The Dark side of Gravity vs MOND/DM

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We review the foundations of the Dark Gravity Theory and find that, inter alia, the strength of the dark side gravity could be enhanced relative to our side gravity in some space-time domains and that a MOND radius arises naturally so that hopefully the best of both MOND and Dark Matter phenomenology may finally be within reach for Dark Gravity.

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1. Introduction

In the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \tag{1}$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [2][3][6][13][14]. See also [4][7][8][5] [29][30][31][32][33][27] for alternative approaches to anti-gravity with two metric fields.

The action treating our two faces of the Janus field on the same footing is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars respectively built from $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ as usual and L and $\tilde{L}$ the Lagrangians for respectively SM F type fields propagating along $g_{\mu\nu}$ geodesics and $\tilde{F}$ fields propagating along $\tilde{g}_{\mu\nu}$ geodesics. This is invariant under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and the field equation satisfied by the Janus field derived from the minimization of the action is:

$$\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}G^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{G}^{\rho\mu} + \mu \leftrightarrow \nu = -8\pi G(\sqrt{g}\eta^{\mu\sigma}g_{\sigma\rho}T^{\rho\nu} - \sqrt{\tilde{g}}\eta^{\nu\sigma}\tilde{g}_{\sigma\rho}\tilde{T}^{\rho\mu} + \mu \leftrightarrow \nu) \tag{3}$$

with $T^{\mu\nu}$ and $\tilde{T}^{\mu\nu}$ the energy momentum tensors for F and $\tilde{F}$ fields respectively and $G^{\mu\nu}$ and $\tilde{G}^{\mu\nu}$ the Einstein tensors (e.g. $G^{\mu\nu} = R^{\mu\nu} - 1/2 g^{\mu\nu} R$). Of course from the Action extremization with respect to $g_{\mu\nu}$ (useful equations are in [3] section 7), we first obtained an equation for the dynamical field $g_{\mu\nu}$ in presence of the non dynamical $\eta_{\mu\nu}$. Then $\tilde{g}_{\mu\nu}$ has been reintroduced using (1) and the equation was reformatted in such a way as to maintain as explicit as possible the symmetrical roles played by the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ of the Janus field which then required to restore by hand the $\mu \leftrightarrow \nu$ symmetry of the lhs and rhs tensors which we had lost by the way doing so. Then we got (3) which value is the manifest anti-symmetry of the lhs under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. Replacing $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$, this translates into the requirement that the lhs be odd to all orders in $\bar{h}_{\mu\nu}$. Having noticed this will prove useful in our section devoted to the analysis of gravitational waves. The contracted form of the DG equation simply is :

$$\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R} = 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T}) \tag{4}$$

In the seventies, theories with a flat non dynamical background metric and/or implying many kinds of preferred frame effects became momentarily fashionable and Clifford Will has reviewed some of them (Rosen theory, Rastall theory, BSLL theory ...) in his book [34]. Because those attempts were generically roughly conflicting with accurate tests of various versions of the equivalence principle, the flat non dynamical background metric was progressively given up. The DG theory we support here is a remarkable exception as it can easily reproduce most predictions of GR up to Post Newtonian order (as we shall remind in the two following sections) and for this reason deserves much attention since it might call into question the assumption behind most modern theoretical avenues: background independence.

It is well known that GR is the unique theory of a massless spin 2 field. However DG is not the theory of one field but of two fields: $g_{\mu\nu}$ and $\eta_{\mu\nu}$. Then it is also well known that there is no viable (ghost free) theory of two interacting massless spin 2 fields. However, even though $\eta_{\mu\nu}$ is a genuine order two tensor field transforming as it should under general coordinate transformations[a], $\eta_{\mu\nu}$ actually propagates no degrees of freedom : it is really non dynamical, not in the sense that there is no kinetic (Einstein-Hilbert) term for it in the action, but in the sense that all it's degrees of freedom were frozen a priori before entering the action and need not extremize the action : we have the pre-action requirement that Riemm($\eta_{\mu\nu}$)=0 like in the BSLL, Rastall and Rosen theories [34]. So DG is also not the theory of two interacting spin 2 fields.

We will later carry out the complete analysis of the stability of the theory however we already found that, at least about a Minkowskian background common to the two faces of the Janus field, any kind of instabilities are trivially avoided

[a]in contrast to a background Minkowski metric $\hat{\eta}_{\mu\nu}$ such as when we write $g_{\mu\nu} = \hat{\eta}_{\mu\nu} + h_{\mu\nu}$, which by definition is invariant since only the transformation of $h_{\mu\nu}$ is supposed to reflect the effect of a general coordinate transformation applied to $g_{\mu\nu}$

because:

- Fields minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.
- The run away issue [9] [10] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [8][5] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves vanishes about a common Minkowski background (we remind in a forthcoming section that DG has a vanishing energy momentum pseudo tensor $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ in this case) avoiding for instance the instability of positive energy matter fields through the emission of negative energy gravitational waves.

In particular the first two points are very attractive so we were not surprised discovering that recently the ideas of ghost free dRGT bimetric massive gravity [35] have led to a PN phenomenology identical to our though through an extremely heavy, unnatural and Ad Hoc collection of mass terms fine tuned just to avoid the so called BD ghost[b]. Anyway, all such kind of bimetric constructions seriously question the usual interpretation of the gravitational field as being the metric describing the geometry of space-time itself. There is indeed no reason why any of the two faces $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$, which describe a different geometry should be preferred to represent the metric of space-time. At the contrary our non dynamical flat $\eta_{\mu\nu}$ is now the perfect candidate for this role.

We think the theoretical motivations for studying as far as possible a theory such as DG are very strong and three-fold : challenge the idea of background independence, challenge the standard understanding of time reversal and bridge the gap between the discrete and the continuous. Basically modern physics incorporates two kinds of laws: continuous and local laws based on continuous symmetries, most of them inherited from classical physics, and discrete and non local rules of the quanta which remain largely as enigmatic today as these were for their first discoverers one century ago. At first sight the latter don't seem to be related to any fundamental symmetry principles. Though there are many ongoing attempts to "unify" the fundamental interactions or to "unify" gravity and quantum mechanics, the unification of the local-continuous with the non-local-discrete laws would be far more fundamental as it would surely come out with a genuine understanding of QM roots. The intuition at the origin of DG is that the Lorentz group which both

[b]Indeed the first order differential equation in [31] is exactly the same as our: see e.g eq (3.12) supplemented by (4.10) and for comparison our section devoted to the linearized DG equations. This is because the particular coupling through the mass term between the two dynamical metrics in dRGT eventually constrains them to satisfy a relation Eq (2.4) which for $\alpha = \beta$ [31] becomes very similar to our Eq (1) to first order in the perturbations which then turn out to be opposite (to first order) as Eq (4.10) makes it clear.

naturally involves discrete P (parity) and T (time reversal) symmetries as well as continuous space-time symmetries is a natural starting point because the structure of this group itself is already a kind of unification between discrete and continuous symmetries. However neither P nor T in the context of QFT seem to imply a new set of dynamical discrete laws. Moreover our investigation in [6] (see also [13] section 3) revealed that following the alternative non- standard option of the Unitary T operator to understand time reversal led to a dead-end at least in flat spacetime. However we concluded that it might eventually be possible to rehabilitate negative energies and relate them to normal positive energies through time reversal but only in the context of a gravitational theory in which the metric itself would transform non trivially under time reversal. This time reversal not anymore understood as a local symmetry (exchanging initial and final states as does the anti-unitary operator) but as a global symmetry implying a privileged time and a privileged origin of time would jump from one metric to it's conjugate. Only such time reversal would retain it's discrete nature inherited from the Lorentz group even in a generally covariant theory because at the contrary to a mere diffeomorphism but rather like an internal symmetry it would really discretely transform one set of inertial coordinates into another non equivalent one (see [3] section 5), i.e. it would transform a metric into a distinct one describing a different geometry. The DG solutions that we shall remind in the first sections in the homogeneous-isotropic case impressively confirm that our seeked privileged time is a cosmological conformal time and that the two faces of the Janus field are just this time reversal conjugate metrics we have been looking for: the conjugate conformal scale factors are indeed found to satisfy $\tilde{a}(t) = 1/a(t) = a(-t)$ (also see [13] section 6.2). The solutions in the isotropic case then also confirm the reversal of the gravific energy as seen from the conjugate metric.

DG is also the straightforward generalization of GR in presence of a background non dynamical metric so either there is no such background and GR is most likely the fundamental theory of gravity or there is one and DG is the most obvious candidate for it. In a sense DG had to reinvent the zero and negative values for the time and mass-energies which only became possible thanks to the pivot metric $\eta_{\mu\nu}$.

Eventually we are aware that we are not yet ready to derive the Planck-Einstein relations from this new framework but in the following we will have to keep in mind what was our initial motivation: understand the origin of the discrete rules of QM from discrete symmetries to not prohibit oneself the explicit introduction of discrete rules and processes any time the development of the theory seems to require them.

The article is organized as follows: in section 2 we remind and complement the results of previous articles as for the global homogeneous solution and in section 3 the local static isotropic asymptotically Minkowskian solutions of the DG equation. In section 4 we discuss the linearized theory about this common Minkowskian background for $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ and the prediction of the theory as for the emission of gravitational waves. In sections 5 and 6, we give up the hypothesis that the two conjugate metrics are asymptotically the same to derive the isotropic static solution again in this more general case and discuss our pseudo Black Hole and new predictions for gravitational waves. It turns out that the theory of one single Janus field can't account for both the global gravity of section 2 and the local gravity of sections 3, 4, 5 and 6. In sections 7, 8 and 9 we are then led to propose a unification scheme for the global and local Janus field theories based on an original quantization postulate, resulting in a renewed understanding of global expansion effects and the Pioneer anomaly. Section 10 explores the MOND like phenomenology of the unified DG theory. Section 11 emphasizes the need for a theory of gravity such as DG which very principles being based on discrete as well as continuous symmetries, for the first time open a natural bridge to quantum mechanics and hopefully a royal road toward a genuine unification. Section 12 discusses all kind of stability issues to conclude that the theory is safe even in the quantum case thanks to it's interactions involving spectator non dynamical fields. Before the last remarks and outlooks (section 14) and conclusion, section 13 establishes the equations for the evolution of cosmological fluctuations and analyses a new plausible Dark Matter candidate within our framework.

2. Global gravity

2.1. *The scalar-tensor cosmological field*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field are required to satisfy the same isometries. However, it is also static so that the only way to save cosmology in the DG framework is to introduce a tensor-scalar Janus field built from a scalar Φ such that $g_{\mu\nu} = \Phi\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \frac{1}{\Phi}\eta_{\mu\nu}$. Then our fundamental cosmological single equation obtained by requiring the action to be extremal under any variation of $\Phi(t) = a^2(t)$ is:

$$a^2\frac{\ddot{a}}{a} - \tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}} = \frac{4\pi G}{3}(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \quad (5)$$

where $\tilde{a}(t) = \frac{1}{a(t)}$. With this scalar cosmology we avoid the other degrees of freedom and corresponding equations which for a spatial curvature k=0, could only be satisfied all together by a static solution for any equations of state[c]. That the two cosmological equations would be incompatible unless in the static case is most easily

[c]In GR cosmology there is for instance the first order equation $H^2(t) = \frac{8\pi G\rho}{3}$ for k=0. Here for $a >> \tilde{a}$ we can neglect $\tilde{a}$ terms in our equation to get an equation that is also valid within GR. For the scale factor in standard time coordinate, it's just: $\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 = \frac{4\pi G}{3}(\rho - 3p)$. Since we only have this second order equation, in principle the initial conditions i.e. a(t) and $\dot{a}(t)$ could be chosen at will at some particular time yielding $H^2(t)$ very different from $\frac{8\pi G\rho}{3}$ at this time. However for negligible pressure the derivative of $H^2(t) = \frac{8\pi G\rho}{3}$ and matter equations of motion lead to $\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 = \frac{4\pi G}{3}\rho$ so any solution of the first order equation is also solution of the second order equation. The converse is not true and the general solution of our second order equation must involve additional integration constants and terms relative to a solution of the first order

checked in the $a(t) = e^{h(t)}$, $\tilde{a}(t) = e^{-h(t)}$ domain of small h(t). The reason for that is that though DG equations are still generally covariant, the Gauge invariance of GR is lost[d]: the equations are not anymore invariant under the transformations of $g_{\mu\nu}$ alone but under the combined transformations of $g_{\mu\nu}$ and $\eta_{\mu\nu}$. As a result we have no equivalent of the Bianchi identities to make the DG equations functionally dependent. It is therefore not surprising to get several independent equations for the scale factor in contrast to GR when the matter and radiation fields equations of motion are satisfied. So we absolutely need the scalar-tensor Janus field to avoid this.

