SPECIAL RELATIVITY AND WAVE-PARTICLE DUALITY.

Enrique Cantera del Río

C/Padre Benito 6-2-E Valladolid (Spain). benarrob@gmail.com

RESUME

UNE SIMETRIE SUR LA TRANSFORMATION DE LORENTZ EST PROPOSE POUR LA VISUALISATION DU PAYSAGE PHYSIQUE DERIVE DE LA RELATIVITE RESTREINTE. BIEN QUE LES ARGUMENTS BASIQUES, LA SIMETRIE MONTRE D'ETROITES RELATIONS AVEC LA DUALITE ONDE-PARTICULE ET L'EXISTENCE DE DEUX ETATS POSSIBLES POUR CE RELATION PHYSIQUE. ON ANALYSE AUSSI LE SPIN ET LA CONTROVERSE D'ABRAHAM-MINKOWSKI SELON CES IDEES ET ENFIN UNE NOUVELLE HYPOTHESE EST PRESENTEE SUR LA DUALITE ONDE-PARTICULE.

ABSTRACT

A SYMMETRY OF LORENTZ TRANSFORMATION IS PROPOSED FOR THE VISUALIZATION OF THE MOST BASIC PHYSICAL LANDSCAPE DERIVED FROM SPECIAL RELATIVITY. ALTHOUGH BASIC ARGUMENTS ,THE SYMMETRY SHOWS A CLOSE RELATIONSHIP WITH WAVE-PARTICLE DUALITY AND THE EXISTENCE OF TWO POSSIBLE STATES FOR THIS PHYSICAL RELATION. THE SPIN AND THE ABRAHAM-MINKOWSKI CONTROVERSY ARE ANALYZED REGARDING THIS IDEAS AND A NEW HYPOTHESIS ON WAVE-PARTICLE DUALITY IS PRESENTED.

KEYWORDS

Special Relativity, Wave-particle duality, Abraham-Minkowski controversy.

1. INTRODUCTION

Imagine a lost explorer in a deserted area. From his position, he may see some mountains; he goes to the nearest, trying to get a broader perspective to see a road, village, river or some other helpful reference. From the top, the winding path he has followed can be seen, as well as other simplest possible paths from the top to the plain, passing close to other roads made by man. Though he carries to the top both good intentions and prejudices, the observer has a chance to get an idea of where he is, where he has to go and what way he has to take. In the case of special relativity, the mountain corresponds to the principle of relativity, while the "eagle's crag" corresponds to the light constant speed principle. We can describe such a high place by means of relativistic transformations between two inertial observers using Cartesian coordinate systems with parallel axes that move relatively along the common axis direction X.

The Lorentz transformation relates the measures of space and time for the same physical event of two observers

$$\Delta t' = (\Delta t - \frac{v}{c^2}\Delta x)\beta^{-1}\ ;\ \Delta x' = (\Delta x - v\Delta t)\beta^{-1};\ \ \Delta y' = \Delta y;\ \Delta z' = \Delta z \quad (1.1)$$

$$\beta = \sqrt{1 - v^2 / c^2}$$

Measures of frequency (ω) and wave vector (k) of the same wave differ between observers according to the relationship

$$\omega' = (\omega - v k_x)\beta^{-1};\ k_x' = (k_x - \frac{v}{c^2}\omega)\beta^{-1};\ k_y' = k_y;\ k_z' = k_z \quad (1.2)$$

From eq 1.2 one can deduce the *Doppler Effect*, so eq 1.2 are valid for a wave in a material media.

Measures of mechanical impulse (P) and energy (E) for the same particle differ between observers according to

$$E' = (E - vP_x)\beta^{-1};\ P_x' = (P_x - \frac{v}{c^2}E)\beta^{-1};\ P_y' = P_y;\ P_z' = P_z \quad (1.3)$$

Measures of densities of charge (ρ) and current (j) are related between inertial observers as

$$\rho' = (\rho - j_x\frac{v}{c^2})\beta^{-1};\ j_x' = (j_x - \rho v)\beta^{-1};\ j_y' = j_y;\ j_z' = j_z \quad (1.4)$$

The electric (Φ) and magnetic (A) potentials, under *Lorenz gauge*, are too components of a vector in Minkowski space

$$\phi' = (\phi - A_x v)\beta^{-1};\ A_x' = (A_x - \phi\frac{v}{c^2})\beta^{-1};\ A_y' = A_y;\ A_z' = A_z \quad (1.5)$$

The usual interpretation of these relationships can be expressed by saying that from the eagle's crag, we see *Minkowski Space*. This is seemingly a paradox, but if he wants to see something concrete from this level of abstraction, our explorer must also imagine abstract questions.

