The Dark side of Gravity vs MOND/DM

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New developments of the Dark Gravity Theory which foundations and some consequences we have detailed in two previous articles predict that the strength of gravity could be enhanced in some space-time domains. A MOND radius also arises naturally in this framework so that hopefully the MOND/DM phenomenology may finally be within reach for Dark Gravity.

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1. Introduction

In the presence of a flat non dynamical background $\eta_{\mu\nu}$, it turns out that the usual gravitational field $g_{\mu\nu}$ has a twin, the "inverse" metric $\tilde{g}_{\mu\nu}$. The two being linked by

$$\tilde{g}_{\mu\nu} = \eta_{\mu\rho}\eta_{\nu\sigma}\left[g^{-1}\right]^{\rho\sigma} = \left[\eta^{\mu\rho}\eta^{\nu\sigma}g_{\rho\sigma}\right]^{-1} \tag{1}$$

are just the two faces of a single field (no new degrees of freedom) that we called a Janus field [1][2][5][12][13]. See also [3][6][7][4] for alternative approaches to Anti-gravity with two metric fields.

The action treating these two faces of the Janus field on the same footing should be invariant under the permutation of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ which is achieved by simply adding to the usual GR and SM (standard model) action, the similar action with $\tilde{g}_{\mu\nu}$ in place of $g_{\mu\nu}$ everywhere.

$$\int d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) + \int d^4x(\sqrt{g}L + \sqrt{\tilde{g}}\tilde{L}) \tag{2}$$

where R and $\tilde{R}$ are the familiar Ricci scalars built from g or $\tilde{g}$ as usual and L and $\tilde{L}$ the Lagrangians for respectively SM F type fields propagating along $g_{\mu\nu}$ geodesics and $\tilde{F}$ fields propagating along $\tilde{g}_{\mu\nu}$ geodesics. This theory symmetrizing the roles of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ is Dark Gravity (DG) and we explained at length why it is the most natural way to rehabilitate and understand time reversal, negative energies and anti-gravity. We found that the theory trivially avoids any kind of instabilities (at least about a Minkowskian background) because :

- Fields minimally coupled to the two different sides of the Janus field never meet each other from the point of view of the other interactions (EM, weak, strong) so stability issues could only arise in the purely gravitational sector.
- The run away issue [8] [9] is avoided between two masses propagating on $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ respectively, because those just repel each other, anti-gravitationally as in all other versions of DG theories [7][4] rather than one chasing the other ad infinitum.
- The energy of DG gravitational waves vanishes avoiding the instability of positive energy fields through the emission of negative energy gravitational waves.

At the end of this article we shall propose an extension of DG allowing to recover the same gravitational waves as in GR while still avoiding instabilities in the gravitational sector. But we first need to come back to the description of our cosmological and isotropic solutions.

2. Global gravity

2.1. *The scalar-tensor cosmological field*

We found that an homogeneous and isotropic solution is necessarily spatially flat because the two sides of the Janus field are required to satisfy the same isometries. However, it is also static so that the only way to save cosmology in the DG framework is to introduce a tensor-scalar Janus field built from a scalar Φ such that $g_{\mu\nu} = \Phi\eta_{\mu\nu}$ and $\tilde{g}_{\mu\nu} = \frac{1}{\Phi}\eta_{\mu\nu}$. Then our fundamental cosmological single equation obtained by requiring the action to be extremal under any variation of $\Phi(t) = a^2(t)$ is:

$$a^2\frac{\ddot{a}}{a} - \tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}} = \frac{4\pi G}{3}(a^4(\rho - 3p) - \tilde{a}^4(\tilde{\rho} - 3\tilde{p})) \tag{3}$$

where $\tilde{a}(t) = \frac{1}{a(t)}$. With this scalar cosmology we avoid the other degrees of freedom and corresponding equations (in GR cosmology there is for instance an additional equation which for k=0 requires Dark Matter to insure that the total density is the critical density) which for a spatial curvature k=0, could only be satisfied all together by a static solution for any physically realistic equations of state. Moreover this field is understood to be genetically homogeneous e.g. the spatially independent $\Phi(t)$ at any scale and sourced by the mean expectation value of the usual sources averaged over space rather than the sources themselves. So there are no scalar waves associated to this field and there is also no scale related to a loss of homogeneity of the background as in GR.

Another independent Janus field will thus be required to describe all other aspects of gravity with all it's usual degrees of freedom, but then a field forced to remain asymptotically static to satisfy all the equations. Thus in DG we have two different fields to describe the background and fluctuations respectively. So for instance the source densities and pressures are $<\rho>(t)$ and $<p>(t)$ for the background and $\rho(\mathbf{x},t)- <\rho>(t)$ and $p(\mathbf{x},t)- <p>(t)$ for the fluctuations, where $<>$ denotes spatial averaging.

