Energy density calculation formula and more for decay based nuclear fuel or battery

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Abstract

This paper presents a convenient calculation formula of energy density for nuclear fuel or nuclear battery that outputs energy by whatever decay. Also a relative formula is deduced for easy comparison between different fuels. At last, with the convenient formula, the energy density comparison and possibility of applying different isomer beta fuels are proactively calculated and aggressively discussed.

Introduction

In a sense, whatever decay, such as **β+** and **β-** decay, those nuclides are "burning" naturally in slow or fast rate. The important parameter **half life** means how long time span from the nuclide's fresh existence to the moment burnt **50%**.

People prefer to use energy density such as watts per **kg** mass when comparing or judging how powerful, let me deduce universal formula in convenience for all interested readers.

Unfortunately, there is not any convenient mathematic formula to calculate the energy density, and probably this situation is caused by the rare application of decay based fuels.

The niche market of such application most likely includes but not limited: spacecraft, medical modality, small power nuclear battery, e.g. NASA uses 238Pu in spacecraft.

Step by step derivation

Now let me deduce it out step by step.

Definition of variables or parameters (if not dimensionless, then unit in brackets):

W = energy density (**watt/kg**); **A** = atom mass; **B** = abundance of designated isotope, for mono-isotope element or full-enriched element, **B= 100%**; **Q** = decay energy (**eV**); $\mathbf{T_{1/2}}$ = half life (**s**).

For **1kg** natural element, the total number of atoms is approximate to **1/(A*amu)**, and the number of one designated isotope atom is **B/(A*amu)**, here **amu** is atom mass unit that is precisely **1/12** of carbon-**12** atom, .i.e. **1 amu = 1.67377*10^{-27}kg**.

By the decay exponential law,

N(t) = N_0* $e^{-\frac{ln2}{T_{½}}t}$, where **N(t)** is the remnant undecayed number of nuclei at time **t**, $\mathbf{N_0}$ is the initial number of nuclei. To calculate power, the differential of **N(t)** is first deduced as:

$$\Delta N/\Delta t = -N_0 * \ln 2/T_{1/2} * e^{-\frac{ln2}{T_{½}}t} \qquad (1)$$

Normalizing $\mathbf{N_0}$ to the number of decaying isotope atoms in **1kg** natural element atoms: **B/(A*amu)**, and considering short time trend: **t** is far less than $\mathbf{T_{1/2}}$, or $\mathbf{t/\ T_{1/2} \approx 0}$, then:

$$\Delta N/\Delta t \approx -N_0 * \ln 2/T_{1/2} = -B * \ln 2/(A * amu * T_{1/2})$$, also then the energy differential in **1kg**, i.e. energy density:

$$W = |\Delta E/\Delta t| = Q * Evj * \Delta N/\Delta t = Q * Evj * B * \ln 2/(A * amu * T_{1/2})$$, where **Evj** is the joule per **eV** energy: **1.602*10^{-19}**.

Inserting all constants to above formula, and calculating expression then reducing to one constant, we get a convenient formula:

$$W = 6.688 * 10^{7} * \frac{B * Q}{A * T_{½}} \ \text{watt/kg}, \qquad (2)$$

just to redeclare the unit: **Q** is **eV** (electron-volt), $\boldsymbol{T_{½}}$ is **s** (second), **A, B** are dimensionless.

Considering beta decay will waste about **40%** energy in neutrinos, thus above formula should be corrected to:

$$W(\beta) = 4*10^7 * \frac{B*Q}{A*T_{½}} \text{ watt/kg}, \qquad (3)$$

Above correction does not apply to alpha and gamma decay.

Calculation exercise

Now let me calculate the energy density for some typical nuclides.

The most known natural radioactive element is the potassium, and only isotope **40K** is radioactive with abundance **B** = **0.0117%**, **Q = Q(β) = 1.504MeV = 1, 504, 000 eV**, $T_{1/2}$ = **$1.251*10^9$** years = **$3.94*10^{16}$s**, **A** = **40.96**, therefore **W = $4.58*10^{-9}$ watt/kg = 4.58nw/kg**.

Obviously potassium energy density is so small, even for pure enriched **40K**, the energy density is still small, though it is greatly increased to **38μw/kg**.

Another example is the widely used strong radioactive pure isotope **60Co** of cobalt: **Q(β-) = 2,822,810eV**, $T_{1/2}$ **= 1925.28** days = **166,344**,**192s**, **A = 60**, so **W = 11,313 watt/kg = 11.3kw/kg**.

Oh, it is really powerful! Isn't it? Of course, if you have such **1kg** chunk of **60Co**, it will be spontaneously 'burning' anywhere and anytime with hot shining surface for at least **5** years.

Relative index of energy density

Now is time to consider relative index computation.

Let natural potassium **K** as the baseline, for contrast, I can deduce out relative index of energy density of other natural radioactive elements.