2. DUALITY OF MOTION.

All motion is a relationship between space intervals (Δx, Δy, Δz) and time intervals (Δt). The simplest relationships are associated with classic vector algebra

$$(\Delta x, \Delta y, \Delta z) = \overline{V}\Delta t \quad ; \ \Delta t = \overline{W} \bullet (\Delta x, \Delta y, \Delta z) \quad (2.1)$$

If we apply eq. 1.1 to these definitions so that the other observers can get the same mathematical form for motion, we have

$$V'_x = \frac{V_x - v}{1 - \frac{vV_x}{c^2}};\ V_y = \frac{\beta V_y}{1 - \frac{vV_x}{c^2}};\ V_z = \frac{\beta V_z}{1 - \frac{vV_z}{c^2}} \quad (2.2)$$

$$W'_x = \frac{W_x - \frac{v}{c^2}}{1 - vW_x};\ W_y = \frac{\beta W_y}{1 - vW_x};\ W_z = \frac{\beta W_z}{1 - vW_x} \quad (2.3)$$

Obviously, the first relationship *(2.2)* corresponds to the transformation of the velocity of a particle between inertial systems of special relativity. However, the second relationship *(2.3)* describes the motion of a wave, namely, the movement of a constant phase state

$$\overline{k} \bullet \Delta\overline{r} - \omega\Delta t = 0 \Rightarrow \frac{\overline{k}}{\omega} \bullet \Delta\overline{r} = \Delta t;\ \ \overline{W} = \frac{\overline{k}}{\omega} \quad (2.4)$$

The reader can check that using eq. 1.2, the k/ω vector transforms between inertial observers as it does the vector W.

We can propose a relation similar to eq. 2.1 for the case of the current density, eqs. 1.4

$$\left(j_x, j_y, j_z\right) = \rho\overline{V} \quad ; \quad \rho = \overline{W} \bullet \left(j_x, j_y, j_z\right) \qquad (2.5)$$

Again the values of *V,W* transforms like expected : eqs 2.2 and 2.3. The first of eqs. 2.5 corresponds to the motion of a *particle stream* ; and following the symetry the second of eqs.2.5 corresponds to the motion of a *wave stream*. Note the reader that wave stream allow a electric current (*j*) with null density of charge (ρ) if *W* is ortogonal to *j*. Evidently it is possible only if *j* is a *component* of a total density current j_{total}. The idea that a wave can transport electric charge fits with the solid state physics phenomena know as *Charge Density Wave (CDW).* In CDW the electron waves in a crystal can combine to produce standing waves creating a spatial modulation in density charge of electrons.

3. DUALITY OF INTERACTION.

We assume a variation of mechanical impulse and / or related energy for any interaction. Similar to the case of movement, we can take

$$\left(\Delta P_x, \Delta P_y, \Delta P_z\right) = \overline{W}\Delta E \quad ; \quad \Delta E = \overline{V} \bullet \left(\Delta P_x, \Delta P_y, \Delta P_z\right) \qquad (3.1)$$

and after applying eq. 1.3, we find that *W* and *V* have the same transformation rule as in the case of motion. The second formula depends on the velocity of a particle and can easily be recognized as the variation of kinetic energy of a constant mass particle. Following the symmetry, we can understand the first expression as energy-impulse exchanged by a wave; therefore

$$\overline{W}\Delta E = \Delta\overline{P};\ \overline{W} = \overline{k}/\omega \Rightarrow \overline{k}\Delta E = \Delta\overline{P}\,\omega \Rightarrow \frac{\Delta E}{\omega} = \frac{\Delta P_x}{k_x} = \frac{\Delta P_y}{k_y} = \frac{\Delta P_z}{k_z} \qquad (3.2)$$