2.2. *Cosmology*

This section is mainly a review of the results already obtained in [13]. At the end a new cosmological alternative is also considered.

The expansion of our side implies that the dark side of the universe is in contraction. Provided dark side terms can be neglected, our cosmological equation reduces to a cosmological equation known to be valid within GR. For this reason it is straightforward for DG to reproduce the same scale factor expansion evolution as obtained within the standard LCDM Model at least up to the redshift of the LCDM Lambda dominated era when something new must have started to drive the evolution in case we want to avoid a cosmological constant term.

A discontinuous transition is a natural possibility within a theory involving truly dynamical discrete symmetries as is time reversal in DG. The basic idea is that some of our beloved differential equations might only be valid piecewise, only valid within space-time domains at the frontier of which new discrete rules apply implying genuine field discontinuities. Here this will be the case for the scale factor.

We postulated that a transition occurred billion years ago as a genuine permutation of the conjugate scale factors, understood to be a discrete transition in time modifying all terms explicitly depending on $a(t)$ but not the densities and pressures themselves in our cosmological equation: in other words, the equations of free fall for our "average source field" did not apply at the discrete transition in time (at the contrary we will later consider other kinds of metric field discontinuities at the frontier between spatial zones in which case it's possible to describe the propagation of the wave function of any particle crossing this frontier just as the Schrodinger equation can be solved exactly in a squared potential well : infinite potential gradients are not actually a nuisance and only potential differences between both sides of such discontinuity matter).

This could trigger the recent acceleration of the universe. This was demonstrated in previous articles assuming the dark side was already dominated by radiation at the time of our side nucleosynthesis so that our side source $\rho - 3p \simeq \rho \propto \frac{1}{a^3(t)}$ in the cold era has driven the evolution up to now, eventually resulting, following the discrete transition, in a recent accelerated expansion regime $(t' - t'_0)^{-2}$ in standard time coordinate with a Big Rip at future time t'_0.

But there is an alternative possibility: following the transition the dark side source might momentarily have started to drive the evolution as far as $a^4(\rho - 3p) \propto a << \tilde{a}^4(\tilde{\rho} - 3\tilde{p}) \propto Const$ for $a(t) << Const << \tilde{a}(t)$ would have been satisfied. Then our cosmological equation simplifies in a different way:

$$\tilde{a}^2 \frac{\ddot{\tilde{a}}}{\tilde{a}} \propto Const \tag{4}$$

with solution $a(t) \propto 1/t$ which translates into an exponentially accelerated expansion regime $e^{t'}$ in standard time coordinate.

In the Big Rip scenario, constraining the age of the universe to be the same as in LCDM the predicted transition redshift is $z_{tr} = 0.27$ in case it occurred everywhere simultaneously otherwise the mean transition redshift should be significantly increased by an expected dispersion of transition redshifts due to inhomogeneities smoothing the observed transition between decelerated and accelerated expansion after averaging over large regions and making the theory difficult to discriminate from the very progressive LCDM transition with observed $z_{tr} = 0.67 \pm 0.1$. The mean measured transition redshift is indeed very sensitive to a smoothing. For instance a fictitious LCDM discrete transition between a purely CDM and a purely Lambda driven expansion regime (the Hubble rate being continuous at the transition) would imply $z_{tr} \approx 0.4$ for the same constrained age of the universe instead of $z_{tr} \approx 0.7$ for the actual progressive LCDM transition. $z_{tr} \approx 0.4$ of course also corresponds to the transition to our exponentially accelerated expansion case if it occurred everywhere simultaneously while a smoothing effect would significantly increase z_{tr} again making this scenario even harder to discriminate from the real LCDM transition.

3. Local gravity

3.1. *The isotropic case*

Another Janus field and it's own separate Einstein Hilbert action are required to describe local gravity with isotropic solution in vacuum of the form $g_{\mu\nu} = (B, A, A, A)$ and $\tilde{g}_{\mu\nu} = (1/B, 1/A, 1/A, 1/A)$

$$A = e^{\frac{2MG}{r}} \approx 1 + 2\frac{MG}{r} + 2\frac{M^2G^2}{r^2} \tag{5}$$

$$B = -\frac{1}{A} = -e^{\frac{-2MG}{r}} \approx -1 + 2\frac{MG}{r} - 2\frac{M^2G^2}{r^2} + \frac{4}{3}\frac{M^3G^3}{r^3} \tag{6}$$

perfectly suited to represent the field generated outside an isotropic source mass M. This is different from the GR one, though in good agreement up to Post-Newtonian order. It is straightforward to check that this Schwarzschild new solution involves no horizon. The solution also confirms that a positive mass M in the conjugate metric is seen as a negative mass -M from its gravitational effect felt on our side.

