosophic group $N(G)$ has the following properties.

1. $N(G)$ is not a group.
2. $G\subset N(G)$.
3. $GI\subset N(G)$.
4. $N(G)$ is a specific composition of G and I.

Lemma 2.4 Let G be any finite group with respect to multiplication and $I^2=I$. Then $G\sim GI$. In particular, $|G|=|GI|$.

Proof. For any finite group G, we have $G\neq GI$ and $GI\not\subset G$. Now define a map $f: G\to GI$ by the relation $f(a)=aI$ for every $a\in I$. Let $a, b\in G$. Then $a=b \Leftrightarrow a-b=0 \Leftrightarrow (a-b)I=0I \Leftrightarrow aI=bI \Leftrightarrow f(a)=f(b)$ .This shows that f is a well defined one-one function. Further, we have

$$\text{Range}(f)=\{f(a)\in GI: a\in G\}$$
$$=\{aI\in GI: a\in G\}=GI.$$

This show that for every $aI\in GI$ at least one $a\in G$ such that $f(a)=aI$.

Therefore, $f: G\to GI$ is one-one correspondence and consequently a bijective function. Hence $G\sim GI$.

Lemma 2.5 Let G be any finite group with respect to multiplication and let $N(G)=\langle G\cup I\rangle$. Then the order of $N(G)$ is $2|G|$.

Proof: We have $GI=\{gI: g\in G\}$. Obviously, $GI\not\subset G$ and $G\not\subset GI$ but $GI\subset N(G)$. It is clear that $N(G)$ is the disjoint union of G and GI .That is,

$$N(G)=G\cup GI \text{ and } G\cap GI=\phi.$$

Therefore, $|N(G)|=|G|+|GI|=2|G|$, since $|G|=|GI|$.

Lemma 2.6 The set GI is not Neutrosophic group with respect to multiplication of group G.

Proof: It is obvious, since $GI\neq\langle G\cup I\rangle$.

Lemma 2.7 The elements in GI satisfies the following properties,

1. $e\cdot gI=gI$
2. $(gI)^2=g^2I$
3. $\underbrace{gI\cdot gI\ldots\cdot gI}_{n \text{ terms}}=g^nI$ for all positive integers n.
4. $(gI)^{-1}$ does not exist, since I^{-1} does not exist.
5. $gI=g'I\Leftrightarrow g=g'$.

Proof: Directly follows from the results of the group $(N(G),\cdot)$.

Theorem 2.8 The structure $(GI, \cdot)$ is a monoid under the operation $(aI)(bI) = abI$ for all a, b in the group $(G, \cdot)$ and $I^2 = I$.

Proof: We know that $GI = \{gI : g \in G\}$. Let aI , bI and cI be any three elements in GI . Then the binary operation $(aI)(bI) = abI$ in $(GI, \cdot)$ satisfies the following axioms.

1. $abI \in GI \Rightarrow (aI)(bI) \in GI$.
2. $[(aI)(bI)](cI) = [(ab)I](cI)$
   $= [(ab)c)]I = [a(bc))]I = aI[(bI)(cI)]$
3. Let e be the identity element in $(G, \cdot)$.Then $eI = I = Ie$ and $I(aI) = aI^2 = aI = (aI)I$.

Remark 2.9 The structure $(GI, \cdot)$ is never a group because I^{-1} does not exist.
Here we obtain lower bounds and upper bounds of the order of the Neutrosophic group $N(G)$. Moreover, these bounds are sharp.

Theorem 2.10 Let G be a finite group with respect to multiplication. Then,

$$1 \le |G| \le n \Leftrightarrow 2 \le |N(G)| \le 2n.$$

Proof. We have,
$|G| = 1 \Leftrightarrow G = \{e\} \Leftrightarrow N(G) = G \cup GI = \{e, I\}$
$\Leftrightarrow |N(G)| = 2$. This is one extreme of the required inequality. For other extreme, by the Lemma [2.4],

$|G| > 1 \Leftrightarrow |GI| > 1$

$\Leftrightarrow |G| + |GI| > 2$ and $|G| + |GI|$ is not odd

$\Leftrightarrow |G| + |GI|$ is even.

$\Leftrightarrow |N(G)| = |G| + |GI| = 2n$.

Hence, the theorem.

3 Basic Properties of Neutrosophic Graph

In this section, our aim is to introduce the notion and definition of Neutrosophic graph of finite Neutrosophic group with respect to multiplication and study on its basic and specific properties such as connectedness, completeness, bipartite, order, size, number of pendent vertices, girth and diameter.