Obviously, the equality of quotients is also valid for the other observer, but there is something more. The reader can see that for eq. 1.2: $k_z = k'_z$, and for eq. 1.3: $\Delta P_z = \Delta P'_z$; therefore, the value of the above quotients is the same for the two observers and then invariant between inertial coordinates, which is necessary in the context of relativity to justify the existence of *Planck's constant* and the *energy quanta*

$$k_z = k_z';\ \Delta P_z = \Delta P_z' \Rightarrow \frac{\Delta E}{\omega} = \frac{\Delta E'}{\omega'} = n\hbar \qquad (3.3)$$

where *n* is the number of basic packages of energy that a wave exchanges. In the black-body problem, Einstein introduced two concepts that fit with this view: the *induced emission* and *induced absorption* of radiation. In induced emission, an electromagnetic wave causes the transition of electrons from one energy level to another, while lower energy levels are available; however, this only happens if emitted photons correspond to the frequency of the inductor wave. Furthermore, the emitted photons are perfectly integrated into the inductive wave. There are not several waves of the same frequency that can interfere with each other; rather, the emitted photons are *in phase* with the inductor wave and it can be said that the wave has absorbed the corresponding amount of energy. This phenomenon is the basis of *LASER*. In induced absorption, a wave loses photons, which are absorbed by electrons, causing the corresponding change in the energy level, while the final level is available. Similarly, the inducing wave maintains the same polarization, phase, frequency and wavelength.

3. DUALITY OF WAVE PACKAGES.

A wave package is a linear combination of several plane waves characterized by a wavelength and frequency dispersion. Writing the dispersions as (Δk, $\Delta\omega$) we can suspect there are two possible relations between them

$$\Delta\omega = \overline{V} \bullet \left(\Delta k_x, \Delta k_y, \Delta k_z\right) \; ; \; \left(\Delta k_x, \Delta k_y, \Delta k_z\right) = \overline{W}\Delta\omega \qquad (4.1)$$

From eq. 1.2 it is inmediate that ***V*** and ***W*** transforms as expected. The first formula of (4.1) corresponds to the wave package *group velocity V* associated to the dispersion. The second formula is for a wave package too, but now the dispersion holds a constant value for *W* ; and we can see in this case that group velocity equals to *phase velocity* (i.e the inverse of *W*)

$$\frac{\Delta\overline{k}}{\Delta\omega} = \frac{\overline{k}}{\omega} \qquad (4.2)$$

so the wave package holds its shape while moving. If two events separated (Δr,Δt) have the same phase state: $\Delta\varphi=0$, this dispersion law holds the phase difference

$$\overline{k} \bullet \Delta\overline{r} - \omega\Delta t = \Delta\varphi \; ; \; \Delta(\Delta\varphi) = \Delta\overline{k} \bullet \Delta\overline{r} - \Delta\omega\Delta t = \Delta\omega\left(\overline{W} \bullet \Delta\overline{r} - \Delta t\right) = \frac{\Delta\omega}{\omega}\left(\overline{k} \bullet \Delta\overline{r} - \omega\Delta t\right)$$

$$\Delta\varphi = 0 \Rightarrow \Delta(\Delta\varphi) = \frac{\Delta\omega}{\omega}\Delta\varphi = 0 \qquad (4.3)$$

5. WAVE-PARTICLE INTERACTION.

From the above, we can theorize regarding the existence of a system in which a particle (subscript *p*) exchanges its energy and momentum with a wave (subscript *w*) so that the energy and mechanical impulse of the system are preserved:

$$\Delta E_p = -\Delta E_w \; ; \; \Delta\overline{P}_p = -\Delta\overline{P}_w \Rightarrow$$
$$\Delta E_p = \overline{V} \bullet \Delta\overline{P}_p = -\overline{V} \bullet \Delta\overline{P}_w = -\overline{V} \bullet \overline{W}\Delta E_w ; \; \rightarrow \; \overline{W} \bullet \overline{V} = 1 \qquad (5.1)$$