Now this field is also understood to be "genetically homogeneous" i.e. the spatially independent $\Phi(t)$ at any scale and sourced by the mean expectation value of the usual sources averaged over space rather than the sources themselves. So there are no scalar waves associated to this field and there is also no scale related to a loss of homogeneity of the background effects as in GR. We strongly support the idea that the homogeneity of the scalar field is fundamental just because we want to rehabilitate field discontinuities: in a sense the field will sometimes need to vary discontinuously just because it cannot vary continuously in space[e]. The Pioneer effect, as we shall see is a perfect signature of what we should expect from spatial discontinuities of the scale factor. Of course in a given domain it is possible to require this fundamental homogeneity in a fully covariant way : the conjugate metrics should share the killing vector of a maximally symmetric sub 3d-space insuring that for each metric there is a coordinate system in which it can be written the way we did and it just remains to assume that in this coordinate system for one of the metrics, we also have $\eta_{\mu\nu} = diag(-1, +1, +1, +1)$ for it to be the common conformal coordinate system for both metrics. The difference with the GR treatment of a cosmological metric is that in the context of GR such symmetry would not prevent the metric to fluctuate in any way it wants i.e. for instance non homogeneously.

Now an independent other Janus field is then of course required to describe all other (other than cosmological) aspects of gravity with all it's usual degrees of freedom, but then a field forced to remain asymptotically static to satisfy all the equations. Thus in DG we have two different fields to separately describe the homogeneous evolution and fluctuations respectively. So for instance the source

equation. However since we know that $a(t) \approx t^{2/3}$ and $H^2(t) \approx \frac{1}{a^3(t)}$ is a solution of the first order equation that correctly fits the data in the cold era we can deduce that $H^2(t) \approx \frac{8\pi G\rho}{3}$ is approximately valid just as in GR in our case for $p \approx 0$ with the same deduction that the baryonic matter is cosmologically not abundant enough to account for the measured Hubble rate: in other words we again have a missing mass issue at the cosmological scale.

[d]So this is rather harmless as compared to theories such as for instance unimodular gravity in their diffeomorphism breaking versions [42][43]

[e]In the future, we might relax this hypothesis to allow a new complete scalar sector, because its G coupling constant could be different and actually much smaller than the gravitational coupling constant G of the separate spin 2 theory. This weakness of the new scalar coupling constant would of course be necessary to satisfy all known observational constraints.

densities and pressures are $\bar{\rho}(t)$ and $\bar{p}(t)$ for the homogeneous scalar-tensor field and $\delta\rho = \rho(\mathbf{x}, t) - \bar{\rho}(t)$, $\delta p = p(\mathbf{x}, t) - \bar{p}(t)$, $\rho(\mathbf{x}, t)\mathbf{v}(\mathbf{x}, t)$ ... for the asymptotically Minkowskian spin 2 field, where "bar" denotes spatial averaging.

2.2. *Cosmology*

This section is mainly a review of results already obtained in [13][14]. A new cosmological alternative is also considered.

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms can be neglected, our cosmological equation reduces to a cosmological equation known to be also valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term. The evolution of our side scale factor before the transition to the accelerated regime is depicted in blue on the top of Figure 1 as a function of the conformal time t and the corresponding evolution laws as a function of standard time t' are also given in the radiative and cold era. Notice however the new behaviour about t=0 meaning that the Big-Bang singularity is avoided.

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is time reversal in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid within space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equation: in other words, the equations of free fall for our "average source field" did not apply at the discrete transition in time.

Let's be more specific. The equations of free fall for the perfect fluids on both sides of course apply as usual before and after the transition and for instance on our side in the cold era dominated by non relativistic matter with negligible pressure, we have $\frac{d}{dt}(\rho a^3) = 0$. Such conservation equation is valid just because it follows from our action for the matter fields on our side. But equations of motion and conservation equations are less fundamental than the symmetry principles of the action they are derived from. Here we not only have the usual invariance of our action under continuous space-time symmetries from which we can derive the corresponding field conservation equations closely related to the continuous field equations of motion. But we also have the invariance of the action under a permutation which is a discrete symmetry. In the same way as continuous symmetries generate continuous evolution and interactions of the fields we here take it for granted that our new permutation symmetry also allows a new kind of process to take place : the actual permutation of the conjugate a and $\tilde{a}$. The process is understood to modify all terms explicitly scale factor dependent in the cosmological equation whereas all density and pressure terms remain unchanged. Because such process is not at all related to the continuous symmetries that generate the continuous field equation there is indeed no reason why the discrete version $(\rho a^3)_{before} = (\rho a^3)_{after}$ of a conservation equation such as $\frac{d}{dt}(\rho a^3) = 0$ should be satisfied by this particular process. Again symmetry principles are more fundamental than conservation equations so we should not be disturbed by a process which violates the conservation of energy since this process is discontinuous and related to a new discrete symmetry for which we have no equivalent of the Noether theorem. Here the valid rule when the permutation of the scale factors occurs is rather $\rho_{before} = \rho_{after}$ and the same for the pressure densities.

This permutation (at the purple point depicted on figure 1) could trigger the recent acceleration of the universe. This was demonstrated in previous articles [13] and [14] assuming our side source $a^4(\rho - 3p)$ term has been dominant and therefore has driven the evolution up to the transition to acceleration. If this term is still dominant after the transition we get an accelerated expansion regime $(t' - t'_0)^{-2}$ in standard time coordinate with a Big Rip at future time t'_0[14]. However this scenario needs densities on the conjugate side much smaller than on our side so may be a more natural possibility is that, following the transition, the dark side source term have started to drive the evolution : $a^4(\rho - 3p) << \tilde{a}^4(\tilde{\rho} - 3\tilde{p})$ resulting from $a(t) << \tilde{a}(t)$ and $\rho - 3p \approx \tilde{\rho} - 3\tilde{p}$. Then the equality of densities would be the perfect trigger for the transition all the more since our equation is actually invariant under the combined permutations of densities and scale factors. As we can't reasonably physically exchange the densities, the scale factor permutation could only occur at the crossing when we have equal density source terms.

Then, we have two possibilities :

- The conjugate side is currently in a radiative regime, so that our cosmological equation simplifies in a different way[f] :

$$\tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}} \approx \frac{4\pi G}{3}\tilde{a}^4(\tilde{\rho} - 3\tilde{p}) = K\tilde{a}^2 \tag{6}$$

The solution $\tilde{a}(t) = C.sh(\sqrt{K}(t - t_0)) \approx C\sqrt{K}(t - t_0)$ for $1/C << \sqrt{K}(t - t_0) << 1$ so $a(t) \propto 1/(t - t_0)$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate. t_0 is determined by demanding the continuity of $H(t) = \frac{\dot{a}}{a} = \frac{-1}{t-t_0}$ after the transition which should match 2/t before the transition. This is not in concordance

[f] That a quantity such as $\tilde{\rho} - 3\tilde{p}$ is expected to follow a $1/\tilde{a}^2$ evolution in the limit where all species are ultra-relativistic can be deduced from Eq (21)-(25) of [39] and the matter and radiation energy conservation equation rewritten as $\tilde{\rho} - 3\tilde{p} = 4\tilde{\rho} + \tilde{a}\frac{d\tilde{\rho}}{d\tilde{a}}$ in a radiation dominated dark side of the universe when $\tilde{\rho}$ and $\tilde{p} \approx 1/\tilde{a}^4(t)$.

with our understanding of time reversal[13] because it requires a shift and redefinition of the origin of time.

- The dark side is also in a cold era at the transition and satisfies $\tilde{\rho} - 3\tilde{p} \approx \tilde{\rho} = \rho - 3p \approx \rho$. Then the continuity of the Hubble rate is automatically satisfied without having to define a new t_0 after transition. For this reason this is still our privileged scenario first introduced in [13]. This leads to a constantly accelerated universe $a(t') \approx t'^2$ in standard coordinate following the transition redshift.

In all scenarii, constraining the age of the universe to be the same as within LCDM the transition redshift can be predicted (see [14] equation 6) and confronted to the measured value $z_{tr} = 0.67 \pm 0.1$.

- For the constantly accelerated universe the prediction is $z_{tr} = 0.78$ in very good agreement with the measured transition redshift.
- In the Big Rip scenario, $z_{tr} = 0.27$. Yet the hypothesis that the transition occurred everywhere simultaneously might not be valid. Otherwise the mean transition redshift should be significantly increased by an expected dispersion of transition redshifts due to inhomogeneities (some domains being already in the accelerating regime while others are still in the decelerating one) smoothing the observed transition between decelerated and accelerated expansion after averaging over large regions and making the theory difficult to discriminate from the very progressive LCDM transition. The mean measured transition redshift is indeed very sensitive to a smoothing. Our interest in this Big Rip scenario is motivated by the anomaly of the best precision "recent" cosmological measurement of $\bar{H}_0 = 73.03 \pm 1.79 (km/sec)/Mpc$ [17][18] over the two last billion years (300 SNe Ia at $z < 0.15$ having a Cepheid-calibrated distance) appearing to be exceeding by three standard deviations the one predicted by LCDM from Planck data. This is noteworthy because an unexpectedly high recent acceleration could of course be the signature of such Big Rip vs LCDM expectations.
- For the exponentially accelerated expansion scenario (hence just like the one produced by a pure cosmological constant) $z_{tr} \approx 0.4$. This again is assuming a transition occurring everywhere simultaneously which is just equivalent to a fictitious LCDM discrete transition between a purely CDM and a purely Lambda driven expansion regime (the Hubble rate still being continuous at the transition).
  The comparison with $z_{tr} \approx 0.7$ predicted for an actual progressive LCDM transition confirms that a smoothing effect would significantly increase the mean observed z_{tr} again making this scenario even harder to discriminate from a LCDM transition.

Whatever the actual scenario we believe that such alternative to the cosmological constant is more satisfactory as it follows from first principles of the theory and eventually should fit the data without any arbitrary parameter, everything being only determined by the actual matter and luminous content of the two conjugate universes, such content so far not being directly accessible for the dark side.

3. Local gravity: the isotropic case about Minkowski

Another Janus field and it's own separate Einstein Hilbert action are required to describe local gravity with isotropic solution in vacuum of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$.

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{7}$$

$$B = \frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{8}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. The detailed comparison will be carried out in section 6. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

4. Local gravity : linear equations about Minkowski

The linearized equations about a common Minkowskian background look the same as in GR, the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2 on the linear lhs:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu} + t_{\mu\nu} - \tilde{t}_{\mu\nu}) \tag{9}$$

however to second order in the perturbation $h_{\mu\nu}$ (plane wave expanded as usual) and given that $\tilde{h}_{\mu\nu} = -h_{\mu\nu} + h_{\mu\rho}h_{\nu\sigma}\eta^{\rho\sigma} + O(3)$ we found that the only non cancelling contributions to $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the rhs, vanish upon averaging over a region of space and time much larger than the wavelength and period (this is the way the energy and momentum of any wave are usually evaluated according [1] page 259). This $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ is standing as usual for the energy-momentum of the gravitational field itself because the Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^{\mu}}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu}) = 0 \tag{10}$$

We can come to the same conclusion that $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ vanishes but now to all orders if we remind ourselves that the geometrical part (lhs) of the DG equation (3) is odd to all orders in $\bar{h}_{\mu\nu}$ (not to be confused with $h_{\mu\nu}$ nor $\tilde{h}_{\mu\nu}$) after making the replacement $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ thus $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$. Then we are free to use the plane wave expansion of this new $\bar{h}_{\mu\nu}$ instead of $h_{\mu\nu}$ and because each term of the perturbative series has an odd number of such $\bar{h}$ factors, such term will always exhibit a remaining $e^{i\mathbf{kx}}$ factor which average over regions much larger than wavelength and period vanishes (in contrast to [1] page 259 where the computation is carried on for quadratic terms for which we are left with some x^μ independent, hence non vanishing, cross-terms).

Our new interpretation is that any radiated wave will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ about Minkowski resulting in a total vanishing radiated energy. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

Actually, we shall show in the next two sections that, since the asymptotic behaviours of the two sides of the Janus field are not necessarily the same, we could both expect from the theory an isotropic solution approaching the GR Schwarzschild one with it's black hole horizon and the same gravitational wave solutions, including the production rate, as in GR but also, whenever some particular yet to be defined conditions are reached, the above DG solutions, with a vanishingly small production rate of gravitational waves and the B=1/A exponential DG Schwarzschild solution without horizon. Both will be limiting cases of a more general solution.

