# Feigenbaum's Constant and the Sommerfeld Fine-Structure Constant

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ABSTRACT

A simple mathematical relationship exists between two unit-less, universal physical constants: the Sommerfeld fine-structure constant and Feigenbaum's delta. This relationship may help to explain the "mystery" that has surrounded the fine-structure constant for many decades.

## FINE-STRUCTURE CONSTANT

The fine-structure constant ($\alpha$) is a dimensionless quantity comprised of the four basic physical constants, electronic charge (e), speed of light in a vacuum (c), the Planck constant (h), and the permittivity of free space ($\varepsilon_0$)[1]. This constant was discovered when it first appeared in connection with the fine structure of atomic spectra. The fine-structure constant is sometimes also referred to as the electromagnetic coupling constant. It characterizes the force of coupling between the elementary electric charge and the electromagnetic field. The equation for the fine structure constant is:

$$\alpha=e^2/2hc\varepsilon_0 \qquad (1)$$

Where:

- e = the charge of an electron: $1.6021917 \times 10^{-19}$ C
- h = Planck's constant: $6.626196 \times 10^{-34}$ J*S
- c = the velocity of light: $2.9979250 \times 10^{8}$ $M*S^{-1}$
- $\varepsilon_0$ = the permittivity of free space: $8.854187817 \times 10^{-12}$ $F*M^{-1}$

therefore: $\alpha = 0.007297353 \approx 1/137$

In some cases, the fine-structure constant is expressed as $\alpha^{-1}$ or 137.

The equation $\alpha = e^2/2hc\varepsilon_0$ contains the fundamental constants of quantum physics (h), of relativity (c) and of electromagnetic theory (e and $\varepsilon_0$). The constant $\alpha$ is also unit-less, as shown here:

$$=C2/(V*C*S)(M*S-1)((C/V)M-1) \quad (2)$$

$$= C2/(V*C*S)(s-1)(C/V)=C2/(V*C)(C/V)=C2/(C*C) = \text{unit-less}$$

A appears in several equations which describe the hydrogen atom, they are:

First Bohr orbit: $a_0 = \lambda'_c/\alpha$ (3)

Ground state energy: $-\omega_0 = -1/2\alpha^2 m_0 c^2$ (4)

Rydberg constant: $R\gamma_\infty = \alpha 2/4\pi\lambda'_c$ (5)

Orbital velocity: $v = \alpha c$ (6)

Where:

$\lambda'_c$ = Compton wavelength divided by $2\pi$

$m_0$ = the electron rest mass

FEIGENBAUMS CONSTANT

The Feigenbaum scaling constant[2], δ = 4.669201609 was discovered by Mitchell Feigenbaum in the mid 1970's while he was doing research at Los Alamos on turbulence problems. He discovered that as a system goes into turbulence (chaos), an infinite sequence of bifurcations or period-doublings occur. Figure 1 is a bifurcation tree illustrating the period doubling sequence. Bifurcations occur simultaneously; each branch is a "scale model" of the previous branch and the scaling is the Feigenbaum constant, δ = 4.6692016…

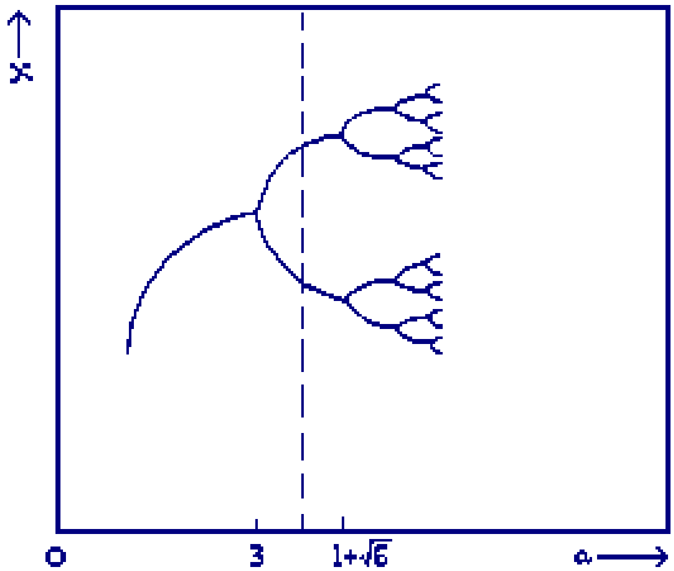


FIGURE 1: Bifurcation tree illustrating the period doubling sequence.

To quote Feigenbaum:

> "What is quite remarkable (beyond the fact that there is always a geometric convergence) is that, for all systems under going this period doubling, the value of δ is predetermined at the universal value δ = 4.6692016…"[3]

## A RELATIONSHIP

A simple mathematical association exists between the fine-structure constant, $\alpha$, and Feigenbaum's scaling factor, $\delta$:

$$\delta' = (1/2\pi\alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609 \quad (7)$$

$$\delta' - \delta = 0.000912 \quad (8)$$

## CONCLUSION

There is a numerical relationship between two remarkable, unit-less constants, Feigenbaum's number and the fine-structure constant. This relationship has an elegant simplicity expressed in the equation $\delta = (1/2\pi\alpha))^{1/2}$ . All terms of this equation are unit-less, therefore the relationship is universal.

## REFERENCES


[1] Dictionary of Physics, Third Edition, pg. 215

[2] Chaos, Making a New Science, James Gleick, Penguin Books, pg. 174

[3] Universal Behavior in Nonlinear Systems, Mitchell J. Feigenbaum, Los Alamos Science1 4-27 (1980)