Definition 3.1 A graph $Ne(G, I)$ associated with Neutrosophic group $(N(G), \cdot)$ is undirected simple graph whose vertex set is $N(G)$ and two vertices x and y in $N(G)$ if and only if xy is either x or y .

Theorem 3.2 For any group $(G, \cdot)$, the Neutrosophic graph $Ne(G, I)$ is connected.

Proof: Let e be the identity element in G .Then $e \in N(G)$, since $G \subset N(G)$. Further, $xe = x$, for every $x \ne e$ in $N(G)$. It is clear that the vertex e is adjacent to all other vertices of the graph $Ne(G, I)$. Hence $Ne(G, I)$ is connected.

Theorem 3.3 Let $|G| > 1$.Then the graph has at least one cycle of length 3.

Proof: Since $|G| > 1$ implies that $|N(G)| \ge 4$. So there is at least one vertex gI of $N(G)$ such that gI is adjacent to the vertices e and I in $Ne(G, I)$,since $eI = I$, $I(gI) = gI^2 = gI$ and $(gI)e = geI = gI$. Hence we have the cycle $e - I - gI - e$ of length 3, where $g \ne e$.

Example 3.4 Since

$$N(G_{10}) = \{2, 4, 6, 8, 2I, 4I, 6I, 8I\}$$

is the Neutrosophic group of the group $G_{10} = \{2, 4, 6, 8\}$ with respect to multiplication modulo 10, where $e = 6$ .The Neutrosophic graph $Ne(G_{10}, I)$ contains three cycles of length 3, which are listed below.

$$C_1 : 6 - I - 2I - 6,$$
$$C_2 : 6 - I - 4I - 6,$$
$$C_3 = 6 - I - 8I - 8.$$

Theorem 3.5 The Neutrosophic graph $Ne(G, I)$ is complete if and only if $|G| = 1$.

Proof: **Necessity.** Suppose that $Ne(G, I)$ is complete. If possible assume that $|G| > 1$, then $|N(G)| \ge 4$. So without loss of generality we may assume that $|N(G)| = 4$ and clearly the vertices

$e, g, I, gI \in V(Ne(G, I))$ .Therefore the vertex g is not adjacent to the vertex I in $Ne(G, I)$, since $gI \neq g$ or I for each $g \neq e$ in G, this contradicts our assumption that $Ne(G, I)$ is complete. It follows that $|N(G)|$ cannot be four. Further, if $|N(G)| > 4$, then obviously we arrive a contradiction. So our assumption is wrong , and hence $|G| = 1$.

Sufficient. Suppose that $|G| = 1$. Then, trivially $|N(G)| = 2$ .Therefore, $Ne(G, I) \cong K_2$, since $eI = I$. Hence, $Ne(G, I)$ is a complete graph.

Recall that $|V(Ne(G, I))|$ is the order and $|E(Ne(G, I))|$ is the size of the Neutrosophic graph $Ne(G, I)$. But,

$$|V(Ne(G, I))| = |N(G)| = 2|G|$$

and the following theorem shows that the size of $Ne(G, I)$.

Theorem 3.6 The size of Neutrosophic graph $Ne(G, I)$ is $3|G| - 2$.

Proof: By the definition of Neutrosophic graph, $Ne(G, I)$ contains $2(|G|-1)^2$ non adjacent pairs. But the number of combinations of any two distinct pairs from $N(G)$ is $\binom{|N(G)|}{2}$. Hence the total number of adjacent pairs in $Ne(G, I)$ is

$$|E(Ne(G, I))| = \binom{|N(G)|}{2} - 2(|G|-1)^2$$

$$= 3|G| - 2.$$

Theorem 3.7 [11] The size of a simple complete graph of order n is $\frac{1}{2}n(n-1)$.

Corollary 3.8 The Neutrosophic graph $Ne(G, I)$, $|G| > 1$ is never complete.

Proof: Suppose on contrary that $Ne(G, I)$, $|G| > 1$ is complete. Then, by the Theorem [3.7], the total number of edges in $Ne(G, I)$ is $\frac{1}{2}(2|G|(2|G|-1)) = |G|(2|G|-1)$, but in view of Theorem [3.6], we arrived a contradiction to the completeness of $Ne(G, I)$.

Theorem 3.9 The graph $Ne(G, I)$ has exactly $|G| - 1$ pendent vertices.