Assuming X_W = energy density ratio to potassium **K**; X_A = atom mass ratio to **K**; X_B = abundance ratio to isotope **40K**; X_Q = decay energy ratio to **40K**; X_T = half time ratio to **40K**, then

$$X_w = X_B X_Q/(X_A X_T) \qquad (4)$$

For examples: Rubidium, of **87Rb** beta decay**,** X_W **= 4.75** times stronger than potassium; Lutetium, of **176Lu** beta decay X_w **= 1.44**; Uranium **U**, of **238U** alpha decay X_w **= 2140**; Thorium

Th, alpha decay $\mathbf{X_w = 662}$; Rhenium, of **187Re** beta decay, $\mathbf{X_w = 0.07}$; Lanthanum, of **138La** beta decay, $\mathbf{X_w = 0.035}$; Indium, of **115In**, $\mathbf{X_W = 0.003}$; etc.

Will some specific nuclear isomers be next potential nuclear fuel?

Nowadays, uranium **235U** is the only commercial nuclear fuel, but its resource is limited, so that humankind should find next candidate elements and utilization methods.

As aforementioned, for **60Co**, it has amazing high energy density **11.3kw/kg**, but unfortunately it is too expensive and not feasible for commercial nuclear fuel, because its natural abundance is zero, and it can only be man-made with accelerator or brooded in fission nuclear reactor.

I propose a new type of would-be nuclear fuel that can be chosen from those stable elements with not low abundance and isomer state of low energy level at which state there is significant branch ratio of greatly shortened half time beta decay.

For example, the dirt cheap element cadmium, of isotope **113Cd**, its sibling abundance **12.22%** is decent, and still stable, though it undergoes extreme slow beta decay at half life $\mathbf{7.6*10^{15}}$ years, however it's yrast isomer of **263keV** undergoes only **14.1** years beta decay with **99.86%** branch ratio versus **0.14%** gamma decay to its ground state, hence such a fact suggests energy density of pure **113Cd** isomer is:

$\mathbf{W_{113m\text{-}Cd} = 4*10^7*586140/(113*14.1*365*24*60*60) = 466W/kg}$.

This is not bad result: for a regular family house, **10kg 113Cd** isomer is probably enough for daily use and winter heating and hot water, if we could make such a use.

In fact, not too much choice for this kind of potential fuels there are, the enumerable other elements: **115In**, **176Lu, 180m-Ta**.

The **180m-Ta** is the only naturally existing isomer isotope of tantalum with sibling abundance merely **0.012%**, and it is perched at **77keV** energy level above **180Ta** ground level at which the half life is only **8.154** hours. By 'shaking off', **180m-Ta** can 'fall' down to ground state, then quickly decay to either **180Hf** at **85%** chance or **180W** at **15%** chance with max total energy

923keV release. Compromised by its low abundance, its energy density is only: **838W/kg**, unless enriched to pure for the max **7MW/kg**.

For indium **115In** with abundance **95.71%**, its isomer state: **336keV**, **β** branch ratio **5%**, half life **4.486** hours, hence, its energy density is calculated out: **898kW/kg**.

For lutetium **176Lu** with abundance **2.59%** (not bad but not decent), its isomer state: **122keV**, **β** branch ration **100%**, half life **3.664** hours, hence its energy density is calculated out: **23MW/kg**.

Do not cheer too early, because pushup from ground state to isomer state is not free! Among all those choices, the lowest excitation energy is **122keV** of lutetium **176Lu**, but this rare earth element is not cheap: its current price is about **35%** of gold!

There are many methods to excite nuclei to isomers: coulomb excitation, neutrons scattering excitation, photons excitation, neutrinos excitation, etc.

All excitations consume energy except by free solar or deep space neutrinos excitation, just like as the Parkhomov experiment with **60Co** sitting at the focus of astronomical telescope (**ref. 1**).

Considering the energy consumption of excitation and efficiency, all above calculated isomers energy density should be discounted in large scale, embarrassingly, even the situation of no commercial value could occur, if too low efficiency of excitation by non-free energy, such as coulomb or photons excitation.

As high energy neutrinos almost no refractive effect, and unconverged neutrinos almost useless, hence the higher the excitation energy, the lesser likely it could be excited by neutrinos.

The **122keV** of **176Lu** is probably relative easy to be excited by converging neutrinos, and it is very close to the **58keV** of **60Co** that is already confirmed of neutrinos excitation by Parkhomov experiment, thus **176Lu** is probably the most hopeful choice for potential nuclear fuel based on greatly expedited isomer beta decay.

Rubidium isotope **87Rb** (sibling abundance **28%**) seems another candidate as its decay energy **Q(β-) = 282keV** is locked by merely **ΔJ = 3** angular quanta that is less than **7** of **176Lu** or **4** of

115In, and the focused low energy neutrino-ray is easy to unlock it. Anyway, I need more experiment time to confirm it. Although rubidium family abundance (natural resource deposit) is quite decent, currently it is very expensive because the low market demand results in low mining activity.

By the way, I even guess that **UFOs** harvest energy of remote star neutrinos to sustain their interstellar travels. Don't believe the fabulous daydream theory of Dyson sphere, as it is just a gedanken experiment.

Last words

Wish the pertinent scientific community love all herein formula and trigger the trend of advanced research and development in new replacement nuclear fuel of the conventional but nasty 235U fission fuel.

References

1. Deviations from Beta Radioactivity Exponential Drop, Alexander G. Parkhomov, DOI:**10.4236/jmp.2011.211162**.