We can apply the above transformations 2.2 and 2.3 to this result and see that the expression $\mathbf{W_xV_x+W_yV_y+W_zV_z=1}$ is invariant between inertial observers. Thus, our system is in a physical state independent of the observer. We will call it *S1*. Suppose the energy interchanged corresponds to a quantum of energy; then, we have

$$\frac{\Delta E}{\omega}\overline{W} \bullet \overline{V} = \Delta\overline{P} \bullet \overline{V}\frac{T}{2\pi} \Rightarrow \Delta\overline{P} \bullet \Delta\overline{r} = 2\pi\frac{\Delta E}{\omega} = 2\pi\hbar \Rightarrow \qquad (5.2)$$
$$\Delta\overline{P} \bullet \Delta\overline{r} = h$$

where Δr is the displacement of the particle for the period *T* of the wave. This result is compatible with *Heisenberg´s uncertainty principle* and therefore any particle can be in this state. In the case of a charge radiating by acceleration, a relationship between energy and the mechanical impulse of radiation is found

$$\Delta\overline{P} = \frac{\overline{V}}{c^2}\Delta E \Rightarrow \overline{W} = \frac{\overline{V}}{c^2} \Rightarrow \overline{W} \bullet \overline{V} = \frac{V^2}{c^2} < 1 \qquad (5.3)$$

The reader can also verify that with the latter definition for *W*, the relationship $\mathbf{W_xV_x+W_yV_y+W_zV_z< 1}$ is also invariant according to 2.2 and 2.3, indicating a physical state other than the *S1* wave-particle balance. We will call it *S2*. However, we should also find a wave that matches the found value *W*; according to quantum mechanics, a free particle can be represented by a wave function ψ with the expected value *W*

$$\psi(\overline{P} \bullet \overline{r} - Et) \rightarrow \overline{W} = \frac{\overline{P}}{E} = \frac{\overline{V}}{c^2} \qquad (5.4)$$

where *P* and *E* are the corresponding relativistic values. If we reproduce the above calculation for the interaction wave-particle in this case, we arrive at

$$\Delta\overline{P} \bullet \Delta\overline{r} < h \qquad (5.5)$$

Therefore, in this state, i.e., *S2,* there is no exchange of energy between the particle and its associated wave. If a wave-particle is in state *S2* and suffers an interaction, the tuning of the couple requires an intermediate state *S1*. A change in the wave-particle from one to another of the aforementioned states implies the emission or absorption of a photon; therefore, this change of state is a non-linear process. This may be the case of *wave function collapse*. The usual de Broglie's formulas[3] in basic texts about quantum physics are compatible with the *S2* state for any inertial observers

$$E = mc^2 = \hbar\omega;\ \overline{P} = m\overline{v} = \hbar\overline{k}$$
$$\overline{P} \bullet \overline{v} = \hbar\overline{k} \bullet \overline{v} = mv^2 < mc^2 \Rightarrow \overline{k} \bullet \overline{v} < \omega \ \rightarrow (S2) \qquad (5.6)$$

Regarding wave packages, the first of eqs. 4.1 is compatible with the De Broglie's formulas as it is well know; so the first of eqs. 4.1 corresponds to a *S2* state where there is a dispersion due to different group and phase velocity. In the wave-particle duality context we can think that eqs. 4.1 tell us that any interaction produces a dispersion into the wave package; and therefore, regarding the De Broglie formulas, the values of energy and mechanical impulse have an additional margin or amplitude in energy and mechanical impulse. In quantum mechanics these amplitudes are related with the mean lifetime of a quantum state and the Heisenberg's principle of indeterminacy.

Imagine the reader a photon incident into a crystal. The photon may be reflected or refracted, but not both. So there is an empty wave which is not interacted by the photon (S2 state) and the other wave which is interacted (S1 state). The *Born's rule* in quantum mechanics requires to maintain the empty wave inside the wave function, because wave function is related with the probability of detection of the photon.

6. DUALITY OF ELECTROMAGNETIC FIELD.

The corresponding relations for eq 1.5 for potential are

$$\overline{A_p} = \phi_p \overline{W} \quad ; \ \phi_w = \overline{V} \bullet \overline{A_w} \qquad (6.1)$$

the first relation corresponds to *Lorenz gauge* potentials for a point charge moving at retarded velocity *V*,*taking W=V/c²* *(Lienárd-Wiechert potentials)* [5]. The second relatión corresponds to *Lorenz gauge* potentials for a plain electromagnetic wave, *taking V=c²W*

$$\overline{A} = \overline{A}_0 \exp(i(k \bullet r - \omega t)) \Rightarrow \nabla \bullet \overline{A} = i\overline{k} \bullet \overline{A} \ ; \quad \phi = \phi_0 \exp(i(k \bullet r - \omega t)) \Rightarrow \frac{\partial\phi}{\partial t} = -i\,\omega\phi$$
$$c^2 \nabla \bullet \overline{A} + \frac{\partial\phi}{\partial t} = 0 \Rightarrow c^2 \overline{W} \bullet \overline{A} = \phi \ ; \overline{W} = \frac{\overline{k}}{\omega} \qquad (6.2)$$