5. Differing asymptotic values

Due to expansion on our side and contraction on the dark side the common Minkowskian asymptotic value of our previous section is actually not a natural assumption.

At the contrary a field assumed to be asymptotically $C^2\eta_{\mu\nu}$ with C constant has its conjugate asymptotically $1/C^2\eta_{\mu\nu}$ so their asymptotic values should differ by many orders of magnitude.

Given that $g_{\mu\nu}^{C^2\eta} = C^2 g_{\mu\nu}^{\eta}$ and $\tilde{g}_{\mu\nu}^{\eta/C^2} = \frac{1}{C^2}\tilde{g}_{\mu\nu}^{\eta}$, where the $< g^\eta, \tilde{g}^\eta >$ Janus field is asymptotically η, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behaviour for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C^2\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C^2}\sqrt{\tilde{g}}\frac{\tilde{G_{tt}}}{\tilde{g}_{tt}} = -8\pi G(C^4\sqrt{g}\delta\rho - \frac{1}{C^4}\sqrt{\tilde{g}}\tilde{\delta\rho}) \tag{11}$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\delta\rho$ is as usual the energy density for matter and radiation density fluctuations. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields. Notice that for no fluctuations, the solutions are Minkowskian as needed, being understood that the background plays it's dynamics in the global Janus field equation rather than in this local Janus field equation.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi G C^2 g_{tt}\delta\rho$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^8$ factor (in the weak field approximation, $G_{tt} = 8\pi G\frac{\delta\tilde{\rho}}{C^6}$). From $g^{\eta}_{\mu\nu}$ we can get back $g^{C^2\eta}_{\mu\nu}$ and then of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we tend to GR : we expect the same gravitational waves emission rate and the same weak field gravitational field. However on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

Of course the roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C^2}G\tilde{g_{tt}}\delta\tilde{\rho}$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^8$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi G C^6\delta\rho$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field).

Only in case C=1 do we recover our local exponential Dark Gravity, with no significant GW radiations and also a strength of gravity ($G_{tt} = -4\pi G\delta\rho$) reduced by a factor $2C^2$ relative to the above GR gravity ($G_{tt} = -8\pi G C^2\delta\rho$).

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system.

Eventually, depending on the local C value in a given space-time domain, a departure from GR predictions could be expected or not both for the gravitational waves radiated power and the local static gravitational field e.g. depending on a context able or not to trigger a reset to C=1, we could get either the DG exponential elements or the GR Schwarzschild solution for the static isotropic gravity; and get either no gravitational waves at all or the same radiated power as in General Relativity[g].

6. Back to Black-Holes and gravitational waves

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild

[g]For $C >> 1$ we also approximately recover the Gauge invariance of GR

radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations tend to GR. However this can't be the case when we approach the Schwarzschild radius because C is finite and the metric elements can grow in such a way that we could not anymore neglect the dark side geometrical term. Therefore presumably the horizon singularity is avoided as well for C $\neq 1$. To check this we need the exact differential equations satisfied in vacuum by C-asymptotic isotropic static metrics of the form $g_{\mu\nu} = (-B, A, A, A)$ in e.g. $d\tau^2 = -Bdt^2 + A(dx^2 + dy^2 + dz^2)$ and $\tilde{g}_{\mu\nu} = (-1/B, 1/A, 1/A, 1/A)$. With $A = C^2 e^a$ and $B = C^2 e^b$, we get the differential equations satisfied by a(r) and b(r):

$$a'' + 2a' + \frac{a'^2}{p} = 0 \tag{12}$$

$$b' = -a' \frac{1 + a'r/p}{1 + 2a'r/p} \tag{13}$$

where $p = 4\frac{e^{a+b}C^4+1}{e^{a+b}C^4-1}$. GR is recovered for C infinite thus p=4. Then the integration is straightforward leading as expected to

$$A = C(1 + U)^{p=4}; \tag{14}$$

$$B = C(\frac{1-U}{1+U})^{(p=4)/2} \tag{15}$$

where $U = GM/2r$ and the infinite C can be absorbed by opting to a suitable coordinate system : then there is no dark side. DG C=1 corresponds to b=-a, p infinite and the integration, as expected, gives $A = e^U$, $B = e^{-U}$.

The integration is far less trivial for intermediary Cs because then p is not anymore a constant, however in the weak field approximation, treating p as the constant $4\frac{C^4+1}{C^4-1}$ the PPN development of the above solutions brings to light a possible departure from GR at the PostPostNewtonian level since:

$$A_{GR} \approx 1 + 4U + 6U^2 \tag{16}$$

$$B_{GR} \approx 1 - 4U + 8U^2 - 12U^3 \tag{17}$$

$$A_{p\neq 4} \approx 1 + pU + \frac{p(p-1)}{2}U^2 \tag{18}$$

$$B_{p\neq 4} \approx 1 - pU + \frac{p^2}{2}U^2 - p\frac{2+p^2}{6}U^3 \tag{19}$$

This makes clear that for $p \neq 4$ redefining the coupling constant to match GR at the Newtonian level, which amounts to replace U by 4U/p in the above expressions, a discrepancy would remain at the PPN level relative to GR predictions.

$$A_{p\neq 4} \approx 1 + 4U + 8(\frac{p-1}{p})U^2 \tag{20}$$

$$B_{p\neq 4} \approx 1 - 4U + 8U^2 - \frac{32}{3}(\frac{2+p^2}{p^2})U^3 \tag{21}$$

For $4 \leq p = 4\frac{1+1/C^4}{1-1/C^4} \leq \infty$ the departure from GR is the greatest for p infinite (C=1) :

$$A_{DG} \approx 1 + 4U + 8U^2 \tag{22}$$

$$B_{DG} \approx 1 - 4U + 8U^2 - \frac{32}{3}U^3 \tag{23}$$

but should hopefully soon become testable with the data from neutron stars or black holes mergers if C is not too big.

In the strong field regime we need to rely on numerical approximation methods to understand what's going on near the Schwarzschild radius. The numerical integration in Geogebra (using NRésolEquaDiff) was carried on and the resulting b(r) are shown in Figure 2 for various C values. It is found that as C increases b(r) will closely follow the GR solution near the Schwarzschild radius over an increasing range of b(r) which can be many orders of magnitude and perfectly mimic the GR black hole horizon, however at some point the solution deviates from GR and crosses the Schwarzschild radius without singularity. Therefore, as far as the numerical integration is reliable our theory appears to avoid horizon singularities (true Black Holes) for any finite C and not only C=1. This means that the collapsed star will only behave as a Black Hole for a finite time after which the external observer will be able to learn something about what's going on beyond the pseudo Horizon. Indeed, the resulting object having no true horizon is in principle still able to radiate extremely red-shifted and delayed light or gravitational waves emitted from inside the object.

The classical picture of a collapse toward a central singularity could therefore also be probed which is interesting because we have another mechanism within our framework that could stop the collapse: when the metric reaches some threshold, the inner region (the volume defined by the star itself) global and local fields could respectively be reset to Minkowski and C=1. This discrete transition would produce a huge discontinuity at a spherical surface with radius very close to the Schwarzschild Radius (because this is where the postulated metric threshold is expected to be reached). This surface would behave like the hard shell of a gravastar [44] and likely produce the same kind of phenomenological signatures such as echoes following BH mergers which might already have been detected [22].

Then at the center of such object, the two faces of the Janus field should get very close to each other just because C=1 and because this is where the own star potential vanishes. The crossing of the metrics is the required condition to allow the transfer of matter and radiation between the star and the conjugate side there. The lost of a significant part of its initial mass along with the strength of gravity being reduced by a factor 2C for DG relative to GR should eventually stop the collapse as it would allow new stability conditions to be reached.

To still behave as a very gravific object while it has lost most of it's matter and gravitational strength, the discontinuity itself must be gravific and behave as an equivalent gravific mass as the original one[h]. This is expected as the discontinuity is at a domain boundary and just needs to "store" the original value of the metric and it's derivative at the surface at the time it became this domain boundary. Then the external Schwarzschild type solution in vacuum is obtained merely thanks to these boundary conditions.

Shocks and matter anti-matter annihilation at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [21]) which we remember is also a bridge toward the dark side and it's presumably anti-matter dominated fluid, could also produce further GWs radiation which would be much less natural from a regular GR Black Hole [22].

Eventually in the vicinity of stars as well as in "Black Holes" we can't exclude a transfer of matter and radiation through the discontinuity at crossing metrics that would proceed in the opposite way feeding them and increasing their total energy : a possible new mechanism to explain the unexpectedly high gravific masses of recently discovered BH mergers but also an attractive simple scenario to explain the six SN like enigmatic explosions of the single massive star iPTF14hls if they resulted from a succession of injections of antimatter from the dark side[40]. Such discontinuities in the vicinity of stars could also block matter accumulating in massive and opaque spherical shells around stars : a possible scenario to explain the reduced light signal from the recently discovered neutron stars merger.

Of course a Kerr type solution also remains to be established in our framework which is postponed for some future paper. But it is already clear that both conjugate metrics as well as the Minkowski metric in between them must be expressed in ellipsoidal coordinates (remind that our theory is generally covariant) hence in the form given by [45] Eq 21 for the Minkowski metric and Eq 22 or similar for the ensatz in input to our differential equations.

7. The unified DG theory

7.1. *Actions and space-time domains*

We earlier explained why the theory must split up into two parts, one with total action made of an Einstein Hilbert action for our scalar-tensor homogeneous and isotropic Janus field added to SM actions for F and $\tilde{F}$ type averaged fields respectively minimally coupled to $\Phi\eta_{\mu\nu}$ and $\Phi^{-1}\eta_{\mu\nu}$. The other part of the theory has an Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g_{\mu\nu}$ for local gravity added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$.

The two theories must remain completely separate. Indeed, to remain asymptot-

[h]or even an event greater one which then might lead to pseudo BHs much more massive than we believed them to be.

ically static, $g_{\mu\nu}$ must be isolated from the scale factor effect. But also as announced earlier the scalar field is spatially independent at all scales so admits only perfectly homogeneous sources. So a unified theory cannot be obtained by mixing the local and global gravity in a Lagrangian term. However it's still possible to add the following global and local actions, being understood that no dynamical field is shared between them.

$$\int_{Global} d^4x(\sqrt{g}(R+L)+\sqrt{\tilde{g}}(\tilde{R}+\tilde{L}))+ \tag{24}$$

$$\int_{Local} d^4x(\sqrt{g}(R+L)+\sqrt{\tilde{g}}(\tilde{R}+\tilde{L})) \tag{25}$$

This confirms that even for the sources, the average background and perturbations are different dynamical fields, the former in the global L and $\tilde{L}$, the latter in the local L and $\tilde{L}$ (yet this total action will be helpful to later establish a non trivial connection between global and local gravity in some particular areas). On the other hand considering the global and local physics of those actions running in parallel totally decoupled and uninterrupted as implied by our above total action leads to another issue. We need to understand then how light, clocks and rods can both feel the effect of global expansion and local gravity being now understood that those light, clocks and rods do not even appear in the global Lagrangians L and $\tilde{L}$ above just because as we already noticed only averaged perfectly homogeneous, perfect fluid densities and pressures are there.

Our proposal for solving this problem is that the asymptotic local static gravity is actually only a constant piecewise function of time rather than rigorously the stationary $\eta_{\mu\nu}$. In other words it is rather $C^2\eta_{\mu\nu}$ which asymptotic value C is piecewise constant, being periodically discontinuously updated to $a(t)$ in such a way that it closely follows the evolution of $a(t)$ through a series of fast discrete transitions on a regular basis. Eventually, clocks and rods coupling to local gravity only but never coupling directly to $a(t)$, can still feel the effects of the continuous global expansion indirectly thanks to this mechanism.

Here as in GR for the isotropic static case, C is a mere integration constant, and as such cannot depend on time, however it can take different values in successive time slots, the differential equations being only valid piece-wise. We understood in a previous section how relevant is this asymptotic value within DG which has no obvious peer within GR. Now we can even point out a striking analogy with what Quantum Field Theory actually describes : the succession of continuous local and discontinuous non local processes respectively described by the propagation of fields according classical wave equations and the annihilation/creation of these fields wherever interactions take place, i.e. respectively propagators and vertices in the Feynman language. We even feel tempted to name our discrete transition of C, a quantization rule even though it is quite an unusual one as it applies to a zero frequency component in contrast to what we learned from the Planck-Einstein relations predicting vanishing quanta in the zero frequency limit.