Proof: Since $N(G) = G \cup GI$ and $G \cap GI = \phi$. Let $x \in N(G)$. Then either $x \in G$ or $x \in GI$. Now consider the following cases on GI and G, respectively.

Case 1. If $x \in GI$, then $x = gI$ for $g \in G$. But $xI = (gI)I = gI^2 = gI = x$ and $ex = egI = gI = x$. This implies that the vertex x is adjacent to both the vertices e and I in $N(G)$ .Hence $\deg(x) \neq 1$ for every $x \in GI$.

Case 2. If $x \in G$, then $ex = x$,for every $x \neq e$ and $egI = gI$, for every $gI \in GI$. Therefore $\deg(e) = |N(G)| - 1 \neq 1$. Now show that $\deg(x) = 1$, for every $x \neq e$ in G. Suppose, $\deg(x) > 1$, for every $x \neq e$ in G. Then there exist another vertex $y \neq e$ in G such that either $xy = x$ or y, this is not possible in G, because G is a finite multiplication group. Thus $\deg(x) = 1$, for $x \neq e$ in G.

From case (1) and (2), we found the degree of each non identity vertex in G is 1. This shows that each and every non identity element in G is a pendent vertex in $Ne(G, I)$. Hence, the total number of pendent vertices in $Ne(G, I)$ is $|G| - 1$.

The following result shows that $Ne(G, I)$ is never a traversal graph.

Corollary 3.10 Let $|G| > 1$. Then $Ne(G, I)$ is never Eulerian and never Hamiltonian.

Proof. It is obvious from the Theorem [3.9].

Theorem 3.11 [11] A simple graph is bipartite if and only it does not have any odd cycle.

Theorem 3.12 The Neutrosophic graph $Ne(G, I)$, $|G| > 1$ is never bipartite.

Proof. Assume that $|G|>1$. Suppose, $Ne(G, I)$ is a bipartite graph. Then there exist a bipartition (G, GI), since $N(G)=G\cup GI$ and $G\cap GI=\phi$. But $e\in G$ and $I\in GI$, where $e\neq I$. So there exist at least one vertex gI in $Ne(G, I)$ such that $e-I-gI-e$ is an odd cycle of length 3 because $eI=I,\ \ I(gI)=gI$ and $(gI)e=gI$.
This violates the condition of the Theorem [3.11].
Hence $Ne(G, I)$ is not a bipartite graph.

Theorem 3.13 The girth of a Neutrosophic graph is 3.

Proof. In view of Theorem [3.3], for $|G|>1$, we always have a cycle $e-I-gI-e$ of length 3, for each $g\neq e$ in G, which is smallest in $Ne(G, I)$.
This completes the proof.

Remark 3.14 Let G be a finite group with respect to multiplication. Then $gir\,(Ne(G, I))=\infty$ if $|G|=1$, since $Ne(G, I)$ is acyclic graph if and only if $|G|=1$.

Theorem 3.15 $\text{Diam}\,(Ne(G, I))\leq 2$.

Proof. Let G be a finite group with respect to multiplication. Then we consider the following two cases.

Case 1 Suppose $|G|=1$. The graph $Ne(G, I)\cong K_2$. It follows that $Ne(G, I)$ is complete, so $\text{diam}\,(Ne(G, I))=1$.

Case 2 Suppose $|G|>1$. Then the vertex e is adjacent to every vertex of $Ne(G, I)$. However the vertex aI is not adjacent to bI for all $a\neq b$ in G, so $d(aI, bI)>1$. But in $Ne(G, I)$, there always exist a path $aI-I-bI$, since $(aI)I=aI$ and $I(bI)=bI$, which gives $d(aI, bI)=2$, for every aI ,$bI\in N(G)$.
Hence, both the cases conclude that:
$\text{Diam}\,(Ne(G, I))\leq 2$.

4 Enumeration of basic Neutrosophic triangles in $Ne(G, I)$

Since $Ne(G, I)$ is triangle free graph for $|G|=1$, we will consider $|G|>1$ in this section.

Let us denote a traingle by (x, y, z) in $Ne(G, I)$ with vertices x, y and z . Without loss of generality we may assume that our triangles (e, I, gI) have vertices e , I and gI , where $g\neq e$ in G . These triangles are called basic Neutrosophic triangles in $Ne(G, I)$, which are defined as follows.