We can see that booth cases correponds to a S2 state for the electromagnetic field ; and this implies no interaction between wave and particle. May be this is the basic problem about the *radiation reaction of an accelerated charge* ; so the electromagnetic field needs a S1 state with interaction between wave and particle. It is evident that eqs. 6.2 are valid for increments, and for *S1 state* we can take

$$\Delta\overline{A_p} = -\Delta\overline{A_w} \ ; \Delta\phi_p = -\Delta\ \phi_w \ \Rightarrow \overline{W} \bullet \overline{V} = 1 \quad (6.3)$$

so in S1 state it is possible that the potentials of particle and wave are balanced and changes are compensated between themselves. So a charged particle in S1 state may be do not emit radiation while it is accelerated.

7. WAVE-PARTICLE DUALITY AND SPIN.

We can manage te equation of S1 state splitting *V* in two components : parallel and perpendicular to *W*.

$$\overline{W} \bullet \overline{V} = \overline{W} \bullet \overline{V_{||}} + \overline{W} \bullet \overline{V_{\perp}} = 1 \quad (7.1)$$

It is evident that the product with $V_\perp$ equals to 0 and do not contributes to the equation (7.1). From eqs 2.5 we can relate eq. (7.1) with two components of the density current tetravector. A component from $V_{||}$ relating to a particle stream and a component from $V_\perp$ relating to a wave stream. The electric current asociated with $V_{||}$ can be seen as an *orbital magnetic momentum* related with the charge's motion and the $V_\perp$ component can be seen as a *Spin magnetic momentum*.

8. THE IMPULSE OF LIGHT IN MATERIAL MEDIA: ABRAHAM-MINKOWSKI CONTROVERSY.

A ray of light falling onto a crystal of a transparent prism is divided into a reflected ray and a refracted one through the prism. There are three significant facts associated with this phenomenon:

1-The frequency of the reflected and refracted waves is the same as the frequency of the incident wave.

2-The speed of propagation of the refracted wave decreases.

3-The process does not cause heating or other energy dissipation in the prism.

The light in the vacuum only can be in *S1* state because the product between *W* and *V* must equal to *1*; and if there is no energy interchanged with the prism, then light holds its *S1* state in the process. If we see the problem in terms of photons, the energy conservation indicates that the incident photons are redistributed between the reflected and refracted wave and, because photons holds its *S1* state, we have

$$\bar{k} \bullet \bar{V} = \omega \Rightarrow V = \frac{\lambda}{T} \qquad (8.1)$$

where *V* is the velocity of the photon and λ,*T* are the length and wave period, respectively. We consider that the velocity of the photon equals the wave velocity, and because the period *T* is not changed, we have a decrease in length of the refracted wave from the incident. This situation seems contradictory in our view because we have two different quanta p_A, p_M for the mechanical impulse of the wave and photon

$$E_A = mc^2\ ,\ E_M = \frac{h}{T},\quad E_A = E_M\ ,\ n = \frac{c}{v} \rightarrow \quad p_A = mv = m\frac{c}{n};\ \ p_M = \frac{h}{\lambda} = mcn \qquad (8.2)$$

where we have used the previous eq. 8.1 related to state S*1* and the *relativistic equivalent mass m* of the photon. The subscripts refer to Minkowski (E_M,p_M) and Abraham (E_A,p_A), as they correspond to energy-impulse that these authors assigned to the electromagnetic field inside the prism and the controversy that bears their names; it seems clear that the mechanical impulse must have a certain value. According to eq. 1.3, an inertial observer exists for which E_M and p_A are cancelled, while there is no inertial observer for which E_A and p_M are null. Thus, equation E_A=E_M is only valid for the observer at rest with respect to the prism.