Another issue is that gravity in the inner part of the solar system as we know it from thorough studies during the last decades exclude that global gravity applied to clocks and rods without being strongly attenuated. Indeed, it would otherwise lead to strongly excluded expansion effects of orbital planetary periods relative to atomic periods: the gravitational constant G would seem to vary at a rate similar to H_0 which is not the case. GR solves this problem because it predicts that significant expansion effects only take place on scales beyond those of galaxy clusters. At the contrary, the theory involving the physics of the global action above would produce expansion effects with the same magnitude at any scale: in fact everywhere the scale factor $a(t)$ evolution takes place and therefore the C evolution according the mechanism explained above. Therefore we are led to the conclusion that the scale factor effects did not apply to the inner part of the solar system at least during the last decades. This is the only possible solution not to conflict with observational constraints: no evidence of expanding planet trajectories so far. This implies the existence of frontiers between space-time domains where the dynamics of the global $\Phi\eta_{\mu\nu}$ field takes place and the $g_{\mu\nu}$ asymptotic value C is step by step discontinuously driven by the scale factor and others where this dynamics is totally absent, for instance in the inner part of the solar system during the last decades. An alternative possibility is that the dynamics of the global field extends over the whole universe but that the mechanism which translates the a(t) evolution into the step by step C(t) evolution is only switched on in some delimited spatial domains: again we expect frontiers between constant C and drifting C(t) domains. We will later be in a better position to decide which alternative is likely the best one.

7.2. *Field discontinuities*

Eventually the theory has cosmological field discontinuities in time as well as at frontiers between spatial zones. Let's stress that those are not related at all to our permutation symmetry and the related discrete cosmological transition process that could trigger the acceleration of the universe but should rather be considered as a quantization rule for the asymptotic field. Now the usual conservation equations for matter or radiation are not violated when crossing such frontiers though in presence of genuine potential discontinuities. Indeed it's possible to describe the propagation of the wave function of any particle crossing this new kind of discontinuous gravitational potential frontier just as the Schrodinger equation can be solved exactly in presence of a squared potential well : we just need to require the continuity of the matter and radiation fields and continuity of their derivatives at the gravitational discontinuity. Since the differential equations are valid everywhere except at the discontinuity itself where they are just complemented by the former matching rules we obviously avoid the nuisance of any infinite potential gradients and eventually only potential differences between both sides of such discontinuity will physically matter. For instance we can now have have $(\rho a^3)_{before-crossing} = (\rho a^3)_{after-crossing}$ in contrast to what we had following the permutation transition $(\rho_{before-crossing} = \rho_{after-crossing})$. Of course clocks and rods must remain insensitive to the latter discrete permutation of the scale factor responsible for the cosmological transition to global acceleration. This is possible if clocks and rods are not sensitive to the corresponding transitions of the local field asymptotic value C i.e $\delta\rho$ and $\delta\tilde{\rho}$ just as $\bar{\rho}$ and $\tilde{\bar{\rho}}$ do not feel the transition to acceleration.

7.3. *Space-time domains and the Pioneer effect*

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a(t)$ in $\Phi\eta_{\mu\nu}$ (still through our indirect mechanism) and another without such effect. The reader is invited to visit the detailed analysis in our previous publication [14] starting at page 71. We shall only remind here the main results. Electromagnetic periods and wavelengths are not impacted in any way during the propagation of electromagnetic waves in the conformal coordinate system where we wrote our cosmological equation even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate H_0. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} \approx 2H_0$ where f_P and f_E stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [11][12]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to global expansion (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

7.3.1. *Back to PLL issues*

As we started to explain in our previous article [14] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically ”follow” a Pioneer like drift in time. We already pointed out that nobody knows how the scale factor (here our

C(t)) actually varies on short time scales: in [14] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [14]. This view is now even better supported since our clocks and rods are understood not to be anymore directly sensitive to the scale factor, but rather indirectly, only through the local field asymptotic value C closely following by a succession of discontinuous steps rather than continuously the evolution of $a(t)$ as the latter is implied by our cosmological differential equation. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is impacted by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

7.4. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate H_0 with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances: this is just the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate. However, according our above analysis this was not locally the case at least during the last decades which did not manifest any cosmological effect (G did not vary) in the inner part of the solar system.

This necessarily implies that earth clocks must have been submitted to alternating static and expanding regimes. It just remains to assume (the full justification will be provided in a forthcoming section) that through cosmological times, not only earth clocks but also all other clocks in the universe, spent exactly half of the time in the expanding regime and half of the time in the static regime, in a cyclic way. It follows that the instantaneous expansion rate $H_0 = 2\bar{H}_0$ of our global field as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2\bar{H}_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the expected field discontinuities at the frontier between regions with different expansion regimes, and likely related effects. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between BBN and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much C drift relative to others.

8. Frontier dynamics

Our next purpose is to understand the physics that governs the location of frontier surfaces between regions identified in the previous sections. Consider the gravitational field total action in a space-time domain where global expansion takes place:

$$\int_{Global} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \tag{26}$$

$$\int_{Local} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) \tag{27}$$

where in the global (resp local) actions the gravitational field is $\Phi\eta_{\mu\nu}$ (resp C-asymptotic $g_{\mu\nu}$). We would like to determine the frontier surface of this domain at the time t the local field asymptotic value C is reset to the scale factor in our domain. Considering the frontier to be stationary between two such successive updates, the frontier position is determined at any time. If such surface is moving because of successive updates it will of course scan a space-time volume as time is running out. To determine this hypersurface we extend the extremum action principle. Not only the total action should be extremum under any infinitesimal field variations which as we all know allows to get the field equations but also the total action is required to be extremum i.e. stationary under any infinitesimal displacement of this hypersurface which is nothing but the frontier of the action validity domain. But the displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{local} = 0 \tag{28}$$

This equation is merely a constraint relating local gravity (terms 3 and 4) to global gravity (terms 1 and 2) at the hyper surface.

Now provided one scale factor dominates the other side one we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} \approx \pm_{a<<\tilde{a}}^{a>>\tilde{a}}(\sqrt{g}R - \sqrt{\tilde{g}}\tilde{R})_{global} \tag{29}$$

and then we can make use of the contracted equation 4 to replace:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} \approx \pm_{a<<\tilde{a}}^{a>>\tilde{a}} 8\pi G(\sqrt{g}T - \sqrt{\tilde{g}}\tilde{T})_{global} \tag{30}$$

in equation 28 and we can do the same for the local part provided one C asymptotic dominates the other side one. Then equation 28 becomes:

$$\pm^{a>>\tilde{a}}_{a<<\tilde{a}}(a^4 < \rho - 3p > -\tilde{a}^4 < \tilde{\rho} - 3\tilde{p} >) \pm^{C>>\tilde{C}}_{C<<\tilde{C}}(C^4 F(r)\Delta(\rho - 3p) - \tilde{C}^4 \tilde{F}(r)\Delta(\tilde{\rho} - 3\tilde{p})) = 0 \tag{31}$$

The $F(r) = e^{\Phi(r)}$ and $\tilde{F}(r) = e^{-\Phi(r)}$ here account for the effect of a local assumed static isotropic gravitational potential $\Phi(r)$. The $<>$ and Δ denote averages and fluctuations. First and third terms currently behave as $a(t)$ in our cold side of the universe while second and fourth terms are expected to follow a $\tilde{a}(t)$ evolution law if the dark side is also cold.

The relative magnitudes of the fluctuations Δ can be very different from the relative magnitudes of the averages $<>$ given the extremely non linear structures in the current universe. Is this enough to make the relative magnitudes of terms 1 and 2 in the opposite way to the relative magnitudes of terms 3 and 4 ? Unlikely at first sight given the huge expected current ratio $a(t)/\tilde{a(t)} = C(t)/\tilde{C(t)} \approx z^2_{crossing} >> 10^{18}$ if $z_{crossing}$ is the redshift of the conjugate scale factors equality probably much greater than the BBN redshift. Moreover before decoupling, beyond z=1000, because $a^4 < \rho - 3p > \propto a^2$, the magnitudes of terms 1 and 3 relative to terms 2 and 4 evolve as z^3 so it is almost impossible to imagine term1 $<<$ term2 yet term3 $>>$ term4 now. But then if term3 $<<$ term4, the equation today (with negligible pressures) simplifies to :

$$\tilde{a}^4 < \tilde{\rho} > + \tilde{C}^4 \tilde{F}(r)\Delta\tilde{\rho} = 0 \tag{32}$$

Such equation is not satisfactory because both term evolve in the same way as a function of time, so this can't lead us to a trajectory r(t) for our hypersurface. Yet we absolutely need drifting frontiers and domains to insure that all clocks will drift in the same way over cosmological times. Moreover in most usual weak field situations, $\tilde{F}(r)$ is very close to unity so it could only be satisfied for small under dense fluctuations $\Delta\tilde{\rho} \approx - < \tilde{\rho} >$.

Therefore the mechanism linking a(t) to C(t) must be slightly more subtle than the first postulated one and involve from time to time a resetting normalization factor : $C(t) = a(t)/a(t_N)$ at some particular times t_N. This allows to kill two birds with one stone as term1 $<<$ term2 and term3 $>>$ term4 now becomes possible yielding:

$$\tilde{a}^4 < \tilde{\rho} >= C^4 F(r)\Delta\rho \tag{33}$$

Therefore, in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ and a quite uniform $\Delta\rho(r)$ so we may neglect it's radial dependency (for instance in the empty space surrounding a star) we are led to:

$$a^{\gamma}(t) \propto e^{\frac{2MG}{rc^2}} \tag{34}$$

Our privileged constantly accelerated scenario has $\gamma = -2$ while the exponential acceleration scenario leads to $\gamma = -3$. This equation gives us nothing but the "trajectory" r(t) of the hypersurface we were looking for. Here obtained in the conformal time t coordinate system, it is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really the DG exponential Schwarzschild solution. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$\gamma 2\bar{H}_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{35}$$

here taking into account that the instantaneous Hubble factor H_0 is actually $2\bar{H}_0$, i.e. twice the average cosmological Hubble parameter that we know from cosmological probes as we explained earlier. From this we learn that in case $\gamma \neq 0$ the frontier between the two domains is drifting at speed:

$$\frac{dr}{dt} = \gamma \frac{\bar{H}_0}{\frac{d\Phi(r)}{dr}} \tag{36}$$

and therefore could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started except that this time the two regions will need to exchange their roles about the moving frontier. In other words if for a given cycle the expanding region is the outer one and the static region the inner one, the next cycle will be with the inner part expanding and the outer part static. After two such complete cycles any area will have spent exactly the same total time static and expanding at $2\bar{H}_0$ resulting in the promised average $\bar{H}_0$. Thus the $\gamma \neq 0$ case must be the correct one if we want to understand both the Pioneer effect, the expansion of the universe, and an expansion dynamics which only takes place in some delimited space-time domains. $\gamma = 0$ at the contrary implies a static frontier in the solar system near the orbit radius of Saturn according the Pioneer effect, leading to the unacceptable result that natural clocks (atoms) in the outer part of the solar system would have accumulated a huge drift of their periods relative to earth atoms over cosmological times. A Geogebra animation in [[16]] helps visualizing the evolution of the local potential over one complete cycle and understand why the resetting normalization factor is indeed mandatory to insure that the global and local term in (33) can remain in contact over many cycles.

An alternative derivation of this phenomenology is possible following the idea that actually the scale factor dynamics applies everywhere in the universe but that this is only reflected on the C(t) evolution in some delimited spatial domains. In domains where C is momentarily constant while a(t) evolves we again get an equation such as (33) in the most natural case i.e. term1 << term2 and term3 << term4 but now the local term is frozen in time so we shall finally obtain $\gamma = 1$. In this case we might do without the upgraded mechanism involving the resetting normalisation factor from time to time because it does not seem absolutely essential anymore.

We may estimate an order of magnitude of the characteristic period of this cyclic drift assuming that the cycle is over when the frontier reaches the deepest potential levels. For collapsed stars such as white dwarfs or neutron stars this would give a far too long cycle exceeding billions of years because their surface potential is so deep and even much worse for black holes. But the majority of stars have very similar surface potentials even though there is a large variability in their masses and sizes. So we may take the value of our sun a-dimensional surface potential which is about 2.10^{-6} as indicative of a mean and common value. To that number we should add the potential in the gravitational field of the Milky Way and the potential to which the local cluster of galaxies is subjected. Knowing the velocities: 220 km/s of the sun about the center of the galaxy and 600km/s of the local cluster vs the CMB, the virial approximation formula $\frac{v^2}{c^2} \approx GM/rc^2$ may lead us to a crude estimation of each contribution and a total potential near 6.10^{-6}. Then the order of magnitude of the period cycle would be in between 10^4 and 10^5 years.