Definition 4.1 A triangle in the graph $Ne(G, I)$ is said to be basic Neutrosophic if it has the common vertices e and I .The set of all basic Neutrosophic triangles in $Ne(G, I)$ denoted by

$$T_{eI}=\{(e, I, gI): g\neq e \text{ in } G\}.$$

A triangle (x, y, z) in $Ne(G, I)$ is called non-basic Neutrosophic if $(x, y, z)\notin T_{eI}$.

The following short table illustrates some finite Neutrosophic graphs and their total number of basic Neutrosophic triangles.

$Ne(G, I)$	$Ne(Z_p^*, I)$	$Ne(C_n, I)$	$Ne(G_{2p}, I)$	$Ne(V_4, I)$
$\lvert T_{eI}\rvert$	$p-2$	$n-1$	$p-2$	3

where $Z_p^*=Z_p-\{0\}$ is a group with respect to multiplication modulo p , a prime,
$C_n=\{1, g, g^2,..., g^{n-1}: g^n=1\}$
is a cyclic group generated by g with respect to multiplication,
$G_{2p}=\{0, 2, 4,..., 2(p-1)\}$
is a group with respect to multiplication modulo *2p* and
$V_4=\{e, a, b, c: a^2=b^2=c^2=e\}$
is a Klein 4-group.

Before we continue, it is important to note that the multiplicative identity e may differ from group to group. However, for simplicity sake we will continue to notate that $e=1$, and we leave it to reader to understand from context of the group for e .

The following results give information about enumeration of basic and non-basic Neutrosophic triangles in the graph $Ne(G, I)$.

First we begin a lemma, which gives a formula for enumerating the number of Neutrosophic triangles in $Ne(G, I)$ corresponding to fixed elements e and I in the Neutrosophic set $N(G)$.

This is useful for finding the total number of non-basic Neutrosophic triangles in $Ne(G, I)$.

Theorem 4.2 Let $|G|>1$. Then the total number of basic Neutrosophic triangles in $Ne(G, I)$ is $|T_{eI}|=|G|-1$.

Proof. Since $N(G)=G\cup GI$ and $G\cap GI=\phi$. It is clear that $e\neq I$. For any $aI\in GI$, the traid $(e, I, aI)\in T_{eI} \Leftrightarrow (e, I), (e, aI),$ and (I, aI) are edges in $Ne(G, I)$

$$\Leftrightarrow eI=I,\ e(aI)=aI, I(aI)=aI$$
$$\Leftrightarrow I, aI\in GI \text{ , where } a\neq e \text{ in } G.$$

That is, for fixed vertices e , I and for each $aI\in GI$, the traid (e, I, aI) exists in $Ne(G, I)$. Further, for any vertex $a\in G$,the vertices e , I and a does not form a triangle in $Ne(G, I)$ because (I, a) is not an edge in $Ne(G, I)$, since $aI\neq a$ or I for all $a\neq e$. So that the total number of triangles having common verities e and I in $Ne(G, I)$ is

$$|T_{eI}|=|N(G)|-(|G|+1)$$
$$=2|G|-(|G|+1)=|G|-1.$$

Theorem 4.3 The total number of non-basic Neutrosophic triangles in $Ne(G, I)$ is zero.

Proof. Suppose that two vertices either x, y or $y,\ z$ or $z,\ x$ are not equal to e and I .

Then the traid $(x,\ y,\ z)$ is a non-basic triangle in

$$Ne(G, I)\Leftrightarrow (x,\ y,\ z)\notin T_{eI}$$
$$\Leftrightarrow xy=x,\ yz=y \text{ and } zx=z$$
$$\Leftrightarrow \text{either } xyzx=x \text{ or } yzxy=y \text{ or } zxyz=z .$$

This is not possible in the Neutrosophic group $N(G)$. Thus there is no any non-basic triangle in the graph $Ne(G, I)$, and hence the total number of non-basic Neutrosophic triangles in $Ne(G, I)$ is zero.

In view of Theorems [3.9] and [4.2], the following theorem is obvious.

Theorem 4.4 The total number of pendent vertices and basic Neutrosophic triangles in $Ne(G, I)$ is same, which is equal to $|G|-1$.

5 Isomorphic properties of Neutrosophic groups and graphs

In this section we consider important concepts known as isomorphism of groups and Neutrosophic groups. But the notion of isomorphism is common to all aspects of modern algebra [14] and Neutrosophic algebra. An isomorphism of groups and Neutrosophic groups are maps which preserves operations and structures. More precisely we have the following definitions which we make for finite groups and Neutrosophic finite groups.