Multiplying eq. 5.1 (*S1* state) by the *relativistic mass m*, Planck's constant and the square of light's velocity c^2, we have for the observer at rest with respect to the prism

$$c^2\hbar\overline{k} \bullet m\overline{V} = mc^2\hbar\omega \Rightarrow c^2\overline{p}_M \bullet \overline{p}_A = E_A^2 \rightarrow E_A^2 = c^2 p_A^2 + c^2\overline{p}_A \bullet \left(\overline{p}_M - \overline{p}_A\right) \rightarrow$$
$$E_A^2 = c^2 p_A^2 + \left(mc^2\sqrt{1-\frac{1}{n^2}}\right)^2 \qquad (8.3)$$

Thus, the discrepancy regarding mechanical impulses in state *S1* justifies the assignment of a relativistic mass and a *rest mass* m_o to the photon inside the prism

$$m\sqrt{1-\frac{1}{n^2}} = m_0 \Rightarrow m = \frac{m_0}{\sqrt{1-\frac{V^2}{c^2}}} \qquad (8.4)$$

where V is the speed of light in the prism for any observer. This way, we can speak of the energy-momentum tetra-vector of Abraham. If we repeat the same calculation for the case of the Minkowski impulse, we should assign an *imaginary mass* to the photon. It seems that this invalidates the Minkowski momentum, but experimental results [1] report situations in which one or more impulses physically act. A rest mass (or energy) means that matter, the prism in this case, is able of energy accumulation. Minkowski momentum, would be related to an internal stress [2] induced in the prism by electromagnetic wave. Even in situations where this internal stress not appears, reference [1] indicates that the Minkowski impulse is the only observed and then, according to our arguments, correspond to photons in the *S2 state* within the crystal.

9. WAVE-PARTICLE-FIELD INTERACTION.

From eq. 5.1(*S1 state)*, we can make

$$mc^2\Delta E_w\overline{W} \bullet \overline{V} = c^2\Delta\overline{P}_w \bullet \overline{P}_p = E_p\Delta E_w \qquad (9.1)$$

where m is the relativistic mass of the particle. Conversely, if m_0 is the rest mass of the particle, for small changes in energy-impulse, we have

$$E_p^2 = c^2P_p^2 + \left(m_0c^2\right)^2 \Rightarrow c^2\overline{P}_p \bullet \Delta\overline{P}_p = E_p\Delta E_p \qquad (9.2)$$

and adding the two expressions, we have

$$c^2 \overline{P}_p \bullet \left(\Delta\overline{P}_w + \Delta\overline{P}_p\right) = E_p\left(\Delta E_w + \Delta E_p\right) \qquad (9.3)$$

If the wave-particle interaction is balanced, then the two sides of the equation are null. However, this equation can be maintained with non-null values on both sides; this fact gives us a chance to include field interaction (subscript *f*) as follows

$$c^2 \overline{P}_p \bullet \left(\Delta\overline{P}_w + \Delta\overline{P}_p + \Delta\overline{P}_f\right) = E_p\left(\Delta E_w + \Delta E_p + \Delta E_f\right) \qquad (9.4)$$

Thus, we suppose that *the field-particle interaction is similar to the wave-particle interaction*. We can see easily that this is true in our theory from eqs. 3.1 that we can write separating wave(*w*) and particle(*p*) components this way

$$\Delta\overline{P}_w = \overline{W}\Delta E_w \quad ; \; \Delta E_p = \overline{V} \bullet \Delta\overline{P}_p \qquad (9.5)$$

We can take the second eq. of 9.5 as the *definition of kinetic energy* ΔE_p ; and in our mechanical context the first eq. of 9.5 should correspond to the *definition of potential energy* ΔE_w . It is easy to see that a classic mechanical wave is only possible in a material context with a defined potential energy. A mechanical wave is a bearer of potential energy and we can think that ΔE_w and ΔE_f are of the same nature.

A particle can have the same interaction with different values for its mechanical impulse; if the above equation is valid for all possible values of a particle´s mechanical impulse, it is evident that

$$\Delta\overline{P}_w + \Delta\overline{P}_p + \Delta\overline{P}_f = 0 \; ; \; \Delta E_w + \Delta E_p + \Delta E_f = 0 \qquad (9.6)$$

Thus, state *S1*, extended with an external field, can conserve the energy-impulse. Evidently, if the state is *S2* (extended), energy-impulse conservation is not possible, and we must expect an absorption/emission of additional energy.