9. Apparent variations of G

Because clocks and rods submitted to local gravity also indirectly felt the effects of global expansion through our quantized (discontinuous step by step) evolution of C, if we could test gravity over the past cycles we would necessarily detect that it's strength was different and has changed in the same proportion as the scale factor itself.

We come to this conclusion by deriving the equation of motion of a body of charge q and mass m orbiting with a quasi circular motion (so we can neglect radial speeds) in an isotropic electrostatic field described by the potential V(r) and gravitational field with metric

$$d\tau^2 = C^2(t)(B(r)dt^2 - A(r)(dx^2 + dy^2 + dz^2)) \tag{37}$$

Where C(t) stands for the evolution of the asymptotic value still following a(t) step by step. In the small speed approximation, following the method of [3] section 18 we get:

$$\ddot{r} = -\frac{1}{2}\frac{B'(r)}{A(r)} + \frac{q}{m}\frac{V'(r)}{C(t)} \tag{38}$$

$$\dot{\phi} = \frac{B(r)}{C(t)A(r)r^2} \tag{39}$$

for the radial acceleration and angular speed. This indeed means that, all else been equal, the scale factor impacts the relative strengths of the electrical and gravitational forces : in other words planet orbits should be seen expanding relative to atoms or any rods governed by atomic physics. Yet, current tests in the solar system and in some strong field binary systems constrain relative variations of G at levels much lower than H_0. On the other hand, a recent publication [24] claiming that galaxies 10 billion years ago were less dark matter dominated might support a long term variation of the strength of gravity in some areas all the more so if those effects are enhanced beyond a MOND radius as we shall argue in the next section. In the inner part of the solar system what we need is either an instantaneous test in the expanding regime (so far inaccessible because we are apparently currently in the stationary half cycle) or a test for multi-millennial variations hence necessarily over much longer time scales than the cycle period to exclude or not a mean variation at the Hubble rate. However, according to [28] "If G were to vary on a nuclear timescale (billions of years), then the rates of nuclear burning of hydrogen into helium on the main-sequence would also vary. This in turn would affect the current sun central abundances of hydrogen and helium. Because helio-seismology enables us to probe the structure of the solar interior, we can use the observed p-mode oscillation frequencies to constrain the rate of G variation." Again the relative variation of G at a rate similar to H_0 is completely excluded the precision being two orders of magnitude smaller.

To escape this new dead-end our understanding of the physics governing field discontinuities must again evolve in a new radical way: high density regions, for instance about stars, cut-out of the rest of the expanding universe, again by a discontinuity at their spherical surface defining a new volume for the global field dynamics which is not anymore submitted to the expanding:

$$d\tau^2 = C^2(t)(dt^2 - d\sigma^2) = dt'^2 - C'^2(t')d\sigma^2 \tag{40}$$

cosmological metric ($d\sigma^2 = dx^2 + dy^2 + dz^2$), but to the new Minkowski metric.

$$d\tau^2 = C^2(t)dt^2 - d\sigma^2 = dt'^2 - d\sigma^2 \tag{41}$$

Notice that this new Minkowski metric (41) is not the same as that of the static half-cycle which remains:

$$d\tau^2 = dt^2 - d\sigma^2 \tag{42}$$

Both (41) and (42) share the crucial property that within a volume subjected to such metrics no expansion effect can be measured, which is what we need to avoid conflicts with the solar system constraints. However, as seen from the (42) metric region (on Earth) the (41) metric atoms (at Pioneer) are blueshifted as needed to get the Pioneer effect.

Eventually the transition from (42) to (41) can be seen as the succession of transition from (42) to (40) we already had in the previous sections but now supplemented by the additional from (40) to (41) but the latter implied discontinuity can only produce hardly noticeable Shapiro delay or deflection of photons crossing it[i].

[i]Very much larger discontinuous barriers might however exist in the vicinity of compact star

With our new understanding the analysis of previous chapters thus remains valid provided the alternating of (42) and (40) being replaced by the alternating of (42) and (41), but valid except for genuine cosmological expansion effects impacting the relative periods of electromagnetic waves and atoms which of course need (40).

This necessarily implies that on the largest scales we still have the alternating (42) and (40) while near denser regions it's the alternating (42) and (41) that takes place so we have an additional frontier and we shall bet that this frontier is located at the MOND radius in the next section.

But now we need to reconsider equation (31) anywhere (41) is the actual metric rather than (40) and find that our results are not modified.

10. The MOND phenomenology

As already pointed out DG crucially differs from GR in the way global expansion and local gravity work together. Any anomaly in the local physics of the solar system or galaxy seemingly pointing to effects related to the Hubble rate is completely puzzling in the context of GR while it may be naturally explained within DG. Not only the Pioneer effect but also MOND phenomenology seem related to $\bar{H}_0$.

We derived in a former section the speed $\frac{dr}{dt} = -\gamma\frac{\bar{H}_0}{d\Phi(r)/dr}$ at which our local vs global frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $\gamma c\bar{H}_0$. For $\gamma = 1$, this appears to be the order of magnitude of the MOND acceleration and the corresponding radius even closer to the MOND radius beyond which gravity starts to be anomalous in galaxies [19][27]. Also remember that we assumed a radially uniform fluctuation to derive the speed formula for our hypersurface which amounts to consider that $d\Phi(r)/dr$ is its leading contribution so such estimation can only be very approximate. We are therefore tempted to suspect that something must be happening near the MOND radius due to frontier discontinuities propagating (and dragging matter) at a speed approaching the speed of light. Our best guess is that this is the radius beyond which the alternating (42) and (40) takes over the alternating (42) and (41) meaning true cosmological expansion responsible for photon redshifts as explained in the previous section.

Another seemingly independent argument is that the mean universe density $\bar{\rho}$ should now be dominated by the conjugate one $\bar{\tilde{\rho}}$ by a $1.7^6 \approx 25$ factor given that equality was reached at the transition redshift $z \approx 0.7$. So at some distance from the center of galaxies we should also expect the local equality between $\tilde{\rho}(r)$ and $\rho(r)$ also defining a crucial radius as this is where the field asymptotic C^2 and

surfaces (white dwarfs, neutron stars or our pseudo Black Holes) because it takes much longer time for the scale factor to scan such star strong gravitational potentials so the expanding and stationary regions on either sides of the border can accumulate a large relative C drift relative to each other over such a long time. Then such discontinuities might start to behave as mirrors able to partly or completely block light in one direction.

$\frac{1}{C^2}$ must exchange their roles just because the cosmological permutation between $a(t)$ and $\tilde{a}(t)$ did not already take place below this radius. This, as we explained in a previous section would result in the gravitational field from the dark side in the region beyond such radius to be enhanced by a huge factor C^8 relative to the gravity due to our side matter in this region.

Eventually this leads to a new picture in which only our side matter can be considered to be significantly gravific below the transition radius while only the dark side matter is significantly gravific beyond this radius.

Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even a slightly under-dense fluctuation[j] on the dark side would result in an anti-anti-gravitational effect on our side. This effect would exclusively originate from beyond the transition radius in such a way that it would be difficult to discriminate from the effect of a Dark Matter hallow! Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [20] or unexpectedly high acceleration effects in the flyby of galaxies [23] are more naturally interpreted in a framework where the gravitational effects from the hidden side are dominant beyond the MOND radius. At last, this would also mean that below the MOND frontiers the scale factor is still evolving according a decelerated expansion law.

11. Discrete symmetries, discontinuities and quantum mechanics

We earlier explained that in a theory with discrete symmetries having a genuine dynamical role to play, here global time reversal relating the two faces of a Janus field [6][13][14], discontinuities are expected at the frontier of space-time domains. All along this article we started to postulate various possible new discrete physical laws assumed to apply there: we can have discontinuous transitions in time when the conjugate scale factors exchange their roles, other kind of discontinuities in space at the frontier between static and expanding spatial regions, and in the expanding regions we also postulated a succession of step by step discontinuous and fast periodic re-actualization of the local field piecewise constant asymptotic value allowing it to follow the evolution of the scale factor. We also already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to highly value as physicists.

[j] though the dark side is in contraction which can boost growing of fluctuations especially on the largest scales, if it's density is similar to our side density at redshifts about 0.8, it's density was for instance much smaller at the time of the CMB emission so it's fluctuations must have started to grow significantly at much lower redshifts and much larger scales than on our side

Discontinuous and global fields as our scalar-tensor field also put into question the validity of the Noether theorem implying the violation of local conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules, discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws. Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are not anymore arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates, might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electrical charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox.

In this perspective, it may be meaningful to notice that our Pseudo Black Hole postulated discontinuity at the pseudo horizon, which would lie at the frontier between approximate GR and DG domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG domain the waves if any, carry no energy while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a step toward a real understanding of the wave function collapse i.e. in line with a realistic view of quantum mechanics. Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain. The latter transition is indeed non local because it is first of all driven by a transition of our global scalar-tensor field which by definition ignores distances.

12. Stability issues about distinct backgrounds: C $\neq$ 1

12.1. *Stability issues in the purely gravitational sector*

Our action for gravity being built out of two Einstein Hilbert terms, each single one is obviously free of Ostrogradsky ghost. This also means that all degrees of freedom have the same sign of their kinetic term in each action.

There might still remain issues in the purely gravitational sector when we add the two actions and express everything in terms of a single dynamical field $g_{\mu\nu}$: everything is all right as we could demonstrate for C=1, but otherwise what we need to insure stability is that in the field equation resulting from the total action, all degrees of freedom will have their kinetic term tilting to the same sign. Again adopting $\bar{h}_{\mu\nu}$ from $g_{\mu\nu} = e^{\bar{h}_{\mu\nu}}$ and $\tilde{g}_{\mu\nu} = e^{-\bar{h}_{\mu\nu}}$ as the dynamical field puts forward that we have exactly the same quadratic (dominant) terms in $t_{\mu\nu}$ and $\tilde{t}_{\mu\nu}$ except that for $C > 1$ (resp $C < 1$) all terms in $t_{\mu\nu}$ are enhanced (resp attenuated) by a C-dependent factor while all terms in $\tilde{t}_{\mu\nu}$ are attenuated (resp enhanced) by a 1/C dependent factor, so that we will find in $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ all such quadratic terms tilting to the same sign, ensuring that the theory is still free of ghost in the purely gravitational sector.

Of course there remains an instability menace whenever $C \neq 1$ in the interactions between matters and gravity which we shall inspect now.

12.2. *Stability issues in the interactions between matter and gravity: the classical case*

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy gravitational terms $t^{\mu\nu}$ and $\tilde{t}^{\mu\nu}$ do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, instabilities would seem unavoidable (see [25] section IV and V for a basic description of the problem and [26] for a more technical approach) and the problem is even worsen by the massless property of the gravitational field.

Yet, the most obvious kind of instability, the runaway of a couple of matter particles with opposite sign of the energy, is trivially avoided in DG theories [4][7][8][5] [29][30][31][32][33][27] in which such particles propagate on the two different sides of the Janus field and just gravitationally repel each other.

It is also straightforward to extend the theory of small gravitational fluctuations to DG in the Newtonian approximation and neglecting expansion: the equations governing the decay or grow of compressional fluctuations are :

$$\ddot{\delta\rho} = v_s^2 \Delta\delta\rho + 4\pi G < \rho > (\delta\rho - \delta\tilde{\rho}) \tag{43}$$

$$\ddot{\delta\tilde{\rho}} = \tilde{v}_s^2 \Delta\delta\tilde{\rho} + 4\pi G < \tilde{\rho} > (\delta\tilde{\rho} - \delta\rho) \tag{44}$$

which in case the speeds of sound v_s and $\tilde{v}_s$ would be the same on both sides allows to subtract and add the two equations with appropriate weights resulting in two new equations governing the evolution of modes $\delta^- = \delta\rho - \delta\tilde{\rho}$ and $\delta^+ = \delta\rho + \frac{<\rho>}{<\tilde{\rho}>}\delta\tilde{\rho}$.

$$\Box_s \delta^- = 4\pi G(< \rho > + < \tilde{\rho} >)\delta^- \tag{45}$$

$$\Box_s \delta^+ = 0 \tag{46}$$

Where $\Box_s$ is a fake Dalembertian in which the speed of sound replaces the speed of light. Because δ^+ does not grow we know that $\delta\rho \approx -\frac{<\rho>}{<\tilde{\rho}>}\tilde{\delta\rho}$ and the two can grow according the growing mode of δ^-. The complete study, involving different sound speeds, attenuation of gravity between the two sides and the effect of expansion (here represented by the evolution of C following the scale factor) will be the subject of the next section. It is already clear that in the linear domain anti-gravity by itself does not lead to a more pathological growth of fluctuations than in standard only attractive gravity: eventually we would expect the growth of a gravitational condensate on one side to proceed along with the corresponding growth of a void in the conjugate side and vice versa[k]. In other words our "instabilities" in the linear domain are nothing but the usual instabilities of gravity which fortunately arise since we need them to account for the growth of matter structures in the universe. These instabilities could be classified as tachyonic (the harmless and necessary ones for the formation of structures), non gradient (fortunately because those instabilities are catastrophic even at the classical level), and ghost (energy unbounded from below which is only catastrophic for a quantum theory) in the terminology of [36] reviewing various kind of NEC violations in scalar tensor theories which confirms that these are acceptable for a classical theory.