Definition.5.1 Two finite groups G and G' are said to be isomorphic if there is a one-one correspondence $f:G\to G'$ such that $f(ab)=f(a)f(b)$ for all $a,b\in G$ and we write $G\cong G'$.

Now we proceed on to define isomorphism of finite Neutrosophic groups with distinct indeterminate, which can be defined over distinct groups with same binary operation. We can establish two main results.

1. Two groups are isomorphic and their Neutrosophic groups are also isomorphic.
2. If two Neutrosophic groups are isomorphic, then their Neutrosophic graphs are also isomorphic.

Definition 5.2 Let $(G, \cdot)$ and $(G', \cdot)$ be two finite groups and let $I\neq I'$ be two indeterminates of two distinct real world problems. The Neutrosophic groups $N(G)=(\langle G\cup I\rangle, \cdot)$ and $N(G')=(\langle G'\cup I'\rangle, \cdot)$ are isomorphic if there exist a group isomorphism φ from G onto G' such that $\varphi(I)=I'$ and we write $N(G)\cong N(G')$.

Definition 5.3 [13] If there is a one-one mapping $a\leftrightarrow a'$ of the elements of a group G onto those a group G' and if $a\leftrightarrow a'$ and $b\leftrightarrow b'$ implies $ab\leftrightarrow a'b'$, then we say that G and G' are isomorphic and write $G\cong G'$. If we put $a'=f(a)$ and $b'=f(b)$ for $a, b\in G$, then $f:G\to G'$ is a bijection satisfying $f(ab)=a'b'=f(a)f(b)$.

Lemma 5.4 $G\cong G' \Leftrightarrow N(G)\cong N(G')$.

Proof. Necessity. Suppose $G\cong G'$. Then there exist a group isomorphism φ from G onto G' such that $\varphi(a)=a'$ for every $a\in G$ and $a'\in G'$. By the definition [12], the relation says that φ sends ab onto $a'b'$, where $a'=\varphi(a)$ and $b'=\varphi(b)$ are the elements of

G' one-one corresponding to the elements a, b in G. We will prove that $N(G) \cong N(G')$. For this we define a map $f: N(G) \to N(G')$ by the relation $f(G) = G'$, $f(I) = I'$ and $f(GI) = G'I'$.

Suppose $x, y \in N(G)$.

Then either $x, y \in G$ or $x, y \in GI$. Now consider the following two cases.

Case 1 Suppose $x, y \in G$.

Then $x \leftrightarrow x'$ and $y \leftrightarrow y'$.

Trivially, $f(x) = x' = \varphi(x)$, for every $x \in G$ and $x' \in G'$, since $G \cong G'$. Thus, $N(G) \cong N(G')$.

Case 2 Suppose $x, y \in GI$.

Then $x = aI$ and $y = bI$ for $a, b \in G$. Obviously, f is one-one correspondence between $N(G)$ and $N(G')$, since $G \cong G'$ and $f(I) = I'$. Further,

$$f(xy) = f((aI)(bI))$$
$$= f(abI) = a'b'I',$$
since $f(GI) = G'I'$
$$= (a'I')(b'I')$$
$$= f(aI) f(bI) = f(x) f(y).$$

Thus f is a Neutrosophic group isomorphism from $N(G)$ onto $N(G')$, and hence $N(G) \cong N(G')$.

Sufficiency. It is similar to necessity, because $\langle G \cup I \rangle \cong \langle G' \cup I' \rangle$ implies that $G \cong G'$ and $GI \cong G'I'$ under the mapping $a \leftrightarrow a'$ and $aI \leftrightarrow a'I'$, respectively.

Theorem 5.5 If $G \cong G'$, then $Ne(G, I) \cong Ne(G', I')$, where $I \neq I'$.

But converse is not true.

Proof. Suppose $N(G) = \langle G \cup I \rangle$ and $N(G') = \langle G' \cup I' \rangle$ be two different Neutrosophic groups generated by G, I and G', I', respectively.