10. DUALITY OF MATHEMATICAL OPERATORS.

We can calculate the transformation rule of the partial derivatives corresponding with the coordinate transformation (eqs 1.1) using the chain rule on an arbitrary function *f'(x'(x,t),y',z',t'(x,t))*

$$\begin{aligned}
\left.\frac{\partial f'}{\partial x}\right|_t &= \left.\frac{\partial f'}{\partial x'}\right|_{t'}\left.\frac{\partial x'}{\partial x}\right|_t + \left.\frac{\partial f'}{\partial t'}\right|_{x'}\left.\frac{\partial t'}{\partial x}\right|_t = \beta\left(\left.\frac{\partial f'}{\partial x'}\right|_{t'} - \frac{v}{c^2}\left.\frac{\partial f'}{\partial t'}\right|_{x'}\right) \\
\left.\frac{\partial f'}{\partial t}\right|_x &= \left.\frac{\partial f'}{\partial x'}\right|_{t'}\left.\frac{\partial x'}{\partial t}\right|_x + \left.\frac{\partial f'}{\partial t'}\right|_{x'}\left.\frac{\partial t'}{\partial t}\right|_x = \beta\left(-v\left.\frac{\partial f'}{\partial x'}\right|_{t'} + \left.\frac{\partial f'}{\partial t'}\right|_{x'}\right) \qquad (10.1)
\end{aligned}$$

To get the Lorentz transformation corresponding to the partial derivatives we should note that *v* is the relative velocity measured in the system (x, y, z, t), and therefore it must be on the side corresponding to the partial derivatives calculated in the system coordinate (x, y, z, t); This is achieved easily by solving for the corresponding terms in the previous system of equations

$$\left.\frac{\partial f'}{\partial x'}\right|_{t'} = \beta\left(\left.\frac{\partial f'}{\partial x}\right|_t + \frac{v}{c^2}\left.\frac{\partial f'}{\partial t}\right|_x\right); \; \left.\frac{\partial f'}{\partial t'}\right|_{x'} = \beta\left(\left.\frac{\partial f'}{\partial t}\right|_x + v\left.\frac{\partial f'}{\partial x}\right|_t\right); \; \left.\frac{\partial f'}{\partial y'}\right| = \left.\frac{\partial f'}{\partial y}\right|; \; \left.\frac{\partial f'}{\partial z'}\right| = \left.\frac{\partial f'}{\partial z}\right| \qquad (10.2)$$

the reader will note that the relative velocity does not appear preceded by a minus sign in these transformations , as in the rest of the transformations that have appeared. In Minkowski space, this corresponds to a *covariant vector transformation*. If we impose these transformations 10.2 have two dual forms, using the gradient operator

$$\nabla = -\overline{W}\frac{\partial}{\partial t} \;\; ; \;\; \frac{\partial}{\partial t} = -\overline{V}\bullet\nabla \;\; ; \;\; \left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right) \equiv \nabla \quad (10.3)$$

Taking the first of eqs 10.3 in Cartesian components it is easy to deduce the wave operator in this form

$$\nabla^2 = W^2\frac{\partial^2}{\partial t^2} \quad (10.4)$$

so we have the motion of a wave in this case. The second equation of 10.3 corresponds to the cancellation of the total time derivative *d/dt* , so it holds for any function *f(x,y,z,t) = constant.* For a position (x,y,z) the value of *t* is determined and therefore there is a relation (*x(t),y(t),z(t)*) corresponding to the motion of a particle.

If we introduce quantum operators for energy and impulse linking it to wave values in eq. 5.1 (*S1* state*)* , we have

$$c^2\overline{P}_p\bullet\Delta\overline{P}_w = E_p\Delta E_w;\;\; \bar{v} = \frac{\overline{P}_p}{E_p} \Rightarrow \bar{v}\bullet\Delta\overline{P}_w - \Delta E_w = 0$$

$$\Delta\overline{P}_w \to -i\hbar\nabla \;\; ; \;\; \Delta E_w \to i\hbar\frac{\partial}{\partial t} \Rightarrow \hbar\left[\frac{\partial}{\partial t} + \bar{v}\bullet\nabla\right]\psi(x,y,z,t) = \hbar\frac{d}{dt}\psi(x,y,z,t) = 0 \qquad (10.5)$$

where we have introduced the total temporal derivative of the quantum state. This result is a necessary condition to the particle moves in phase with *ψ(x,y,z,t)*, the quantum state.

