Poulet numbers which can be written as x^3±y^3

Marius Coman
email: mariuscoman13@gmail.com

Abstract. It is well known the story of the Hardy-Ramanujan number, 1729 (also a Poulet number), which is the smallest number expressible as the sum of two cubes in two different ways, but I have not met yet, not even in OEIS, the sequence of the Poulet numbers which can be written as x^3±y^3, sequence that I conjecture in this paper that is infinite. I also conjecture that there are infinite Poulet numbers which are centered cube numbers (equal to 2*n^3 + 3*n^2 + 3*n + 1), also which are centered hexagonal numbers (equal to 3*n^2 + 3*n + 1).

Conjecture 1:

There exist an infinity of Poulet numbers which can be written as x^3 + y^3.

The first fifteen such Poulet numbers:

: 341 = 5^3 + 6^3;
: 1729 = 1^3 + 12^3 = 9^3 + 10^3;
: 10261 = 10^3 + 21^3;
: 15841 = 6^3 + 25^3;
: 46657 = 1^3 + 36^3;
: 126217 = 25^3 + 48^3;
: 188461 = 45^3 + 46^3;
: 228241 = 48^3 + 49^3;
: 617093 = 29^3 + 84^3;
: 688213 = 42^3 + 85^3;
: 1082809 = 81^3 + 82^3;
: 1157689 = 4^3 + 105^3;
: 1773289 = 12^3 + 121^3;
: 2628073 = 1^3 + 138^3;
: 2867221 = 40^3 + 141^3.

Note that the numbers 341, 1729, 188461, 228241, 1082809 are centered cube numbers (equal to 2*n^3 + 3*n^2 + 3*n + 1, see the sequence A005898 in OEIS). I conjecture that there are infinite Poulet numbers which are also centered cube numbers. I also conjecture that there are infinite Poulet numbers of the form n^3 + 1.

Conjecture 2:

There exist an infinity of Poulet numbers which can be written as $x^3 - y^3$.

The first twenty such Poulet numbers:

: $1387 = 22^3 - 21^3$;
: $4681 = 40^3 - 39^3$;
: $7957 = 52^3 - 51^3$;
: $8911 = 24^3 - 17^3 = 55^3 - 54^3$;
: $13741 = 29^3 - 22^3$;
: $14491 = 70^3 - 69^3$;
: $63973 = 40^3 - 3^3$;
: $93961 = 46^3 - 15^3$;
: $115921 = 49^3 - 12^3$;
: $126217 = 81^3 - 74^3$;
: $172081 = 94^3 - 87^3 = 240^3 - 239^3$;
: $341497 = 100^3 - 87^3$;
: $488881 = 84^3 - 47^3$;
: $748657 = 145^3 - 132^3$;
: $873181 = 540^3 - 539^3$;
: $1397419 = 683^3 - 682^3$;
: $2113921 = 129^3 - 32^3 = 166^3 - 135^3 = 202^3 - 183^3$;
: $2455921 = 145^3 - 84^3 = 217^3 - 198^3$;
: $2628073 = 144^3 - 71^3 = 172^3 - 135^3$;
: $2867221 = 373^3 - 366^3$.

Note that the numbers 1387, 4681, 7957, 8911, 14491, 172081, 873181, 1397419 are centered hexagonal numbers (equal to $3*n^2 + 3*n + 1$, see the sequence A003215 in OEIS). I conjecture that there are infinite Poulet numbers which are also centered hexagonal numbers.