From this it appears that DG is not less viable than GR in the linear domain as a classical theory and that the real concern with all DG models proposed to this date will actually arise for the quantized DG theories for which ghost instabilities are of course prohibitive, and may be in the strong field regime for the classical theories. Only then the real energy exchange between the gravitational field itself (it's kinetic energy quadratic terms) and other fields kinetic energies should start to become significant relative to the Newtonian like energy exchange between kinetic energy of the fields and their gravitational potential energy that drives the evolution of the compressional modes according Eq 43 and 44. In the strong field regime the problem is thus related to the radiation of gravitational waves when they are carrying non zero energy (for C≠1) while they can couple to matter sources with both positive and negative energies[l]. However the avoidance of black-hole Horizon singularities even for $C \neq 1$ in DG makes the classical instabilities at least less serious than within GR while the transition to C=1 should completely fix the issue even in the strong field regime.

[k]The situation is less dramatic than Ref [25] section IV might have led us to think mainly because our leading order terms are linear in a gravitational field perturbation h whereas the leading order coupling term is quadratic in the lagrangian (22) of [25] leading to equations of motion of the form $\ddot{\Psi} \propto \Psi^3$.

[l]This remains true even when great care is being taken to avoid the so-called BD ghost in the massive gravity approach particularly when the perturbations of the two metrics about a common background have different magnitudes i.e. when one parameter of the couple α, β dominates the other in [31]. By the way there is a much worse problem in models having two independent differential equations instead of one to describe the dynamics of two fields assumed independent, i.e. not related from the beginning by a relation such as Eq (1). Then the energy losses through the generation of gravitational waves predicted by each equation are different so that such models are inconsistent [4][7][8][5] [29] as shown in [15].

12.3. *Stability issues in the interactions between matter and gravity: the quantum case*

12.3.1. *Problem statement*

The next step is therefore to try to understand how we might solve stability issues in the quantum case. In the quantized theory the problematic couplings would produce divergent decay rates by opening an infinite space-phase for for instance the radiation of an arbitrary number of negative energy gravitons by normal matter (positive energy) particles. To avoid such instabilities may be the most natural way would be to build the quantum Janus field operator also as a double-faced object, coupling it's positive energy face to usual positive energy particles and it's negative one (from our side point of view) to the negative energy particles (from our side point of view) of the dark side thereby avoiding any kind of instabilities. However the picture described by our classical Janus field equation which in principle really allows the direct exchange of energy between GW (with a definite sign of the energy depending on C > 1 or $C < 1$) and matter fields with different signs of the energy does not actually fit into such quantization idea.

Fortunately, as we shall see now, a closer examination of DG reveals the fundamental role played by spectator non dynamical fields and homogeneous fields in our framework leading us to understand that the serious quantum instability issue is actually already avoided in DG.

12.3.2. *A standalone action for matter and radiation*

The development of DG obliged us to introduce distinct Einstein-Hilbert actions for the background scalar-tensor gravitational field and the spin 2 local gravitational field. This in turn implies that the source terms for the scalar tensor field i.e. $\bar{\rho}, \bar{p}, \tilde{\bar{\rho}}, \tilde{\bar{p}}$ and the source terms for the spin 2 field, $\delta\rho, \delta p, \bar{\rho}\mathbf{v}$ and dark side equivalents must belong to and be isolated in well separated source actions. For instance, even though $\bar{\rho}\mathbf{v}$ explicitly includes $\bar{\rho}$, not only this $\bar{\rho}$ can't be dynamical in the source action for the spin 2 field but it also needs to be frozen as, as we explained earlier, the dynamics of the spin 2 field actions must be completely isolated from the effects of the scale factor. Again, in dynamical equations such as (11) the scale factor is replaced by an integration constant so it can't depend on time and the equation itself is only valid on a short time slot in between two discrete transitions of C.

However at some point when we want to study the evolution of fluctuations, we of course can't do without the conservation equations of the total energy momentum for matter and radiation fields (total means including both background and perturbations) respectively minimally coupling to each side of the conjugate metrics mixing local and global gravity :

$$d\tau^2 = C^2(t)((1 + 2\Psi)dt^2 - (1 - 2\Psi)d\sigma^2) \quad (47)$$

$$d\tilde{\tau}^2 = \tilde{C}^2(t)((1 + 2\tilde{\Psi})dt^2 - (1 - 2\tilde{\Psi})d\sigma^2) \quad (48)$$

in which C(t) and $\tilde{C}(t)$ follow and can be replaced by $a(t)$ and $\tilde{a}(t)$ on the mean since the actual step by step evolution does not matter when integrated over cosmological times. We need such conservation equations written as usual as: $\tilde{\partial}_{;\mu}\tilde{T}^{\mu}_{\nu} = 0$ and $\partial_{;\mu}T^{\mu}_{\nu} = 0$ because they imply the same total Euler and continuity equations as in GR but now along with their dark side equivalents. Of course the Euler and continuity equations mix the background and perturbations in the same matter and radiation fields leading for instance to a well known perturbative first order continuity equation such as:

$$\dot{\delta\rho} = -3H(\delta\rho + \delta p) - \nabla q + 3\dot{\Phi}(\bar{\rho} + \bar{p}) \tag{49}$$

Therefore we need a third action where the dynamics of the matter and radiation fields background is entangled with the dynamics of their perturbations: just as in GR this is the action for complete matter and radiation fields except that it's now a standalone action (not summed to any Einstein Hilbert action) and different from the source actions for our spin2 and scalar tensor theories which isolate background and perturbations of the matter and radiation fields.

We are then led to acknowledge two important seemingly problematic points, their solutions, and how by the way quantum instabilities are avoided in the next two subsections.

12.3.3. *Homogeneous fields and quantum instabilities*

Of course in our scalar-tensor field source action, the dynamical $\bar{\rho}_{ST}$ which evolves continuously as a function of a(t) can't be the same as dynamical $\bar{\rho}_{MR}$ in the newly introduced complete (background and perturbation) matter and radiation field action which evolves according C(t) (such action is actually only valid piece-wise .ie. in C-static time slots in between discrete transitions of C) hence discontinuously step by step or sometimes frozen to a constant.

It remains that the scalar-tensor action in which everything is dynamical (ρ_{ST}, p_{ST} ... terms and scalar-tensor field) but at the same time genetically homogeneous, obviously avoids quantum instability issues: no integration over an infinite phase-space of frequency levels just because this action exclusively deals with the zero frequency level. The only actually related quantum law is of the new kind that we introduced earlier: the one that makes a(t) drive C(t) step by step.

The payoff is that we can reasonably expect that only $\bar{\rho}_{MR}$ is to be affected by the vacuum expectation value of quantum fluctuations while $\bar{\rho}_{ST}$ if defined as the spatial average of the energy density of the corresponding classical field, is likely not and then we can forget the old cosmological constant problem[m].

[m] May be there is also a difference between $\bar{\rho}_{ST}$ and $\bar{\rho}_{MR}$ which is just what we usually interpret as a missing mass issue at the cosmological scale. In the following we shall not exploit this possibility and instead shall try to convince the reader that we have a natural Dark Matter candidate within our framework.

Ultimately DG has isolated the zero spatial frequency component of matter and radiation classical fields as a new dynamical homogeneous field which remains classical while all the non zero frequency components make up the quantum field and therefore acquire a possibly huge vacuum expectation value which does not source gravity.

12.3.4. *Spectator fields and quantum instabilities*

Another disturbing point is that for instance $\bar{\rho}_{MR}$ both appears in the source action for our spin 2 field (in a term such as $\bar{\rho}_{MR}\mathbf{v}$) as we already knew but now also in the third standalone action. As well $\delta\rho_{MR}$ appears in both the source action for our spin 2 field and in our newly introduced standalone action.

This looks disturbing but actually turns out not to be prohibitive as soon as we understand that we perfectly can have several "parallel" actions involving the same field (for instance here the $\bar{\rho}_{MR}$ term) without running into inconsistencies as soon as we admit that the field does not need to be dynamical in all these actions : here the matter and radiation fields are only dynamical in their newly introduced third standalone action mixing their background and perturbation as usual and this action alone and exclusively determines the equations satisfied by $\delta\rho_{MR}$, $\bar{\rho}_{MR}$ ... But once we have determined their space-time evolution we can introduce them as spectator non dynamical fields in the source actions for our spin 2 field[n]. The same game is played in the reversed way for the spin 2 field which is only dynamical in it's own E-H + source action while it is introduced as a spectator non dynamical field in the standalone matter and radiation action[o]. Eventually in these actions we have either the dynamical gravitational field interacting with spectator non dynamical matter and radiation fields or the converse i.e. dynamical matter and radiation fields interacting with a now non dynamical spectator gravitational field.

Therefore what just remains to be understood is merely that whenever a field is involved non dynamically in an action, it naturally does not need to participate to such action as a quantum field. There are probably several ways leading to such picture but the good new is that at least one of them is already well known and

[n]Notice that since a spectator field in an action does not necessarily satisfy the same field equations as the one it would satisfy if it was on shell with respect to such action (if it was extremizing the action), the requirement that the action be a scalar under general coordinate transformations also does not necessarily lead to the same covariant conservation equations for such field as for an on-shell field.

[o]By the way GR total action itself could be considered an action for the dynamics of the gravitational field alone, the matter and radiation fields entering this source action as non dynamical entities i.e varying such action with respect to the metric of course leads to the usual Einstein equations. But then we of course need to introduce an extra standalone equation for the matter and radiation fields to play their own dynamics on the metric field now spectator and non dynamical that follows from the resolution of the Einstein equation. Of course there can't be anything wrong with this as eventually this results in exactly the same set of equations as in GR. This way of playing with non dynamical fields is useless in the context of GR however proves not only extremely useful in our case but just mandatory.

practicable: Semi-classical gravity indeed treats matter fields as being quantum and the gravitational field as being classical, which is not problematic as far as we just want to describe quantum fields propagating and interacting with each others in the gravity of a curved space-time (within GR) considered as a spectator background. To describe the other way of the bidirectional dialog between matter and gravity i.e how matter fields source gravity, semi-classical gravity promotes the expectation value of the energy momentum tensor of quantum fields as the source of the Einstein equation. Here we would just need to extend the idea to the case corresponding to non dynamical spectator matter and radiation fields interacting with a dynamical gravitational field. This time only the gravitational field may need quantization but exclusively applied to it's self interaction while the spectator matter and radiation fields would be described by their expectation value.