11. ENERGY-IMPULSE OF THE WAVE IN THE S1 STATE.

The S1 state verifies $\omega = k^*v$, and for the inertial rest coordinate system of the particle eqs. 1.2 are solved as

$$\omega = \bar{k}\bullet\bar{v} \Rightarrow \omega_0 = 0;\;\; \lambda_0\sqrt{1-\frac{v^2}{c^2}} = \lambda \qquad (11.1)$$

where the zero subscript refers to the coordinate system when the particle is at rest instantaneously. It seems proper to take λ_0 as the *Compton wavelength* of the particle; so we can use eq. 1.3 to calculate the value of momentum and energy of the wave (*w*) in the S1 state and to compare it with particle(*p*)

$$\left.\begin{array}{l} E_w = -mcv\beta^{-1} \\ P_w = mc\beta^{-1} \\ E_w^2 = c^2P_w^2 + \left(imc^2\right)^2 \end{array}\right\};\qquad \left.\begin{array}{c} E_p = mc^2\beta^{-1} \\ P_p = mv\beta^{-1} \\ E_p^2 = c^2P_p^2 + \left(mc^2\right)^2 \end{array}\right\} \qquad (11.2)$$

where we see that the wave energy-momentum are associated with an imaginary mass. We see that energy-impulse components in wave and particle are interchanged. Into the wave-particle interaction, the momentum of the particle becomes energy of the wave and energy of the particle becomes impulse of the wave. Something similar happens when passing the event horizon of a *black hole* in the Schwartschild metric: space takes time characteristics and vice-versa. Equations 11.1-11.2 , with a different definition of λ_0, are also compatible with the existence of an observer at rest with respect to a photon moving inside a dielectric crystal discussed in Section 8 about the Abraham-Minkowski controversy.

12. CONCLUSIONS AND HYPOTHESIS.

The wave-particle duality implies the existence of waves *and* particles; the Abraham-Minkowski paradox shows this aspect of duality. Physical states that maintain an internal stress between wave and particle are possible. This internal stress may explain the absence of radiation from the atomic electrons. Physical states with no interaction between wave and particle corresponds to De Broglie's waves.

From the concept of a interaction between wave and particle we can propose, from De Broglie's[4], the *hypothesis of Pilot Wave* : *In the wave-particle duality context, the particle can interact directly only with its wave*. This idea implies that the wave must be physically influenced by external fields and other forces like obstacles (double slit) or measure apparatus. The Schrödinger equation expresses this influence and must include *all possible classic interactions of the particle*, so it produces the wave with the correct geometry and dynamic. According to this hypothesis, the associated wave is something like a *quantum censorship* to avoid a *naked singularity* for the particle. This quantum censorship *restricts information* about the particle as we can see in Heisenberg's principle of indeterminacy, the Born's rule or the double slit experience. *Non-local* and *non-causal* phenomena, as shown in *Aharonov-Bohm* or *EPR experiences*, can be also attributed to the quantum censorship. But this phenomena, due to non-causality, can not be used to transmit information beyond light's velocity. In this way quantum censorship holds the relativistic limit of light's velocity. A De Broglie wave, with no interaction with the particle, have no limits or external interaction, so it corresponds to the quantum version of the inertia principle for a free particle.

References.

[1]Zhang, Li; She, Weilong; Peng, Nan; Leonhardt, Ulf (2015). "Experimental evidence for Abraham pressure of light". New Journal of Physics 17: 053035.

[2] J. Guck et al., "The Optical Stretcher: A Novel Laser Tool to Micromanipulate Cells," Biophys. J. 81, 767 (2001)

[3] L. de Broglie, "On the Theory of Quanta" A translation of : RECHERCHES SUR LA THEORIE DES QUANTA (Ann. de Phys., 10e serie, t. III (Janvier-Fevrier 1925). by: A. F. Kracklauer. ©AFK, 2004

[4] L. de Broglie, Interpretation of quantum mechanics by the double solution theory. Annales de la Fondation Louis de Broglie, Volume 12, no.4, 1987

[5] Wangsness,R.K., (1979),Electromagnetic Fields, John Wiley and Sons.

Author : Enrique Cantera del Río is Graduate *in Physical Sciencies and Technical Engineer in Telecommunications by Valladolid University (Spain).*