On synchronization and the relativity principle

Mohammed Mezouar
Email : laud.prim@yahoo.com
Haï Benhamidi, Sabra, 13011, Algeria

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ABSTRACT

Lorentz transformation allows two ways to compare time measures from two moving clocks. We show that the more realistic way leads to discover that absolute rest plays a hidden role and prescribes a restriction on the relativity principle.

Let K_1 and K_2 two cartesian frames moving along the x axis with the speeds v_1 and v_2 in a third frame K_0 at rest.

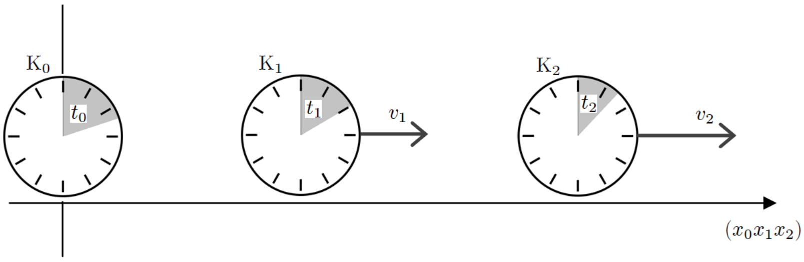

Fig. 1

If (x_0, y_0, z_0, t_0) are the space-time coordinates of an event in the frame K_0, it's coordinates (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) in the frames K_1 et K_2 are deducted via Lorentz transformations :

$$\begin{cases} x_1 = \dfrac{x_0 - v_1 t_0}{\sqrt{1-(v_1/c)^2}} \\ t_1 = \dfrac{t_0 - (v_1 x_0/c^2)}{\sqrt{1-(v_1/c)^2}} \end{cases}, \begin{cases} x_2 = \dfrac{x_0 - v_2 t_0}{\sqrt{1-(v_2/c)^2}} \\ t_2 = \dfrac{t_0 - (v_2 x_0/c^2)}{\sqrt{1-(v_2/c)^2}} \end{cases}, \tag{1}$$

with $y_0 = y_1 = y_2$ and $z_0 = z_1 = z_2$.
The inverse transformation of the first one in (1) is :

$$\begin{cases} x_0 = \dfrac{x_1 + v_1 t_1}{\sqrt{1-(v_1/c)^2}} \\ t_0 = \dfrac{t_1 + (v_1 x_1/c^2)}{\sqrt{1-(v_1/c)^2}} \end{cases}. \tag{2}$$

The replacement of x_0 and t_0 from (2) in the second transformation of (1) gives :

$$\begin{cases} x_2 = \dfrac{x_1 - q t_1}{\sqrt{1-(q/c)^2}} \\ t_2 = \dfrac{t_1 - (q x_1/c^2)}{\sqrt{1-(q/c)^2}} \end{cases}, \tag{3}$$

$$\text{where } q = \frac{v_2 - v_1}{1-(v_2 v_1/c^2)}. \tag{4}$$

Since the clock linked to the origin of K_2 is characterized by $x_2 = 0$, which means that $x_1 = q t_1$, by replacing in the expression of t_2 in (3) one finds :

$$\frac{t_2}{t_1} = \sqrt{1 - \frac{q^2}{c^2}} \tag{5}$$

Let suppose that $v_1 = -v_2$. Consequently, relations (4) and (5) lead to :

$$\frac{t_2}{t_1} = \sqrt{1 - \left(\frac{2 v_2 c}{c^2 + v_2^2}\right)^2}. \tag{6}$$

From Eq. (6), the synchronization is conserved ($t_2 = t_1$) only in the case $v_2 = v_1 = 0$.

But according to Special Relativity results (chap. I,4 in [1]), if the clock linked to the origin of K_0 measure a duration t_0, the moving clocks linked to K_1 and K_2 origins must measure :

$$t_1 = t_0\sqrt{1-\frac{v_1^2}{c^2}},\ t_2 = t_0\sqrt{1-\frac{v_2^2}{c^2}}. \tag{7}$$

From the law of speed addition, the speed of the K_2-clock in relation to the K_1-clock is :

$$u = \frac{v_2 - v_1}{1-(v_1 v_2/c^2)} = q. \tag{8}$$

Eqs. (7) give :

$$\frac{t_2}{t_1} = \sqrt{\frac{1-(v_2/c)^2}{1-(v_1/c)^2}}. \tag{9}$$

The synchronization is conserved ($t_2 = t_1$) if $v_2 = \pm v_1$ with the possibility of v_1(and v_2) $\neq 0$: When the two clocks are moving with the same non null speed in relation to another frame but in two opposite directions. Clearly, this logical issue is very realistic. One can imagine two synchronous clocks starting motion in opposite directions and returning after the same journey length at the same speed. Obviously, their relative speed is not necessary null :

$$u = \frac{-2v_1}{1+(v_1/c)^2} = \frac{2v_2}{1+(v_2/c)^2}$$

It is very natural to expect that both the two clocks will measure the same duration. This situation is in disagreement with Einstein's deduction : *All* synchronous clocks do not remain so after the accomplishment of galilean relative motions (chap. I,4 in [1]).

Moreover, together Eqs. (8) and (9) lead to :

$$\frac{t_2}{t_1} = \frac{\sqrt{1-(u/c)^2}}{1+(uv_1/c^2)} = \frac{1-(uv_2/c^2)}{\sqrt{1-(u/c)^2}}. \tag{10}$$

Clearly, relation (5) is recovered from Eqs. (10) only in the case $v_1 = 0$ with $t_1 > t_2$ and the case $v_2 = 0$ corresponds to $t_1 < t_2$.

Eqs. (10) are equivalent to :

$$v_1 = \frac{c^2}{u}\left(1-\frac{t_1}{t_2}\sqrt{1-(u/c)^2}\right),\ v_2 = \frac{c^2}{u}\left(1-\frac{t_2}{t_1}\sqrt{1-(u/c)^2}\right). \tag{11}$$

From Eq. (8) a fixed value of u is possible with an infinity choices of (v_1, v_2). But according to relations (11), for *any fixed value* of t_2/t_1 and u, the values of v_1 (and v_2) are *unique*. Thus the comparison of the measures t_1 and t_2 does not depend only on the relative speed u (equal to q) as from Eq. (5), but also on the speed v_1 (or v_2) whose uniqueness significate that they are in relation to the absolute rest frame. Interestingly, the absolute rest reappears with his fundamental importance, so hidden in Einstein's theory.

As a conclusion, the "totaly motionless" frame is always needed to compare time measures and, thus, is not actually equivalent to "moving" galilean frames. This fact prescribes a neat restriction on Einstein's equivalence postulate (relativity principle).

Reference

[1] A. Einstein, *Zur Elektrodynamik bewegter Körper*. Annalen der Physik 17, 891 (1905). "On the electrodynamics of moving bodies" translated in *The Principle of Relativity*, Dover, NY, (1952) pp.35-65.