12.3.5. *The Janus field and the Quantum*

One often raised issue with semi-classical gravity is that this is incompatible with the Multi Worlds Interpretation (MWI) of QM since within the MWI the other terms of quantum superpositions which are still alive and represent as many parallel worlds would still be gravific as they contribute to the energy momentum tensor expectation value and should therefore produce large observational effects in our world. The MWI, considered as a natural outcome of decoherence is adopted by a large and growing fraction of physicists mainly because is considered the only alternative to avoid the physical wavefunction collapse. For this reason incompatibility with the MWI is often deemed prohibitive for a theory. Since we have nothing against a physically real wave function collapse (our theory even has opened new ways to hopefully understand it; discontinuity and non locality are closely related) we are not very sensitive to such argument. The wave function collapse might eventually be triggered at the gravitational level: a simple achievement of something similar to the Penrose idea (gravitationally triggered collapse) seems within reach in our framework, thanks to a transition to C=1 which is tantamount to a gravitational wave collapse. We are all the more supported in considering semi-classical gravity and the Schrodinger-Newton equation it implies [38] as the correct answers, as the usual arguments based on the measurement theory often believed to imply that gravity must be quantized have recently been re-investigated in [37] and the authors to conclude that "Despite the many physical arguments which speak in favor of a quantum theory of gravity, it appears that the justification for such a theory must be based on empirical tests and does not follow from logical arguments alone." This has even reactivated an ongoing research which has led to experiment proposals to test predictions of semiclassical gravity, for instance the possibility for different parts of the wave functions of a particle to interact with each other non linearly according classical gravity laws. However "together with the standard collapse postulate, fundamentally semi-classical gravity gives rise to superluminal signalling" [37] so the theoretical effort is toward suitable models of the wavefunction collapse that would avoid this superluminal signalling. From the point of view of the DG theory this effort is probably unnecessary because superluminal signalling would not lead to inconsistencies as long as there exists a unic privileged frame for any collapse and any instantaneous transmission exploiting it. We indeed have such a natural privileged frame since we have a global privileged time to reverse, so it is natural in our framework to postulate that this frame is the unic frame of instantaneity. Then the usual gedanken experiments producing CTCs (closed timelike curves) do not work any more : the total round trip duration is usually found to be possibly negative only because these gedanken experiments exploit two or more different frames of instantaneous signaling. Let's be more specific : Does instantaneous hence faster than light signalling unavoidably lead to causality issues ? : apparently not if there is a single unic privileged frame where all collapses are instantaneous. Then i (A) can send a message to my colleague (B) far away from me instantaneously and he can send it back to me also instantaneously still in this same privileged frame using QM collapses (whatever the relative motions and speeds of A and B and relative to the global privileged frame): the round trip duration is then zero in this frame so it is zero in any other frames according special relativity because the spatial coordinates of the two end events are the same: so there is no causality issue since there is actually no possible backward in time signalling with those instantaneous transmissions... in case there is some amount of time elapsed between B reception and re-emission, eventually A still receives it's message in it's future: no CTC here.

13. Evolution of fluctuations

13.1. *Evolution equations for negligible dark side gravity*

In case we could neglect all dark side terms it would be straightforward to actually derive the whole set of equations satisfied by the gravitational potential during the static time slot preceding a transition: these are just the same as in GR but for fluctuations about a k=0 static hence empty universe background. Let's remind the first order cosmological perturbation GR equations for k=0: (4.4.169;4.4170;4.4.171 from [41]):

$$\nabla^2\Psi - 3H(\dot{\Psi} + H\Psi) = 4\pi G a^2 \delta\rho \tag{50}$$

$$\dot{\Psi} + H\Psi = -4\pi G a^2 (\bar{\rho} + \bar{p})\mathbf{v} \tag{51}$$

$$\ddot{\Psi} + 3H\dot{\Psi} + (2\dot{H} + H^2)\Psi = 4\pi G a^2 \delta p \tag{52}$$

For H=0 they give us our equations during the static time slot:

$$\nabla^2\Psi = 4\pi G a^2 \delta\rho \tag{53}$$

$$\dot{\Psi} = -4\pi G a^2 (\rho + p)\mathbf{v} \tag{54}$$

$$\ddot{\Psi} = 4\pi G a^2 \delta p \tag{55}$$

having replaced $\bar{\rho}$ and $\bar{p}$ by the total ρ and p to account for the fluctuation at any order beyond zero and not just the first order perturbation. Though the here included contribution of nonzero $\bar{\rho}$ or $\bar{p}$ terms does not seem to make any sense (as we pointed out that the local gravitational field we are here talking about has actually no background which is of course a necessary condition for it to be asymptotically static), remind that $\delta\rho, \delta p, (\rho{+}p)\mathbf{v}$ here are not dynamical but external contributions which dynamics and conservation laws were obtained from the stand alone actions of matter and radiation fields including perturbations and background terms such as $\bar{\rho}, \bar{p}$.

Equations (53)(54)(55) valid during a time slot need to be complemented by discrete rules specifying how the potential Ψ should be affected at the transition between two time slots. All effects being integrated over a long time duration we would eventually expect effective equations similar but different from the above GR ones to be valid with new terms involving the Hubble parameter. Fortunately, we might actually not always need to determine those equations because in GR below the Hubble scale we just need the Poisson equation which in this case simplifies to

$$\nabla^2 \Psi = 4\pi G a^2 \delta\rho \tag{56}$$

and should also be valid in our framework whenever we as well can neglect Hubble terms e.g. below the Hubble scale. This as usual allows to eliminate the potential from the Euler equation (same as in GR) and then using the continuity equations (same as in GR) get the same differential equations satisfied by the fluctuations as in GR cosmology. Again the whole derivation of these evolution equations for the fluctuations is only valid because the dark side terms were neglected from the beginning which is for sure a valid assumption in the radiative era and in the early stage of the matter dominated era as we shall see. Of course we can also as usual determine the evolution of the potentials directly from the same Poisson equation once we know the evolution of perturbations. However, new behaviour of the fluctuations might occur at superhorizon scale with respect to GR: it depends on how closely our whole set of equations for the potential will resemble the first order perturbation equations of GR.

Eventually, the main noticeable difference with respect to GR as for the theory of cosmological perturbations, is that the background and perturbations of the gravitational field itself here don't mix non linearly as in GR but play their dynamics as independent fields having their own separate E-H actions. This of course is necessary to avoid for instance the GR quasi-extinction of background effects in the non linear domain of matter perturbations and get effects having the magnitude of the Hubble rate in the solar system or within galaxies.

13.2. *Evolution equations with dark side gravity*

We already pointed out that the evolution of the background (our homogeneous scalar field) before the transition to acceleration seems to require Dark Matter just as in the standard model to reach the cosmological critical density implied by k=0 and the measured value of the Hubble expansion rate. Presumably, this Dark Matter does the same good job as within LCDM to help the formation of potentials already in the radiative era and then thanks to these potentials the growth of baryonic fluctuations falling into these potentials. We then have potentially all the successes of CDM phenomenology on the largest scales with the bonus that we have a new natural candidate for Dark Matter and shall present it in the next section.

Thus it remains to investigate whether the conjugate fluctuations from the dark side could now add new contributions on the smaller scales of galaxies to have the additional successes of MOND phenomenology there.

The dark side is also in a cold state with the same density as on our side at the transition redshift, but in contraction and therefore having started from a very low and presumably highly homogeneous mean density at z=1000. Therefore the radiative era is essentially the same as in LCDM (we have no effects related to the dark side at this epoch) and for instance we naturally have almost the same sound horizon even though a true singularity is avoided at t=0.

The dark side fluctuations could of course be boosted by the contracting scale factor especially on the largest scales but since the mean density was extremely small at high redshift with $\tilde{\rho} \approx z^{-6}\rho \ = 10^{-18}\rho$ at $z \approx 1000$, it is obvious that the growth of our side fluctuations starting from $\frac{\delta\rho}{\rho} \approx 10^{-5}$ of the CMB, could not be helped at high z. At low z, on the other hand, it is the weakness of the source term $\tilde{a^4}\tilde{\rho} \propto 1/a$ relative to $a^4\rho \propto a$ which makes the gravity from the dark side negligible with respect to our side matter gravity.

So we entirely need to rely on the extremely efficient new mechanism we introduced in the section devoted to the MOND phenomenology to see the gravitational effect of dark side fluctuations (voids) starting to play a significant role and produce the MOND empirical laws in galaxies.

What's indeed really new is that in accordance with what we also explained earlier each fluctuation has two regions : one central region where the gravity from our side $\delta\rho_{in}$ is hugely enhanced over the gravity from the dark side $\delta\tilde{\rho}_{in}$ and a peripheral one where at the contrary it is the gravity from the dark side $\delta\tilde{\rho}_{out}$ that hugely dominates that from $\delta\rho_{out}$. Moreover the permutation of the scale factors results in the same strength for $\delta\rho_{in}$ and $\delta\tilde{\rho}_{out}$ gravity in each equation.

As in LCDM, for the evolution of fluctuations the background evolution only becomes important in the matter dominated era arising as usual as an additional friction term $H\dot{\delta\rho}$ where H is the Hubble rate. So we can readily rewrite Eq (43) and (44) taking into account all non negligible effects depending on the scale factor.

$$\ddot{\delta\rho} + H\dot{\delta\rho} = 4\pi G < \rho > a^2(\delta\rho_{in} - \delta\tilde{\rho}_{out}) \tag{57}$$

$$\ddot{\delta\tilde{\rho}} - H\dot{\delta\tilde{\rho}} = 4\pi G < \tilde{\rho} > a^2(\delta\tilde{\rho}_{out} - \delta\rho_{in}) \tag{58}$$

We see that the interaction between the dark side and our side fluctuations can only be significant when $\tilde{\rho}$ is not too much smaller than ρ. So MOND like phenomenology would not be expected to arise well before the transition redshift. After the transition redshift on the other hand, even small fluctuations $\tilde{\delta\rho}$ in the dark side distribution relative to the dark side average density can lead to gravific effects much larger than what our side fluctuations $\delta\rho$ are able to do and all the effects of their dominant gravity is probably wrongly attributed to Dark Matter Halos within LCDM.

13.3. *Cosmological Dark Matter reinterpretation*

We already pointed out that $H^2(t) \approx \frac{8\pi G\rho}{3}$ is also (this is an approximate version of the GR exact second Friedmann equation) valid according the DG cosmological equation (5) provided our side scale factor dominates the dark side one for $p \approx 0$. Therefore baryonic matter is, just as within GR, cosmologically not abundant enough to account for the measured Hubble rate, and we still need a "Dark Matter" cosmological density $\bar{\rho}_{DM}$. Primordial Black Holes (PBH) were recently considered a possible candidate for Dark Matter because these are collisionless, stable, and at least in the mass range of BH mergers discovered by LIGO, not yet completely ruled out by astrophysical and cosmological constraints. But much more likely is the possibility that the Dark Matter is made of our pseudo Black Holes and their remnants after death i.e. after the transition to C=1 we described earlier as a mechanism to stop the collapse to singularity. Indeed we could have many of these compact objects just as gravific as the, primordial or not, pseudo BH they originated from except that it's now their discontinuity that is gravific. Just as Black Holes, these could exist in any size and remain stable for ever as these would not "evaporate" into Hawking radiation. The important difference with true PBH is that the smallest ones would now escape the two observational constraints on the fraction of PBH: both the microlensing signature (too small to be detected for small objects) and the Hawking radiation flashes.

In previous papers we also described objects called micro lightning balls (mlb) that would also be collisionless in their collapsed state (they would "decouple" from the baryon photon fluid due to their small "cross-section") and deserve much attention since these as well might be perfect Dark Matter candidates. Some of those object, as well as pseudo BH, might have been created as the result of density fluctuations producing a gravitational potential rising above a fundamental threshold triggering the discontinuous potential trapping and stabilizing the object. Some are likely to behave as miniature stars, presumably as dense and cold as black dwarfs and extremely difficult to detect either through their black body radiation of an extremely cold object, their gravitational lensing given their surface gravity much smaller than that of a pseudo Black Hole of the same size and again the absence of Hawking radiation even for the smallest of these objects. Of course a much more detailed characterization of long living micro lightning balls would be needed to make firm predictions as for both their spatial and mass distribution and the best way to detect them.

At last it is worth mentioning that discontinuities should help the fast formation of stars in general and large mass ones in particular leading to many large mass pseudo BH such as the ones recently discovered by Ligo or giant black holes at the centers of large galaxies. This is because the dragging effect of drifting discontinuities is presumably an effective mechanism to concentrate matter at all scales or to merge already formed pseudo BHs or their remnants.

Most such objects would need to be primordial to account for the total matter density inferred from the CMB. Those that started to arise later must have resulted in a drop of the total detectable baryonic matter (the matter which is still free i.e. not captured by pseudo-BH, their remnants or micro lightning balls) hence a missing baryons effect.

On the other hand if these objects or discontinuities in general allow a significant transfer of matter from our side to the dark side, then it's the total gravific density on our side that could drop significantly producing a universe slightly less decelerating just before the transition redshift and slightly more and more accelerating after the transition redshift. This is again an interesting alternative idea to help explain a recent acceleration higher than expected if this anomaly (recently measured high H_0 value) were to be confirmed.

14. Last remarks and outlooks

We already pointed out that none of the faces of our gravitational Janus field could be seriously considered as a candidate for the spacetime metric. Yet, though the gravitational field loses this very special status (be the spacetime metric) it had within GR, it acquires another one which again makes it an exceptional field : it is the unic field that makes the connection between the positive and negative energy worlds (this definition is relative: for any observer the negative field is the one that lives on the other side), the only one able to couple to both the dark side SM fields and our side SM fields.