Let φ be an isomorphism from G onto G'. Then φ is one-one correspondence between the graphs $Ne(G, I)$ and $Ne(G', I')$ under the relation $\varphi(x) = x'$ for every $x \in N(G)$ and $x' \in N(G')$. Further to show that φ preserves the adjacency. For this let x and y be any two vertices of the graph $Ne(G, I)$, then x, $y \in N(G)$. This implies that

$$(x, y) \in E(Ne(G, I)) \Leftrightarrow xy = x$$
$$\Leftrightarrow \varphi(x)\varphi(y) = \varphi(x) \Leftrightarrow x'y' = x'$$
$$\Leftrightarrow (x', y') \in E(Ne(G', I')).$$

Hence, G and G' are adjacent in $Ne(G', I')$. similarly, φ maps non-adjacent vertices to non-adjacent vertices. Thus, φ is a Neutrosophic graph isomorphism from $Ne(G, I)$ onto $Ne(G', I')$, that is, $Ne(G, I) \cong Ne(G', I')$.

The converse of the Theorem [5.5] is not true, in general. Let $G = V_4$ and let $G' = Z_5^*$. Clearly, $Ne(G, I) \cong Ne(G', I')$, but V_4 is not isomorphic to Z_5^*.

This is illustrated in the following figure.

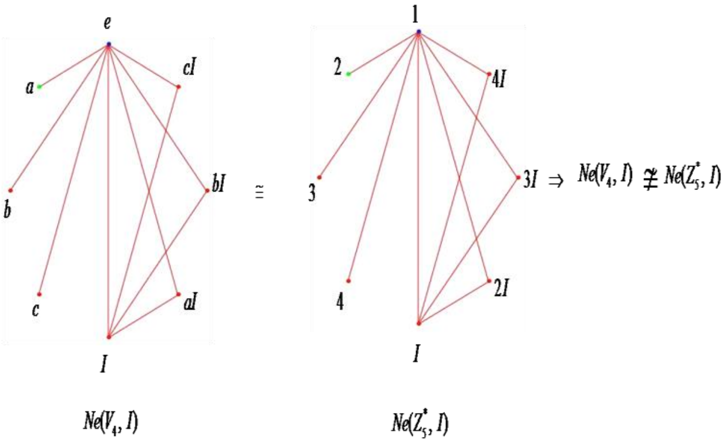

Figure Neutrosophic graphs of V_4 and Z_5^*

Acknowledgments

The authors express their sincere thanks to Prof.L. Nagamuni Reddy and Prof.S.Vijaya Kumar Varma for his suggestions during the preparation of this paper. My sincere thanks also goes to Dr. B. Jaya Prakash Reddy.

References

[1] L.A. Zadeh, Fuzzy sets, Inform and Control. 8 (1965), 338–353.
[2] E. T. Jaynes, G. Larry Bretthorst. Probability Theory: The Logic of Science. Cambridge University Press(2003).
[3] K. Atanassov. More on intuitionistic fuzzy sets, Fuzzy Sets and Systems 33 (1989), 37–46.
[4] Z. Pawlak: Rough sets, International Journal of Computer and Information Sciences, 11(1982) 341-356.

[5] Florentin Smarandache. A unifying field in logics. Neutrosophy: Neutrosophic probability, set and logic. Americal research press, Rehoboth (1999).
[6] Florentin Smarandache, Unmatter, mss. Archives Vâlcea (1980).
[7] W.B. Vasantha Kandasamy, Florentin Smarandache. Basic Neutrosophic Algebraic Structures and their Applications to Fuzzy and Neutrosophic Models, Hexis, Church Rock (2004).
[8] A.A.A. Agboola, A.O. Akwu, Y.T. Oyebo, Neutrosophic Groups and Neutrosophic Subgroups, Int. J. of Math. Comb.3 (2012)1-9.
[9] W. B. Vasantha Kandasamy, Ilanthenral K, Florentin Smarandache . Neutrosophic Graphs: A New Dimension to Graph Theory. EuropaNova (2015).
[10] W.B. Vasantha Kandasamy, F. Smarandache, Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures, Hexis, Phoenix, Arizona (2006).
[11] David Joyner. Adventures in Group Theory: Rubik's Cube, Merlin's Machine, and Other Mathematical. Toys. 2nd edition, JHU Press (2008).
[12] I.Vitaly, I. Voloshin. Introduction to graph theory, Nova Science Publishers. Inc. New York (2009).
[13] Kiyosi Itô.: Encyclopaedic Dictionary of Mathematics, Volume 1. The MIT Press. 2nd edition (2000).
[14] Linda Gilbert. Elements of Modern Algebra. 8th edition Cengage Learning, (2014).

Received: January 23, 2017. Accepted: February 13, 2017.