This special status alone implied that the gravitational interaction might need a special understanding and treatment avoiding it to be quantized as the other interactions. Avoiding ghost instabilities related to the infinite phase space opened by any interaction between quantum fields that do not carry energies with the same sign, is a requirement which also confirms that the gravitational Janus field in Eq (1) and (2) could not interact with matter as a quantum field. So the old question whether it is possible to build a theory with a classical gravitational field interacting with all other fields being quantum, was back to the front of the stage just because the usual answer "gravity must be quantized because everything else is quantum" fails for the Janus theory of the gravitational field. And of course understanding the crucial role played by spectator fields in our actions has confirmed this and definitely settled the issue.

Eventually what also deserves attention is that we repeatedly had to make use of the following very helpful non standard idea : not all field degrees of freedom should be considered a priori completely dynamical in an action. First we had the most extreme case of our $\eta_{\mu\nu}$ metric which was completely fixed before the action so there is obviously no ghost menace from such metric. We also encountered the case of a metric which has already played it's dynamics in one action and could enter a new action as a completely non dynamical metric, which needs not extremize the action, hence again avoiding ghost issues for this metric in the most trivial way (we could even stuckelbergize the field [35], there would be no Action extremization hence no field equation hence no propagator associated to the kinetic terms generated this way). In the meantime, an even more interesting case was the action for our global scalar field when we demanded that the field should be spatially maximally symmetric not to reflect the fact that the source tends to be homogeneous and isotropic on the largest scales as in usual GR cosmology but already before entering the action which then forced the source to be a purely homogeneous one. This pre-action requirement for the field could of course be expressed in a fully covariant way using the language of killing vectors. This is why such field actually does not admit any non homogeneous perturbation.

The new rules of that game we have been playing all along this article may appear very unnatural if one did not completely figure out that the permutation symmetry linking the two faces of a Janus field has an interpretation in terms of a discrete global privileged time reversal symmetry and that such global symmetry also constrains the metrics in a non trivial way: they should share the same isometries, the form B=A of our scalar tensor field and B=1/A for asymptotically Minkowskian C=1 isotropic metrics are actually necessary ones, and once we have a global privileged coordinate system, other unusual symmetry properties linking space and time coordinates become meaningful and so on ... (see [3] section 6).

By way of prospects an interesting open issue is that of renormalizability of the Quantum Janus field theory coupled to spectator hence non quantum matter and radiation fields: we expect the self interaction of the Janus field to lead to simplifications with respect to GR even when $C \neq 1$ may be thanks to the same kind of compensations that suppressed the Horizon singularity.

15. Conclusion

New developments of DG not only seem to be able to solve the tension between the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer effect as well as the recent cosmological acceleration. An amazing unification of MOND and Dark Matter phenomenology seems also at hand. The most important theoretical result is the avoidance of both the Big-Bang singularity and Black Hole horizon.

References

1. Weinberg, S. 1972, Quantum Field Theory, Vol 1., Cambridge University Press
2. Henry-Couannier, F. 2004, Int.J.Mod.Phys, vol. A20, no. NN, pp. 2341-2346, 2004.
3. Henry-Couannier, F. 2013, Global Journal of Science Frontier Research A. Vol 13, Issue 3, pp 1-53.
4. Petit, J. P. 1995, Astr. And Sp. Sc. Vol. 226, pp 273.
5. Hossenfelder, S. 2008, Phys. Rev. D 78, 044015.
6. Henry-Couannier, F. 2012, Dark Gravity, Global Journal of Science Frontier Research F. Vol 12, Issue 13, pp 39-58.
7. Petit, J.P. 1977, C. R. Acad. Sci. Paris t.285 pp. 1217-1221.
8. Petit, J.P. and D'Agostini, G. 2014, Astr. And Sp. Sc. 354: 611-615.
9. Bondi H., Rev. of Mod. Phys., Vol 29, N3 (1957)12.
10. Bonnor W.B., General Relativity and Gravitation Vol.21, N11 (1989)
11. Anderson, J D., Laing, P A., Lau, E L., Liu, A S., Nieto M M., Turyshev, S G. , 2002 Phys.Rev.D65:082004,2002
12. Anderson, J D., Feldman, M R., 2015 Int.J.Mod.Phys, vol. D24 1550066
13. Henry-couannier, F., 2015, J. Condensed Matter Nucl. Sci. 18 (2016) 1–23
14. Henry-couannier, F., 2016, J. Condensed Matter Nucl. Sci. 21 (2016) 59–80
15. http://www.darksideofgravity.com/Consistency.pdf
16. Geogebra animation at https://ggbm.at/EX2DSukC
17. A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones, J. M. Silverman, R. Chornock, P. Challis, W. Yuan, P. J. Brown, and R. J. Foley, APJ 826, 56 (2016), arXiv:1604.01424.
18. V. Bonvin, F. Courbin, S. H. Suyu, P. J. Marshall, C. E. Rusu, D. Sluse, M. Tewes, K. C. Wong, T. Collett, C. D. Fassnacht, T. Treu, M. W. Auger, S. Hilbert, L. V. E. Koopmans, G. Meylan, N. Rumbaugh, A. Sonnenfeld, C. Spiniello, 2016, arXiv:1607.01790
19. S. McGaugh, F. Lelli, J. Schombert, Physical Review Letters, Volume 117, Issue 20, arxiv:1609.05917
20. M. A. Beasley, A. J. Romanowsky, V. Pota, I. M. Navarro, D. M. Delgado, F. Neyer, A. L. Deich The Astrophysical Journal Letters, Volume 819, Issue 2, article id. L20, 7 pp. (2016) arxiv:1602.04002
21. Daylan, T., Finkbeiner, D. P., Hooper, D., Linden, T., Portillo, S. K. N., Rodd, N. L., Slatyer, T. R. 2016, Physics of the Dark Universe, 12, 1 astro-ph:1402.6703
22. Abedi, Jahed, Dykaar, Hannah and Afshordi, Niayesh, "Echoes from the Abyss: Evidence for Planckscale structure at black hole horizons" (2016). arXiv:1612.00266
23. Indranil Banik, Hongsheng Zhaoa, Anisotropic Distribution of High Velocity Galaxies in the Local Group astro-ph:1701.06559
24. R. Genzel, N.M. Förster Schreiber, H. Übler, P. Lang, T. Naab, R. Bender, L.J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, A. Beifiori, S. Belli, G. Brammer, A. Burkert, C.M. Carollo, J. Chan, R. Davies, M. Fossati, A. Galametz, S. Genel, O. Gerhard, D. Lutz, J.T. Mendel, I. Momcheva, E.J. Nelson, A. Renzini, R. Saglia, A. Sternberg, S. Tacchella, K. Tadaki, D. Wilman, Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago Accepted for publication in Nature, astro-ph:1703.04310
25. S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68 (2003) 023509 arxiv:0301273
26. R. J. Gleiser, G. Dotti, Class.Quant.Grav. 23 (2006) 5063-5078 arXiv:0604021
27. M. Milgrom, Monthly Notices Roy. Astronomical Soc. 405 (2) (2010) 1129–1139 arXiv:1001.4444v3
28. D.B. Guenther, L.M. Krauss, P. Demarque, Astrophys. J., 498, 871–876 (1998).

29. L. Bernard and L. Blanchet, Phys. Rev.D91, 103536 (2015), arXiv:1410.7708
30. L. Blanchet and L. Heisenberg, Phys. Rev.D91, 103518 (2015), arXiv:1504.00870
31. L. Blanchet and L. Heisenberg, JCAP 1512, 026 (2015) arXiv:1505.05146
32. L. Blanchet and L. Heisenberg, arXiv:1701.07747
33. L. Bernard, L. Blanchet and L. Heisenberg Contribution to the proceedings of the 50th Rencontres de Moriond, "Gravitation: 100 years after GR" arXiv:1507.02802
34. C.M. Will Theory and experiment in gravitational physics
35. C d Rham Living Rev. Relativity, 17, (2014), 7
36. V.A. Rubakov Physics-Uspekhi, Volume 57, Number 2 (2014) arXiv:1401.4024
37. M. Albers, C. Kiefer, M. Reginatto Phys.Rev.D78:064051,2008 arXiv:0802.1978
38. M. Bahrami, A. Großardt, S. Donadi, A. Bassi, New Journal of Physics, Volume 16, November 2014 arXiv:1407.4370
39. E. Komatsu et al Astrophys.J.Suppl.192:18,2011 arXiv:1001.4538
40. I. Arcavi et al, Nature 551, 210–213 09 November 2017 arxiv:1711.02671
41. Cosmology course at http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter5.pdf
42. R. Bufaloa, M. Oksanena, A. Tureanu Eur. Phys. J. C 75 (2015) 477
43. T. Josset, A. Perez, D. Sudarsky Phys. Rev. Lett. 118, 021102 (2017)
44. P. O. Mazur, E. Mottola Gravitational Condensate Stars: An Alternative to Black Holes (2001) arXiv:gr-qc/0109035
45. B. D. Nikolić, M. R. Pantić A Possible Intuitive Derivation of the Kerr Metric in Orthogonal Form Based On Ellipsoidal Metric Ansatz (2013) arXiv:1210.5922

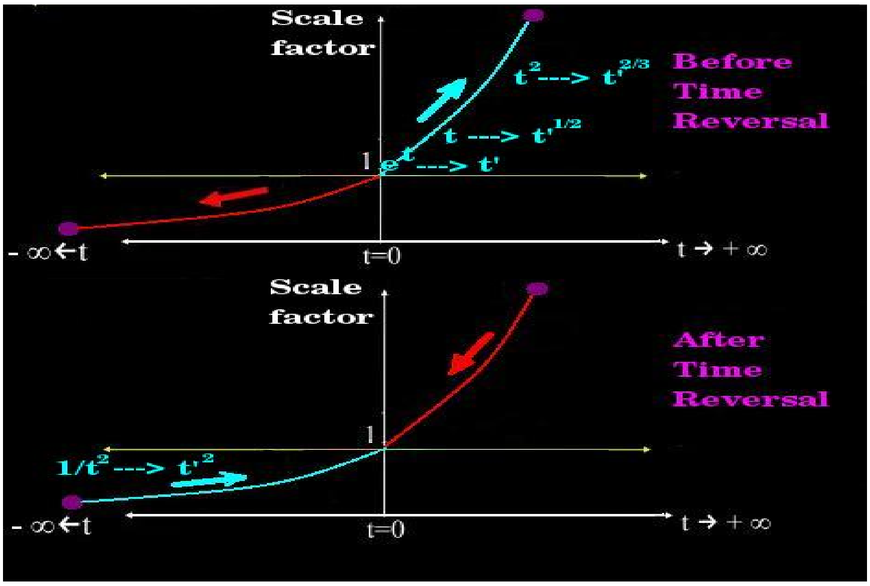

Fig. 1. Evolution laws and time reversal of the conjugate universes, our side in blue

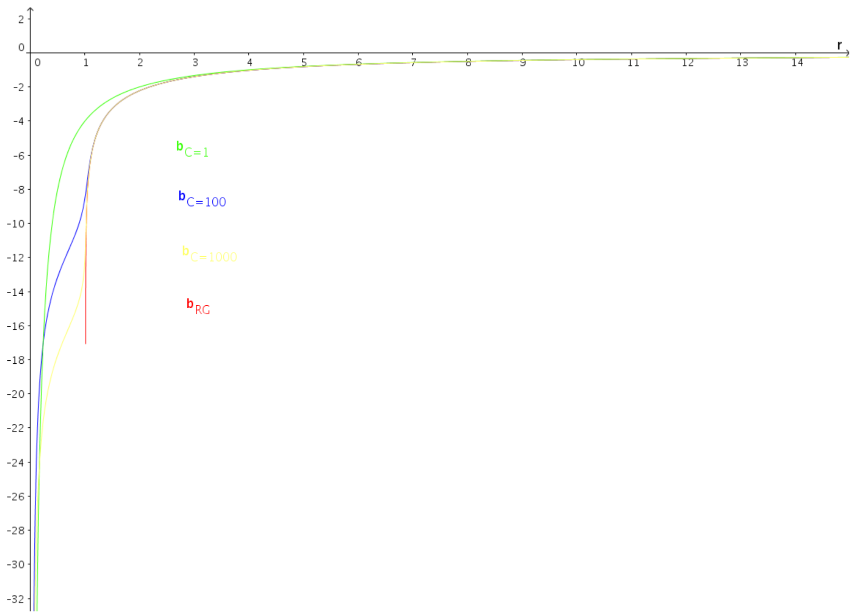

Fig. 2. b(r) near the Schwarzschild radius (r=1) for various C values