3.2. *Gravitational Waves*

The linearized equations look the same as in GR the main differences being the additional dark side source term $\tilde{T}_{\mu\nu}$ and an additional factor 2:

$$2(R^{(1)}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}R^{(1)\lambda}_{\lambda}) = -8\pi G(T_{\mu\nu} - \tilde{T}_{\mu\nu}) \tag{7}$$

however this equation is also valid to second order in the perturbation $h_{\mu\nu} = -\tilde{h}_{\mu\nu}$ because to this order the additional quadratic term $t_{\mu\nu} - \tilde{t}_{\mu\nu}$ on the right side standing as usual for the energy-momentum of the gravitational field itself has two cancelling contributions since $t_{\mu\nu} = \tilde{t}_{\mu\nu}$ to second order in small plane wave perturbations. The Linearized Bianchi identities are still obeyed on the left hand side and it therefore follows the local conservation law:

$$\frac{\partial}{\partial x^\mu}(T^{\mu\nu} - \tilde{T}^{\mu\nu} + t^{\mu\nu} - \tilde{t}^{\mu\nu}) = 0 \tag{8}$$

Our new interpretation is that any radiated wave will both carry away a positive energy in $t^{\mu\nu}$ as well as the same amount of energy with negative sign in $-\tilde{t}^{\mu\nu}$ to second order resulting in a total vanishing radiated energy to this order. Thus the DG theory, so far appears to be dramatically conflicting with both the indirect and direct observations of gravitational waves.

Actually, we shall show in a forthcoming section that the theory is naturally extended in such a way that we can both expect an isotropic solution approaching the GR Schwarzschild one with it's black hole horizon and the same gravitational wave solutions, including the production rate, as in GR but also, whenever some particular yet to be defined conditions are reached, the above DG solutions, with a vanishingly small production rate of gravitational waves and an exponential Schwarzschild solution without horizon. Both will be limiting cases of a more general solution.

4. The unified DG theory

4.1. *Actions and space-time domains*

Eventually the theory splits up into two parts, one with total action made of an Einstein Hilbert action for our scalar-tensor homogeneous and isotropic Janus field added to SM actions for F and $\tilde{F}$ type averaged fields respectively minimally coupled to $\Phi\eta_{\mu\nu}$ and $\Phi^{-1}\eta_{\mu\nu}$. The other part of the theory has an Einstein Hilbert (EH) action for the asymptotically Minkowskian Janus Field $g_{\mu\nu}$ for local gravity added to SM actions for F and $\tilde{F}$ type fields respectively minimally coupled to $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$.

The two theories must remain completely separate. Indeed, to remain asymptotically static, $g_{\mu\nu}$ must be isolated from the scale factor effect. But also as announced earlier the scalar field is spatially independent at all scales so admits only perfectly homogeneous sources. So a unified theory cannot be obtained by mixing the local and global gravity in a Lagrangian term. However it's still possible to add the following global and local actions, being understood that no dynamical field is shared between them.

$$\int_{Global} d^4x(\sqrt{g}(R+L) + \sqrt{\tilde{g}}(\tilde{R}+\tilde{L})) + \tag{9}$$

$$\int_{Local} d^4x(\sqrt{g}(R+L) + \sqrt{\tilde{g}}(\tilde{R}+\tilde{L})) \tag{10}$$

This confirms that even for the sources, the average background and perturbations are different dynamical fields, the former in the global L and $\tilde{L}$, the latter in the local L and $\tilde{L}$ (yet this total action will be helpful to later establish a non trivial connection between global and local gravity in some particular areas). We could also have assumed that global and local gravity never apply at the same place and time so that only an alternating of the two would remain conceivable. But it would be difficult in this case to find out non arbitrary rules linking the unconnected successive time slots of both global and local evolutions. Moreover, this would require the introduction of arbitrary parameters for the global and local slots durations.

On the other hand considering the global and local physics of those actions running in parallel totally decoupled and uninterrupted as implied by our above total action leads to another issue. We need to understand then how light, clocks and rods can both feel the effect of global expansion and local gravity being now understood that those light, clocks and rods do not even appear in the global Lagrangians L and $\tilde{L}$ above just because as we already noticed only the averaged perfectly homogeneous over the whole universe, perfect fluid densities and pressures are there.

Our proposal for solving this problem is that the asymptotic local static gravity is actually only a constant piecewise function of time rather than rigorously the stationary $\eta_{\mu\nu}$. In other words it is rather $C\eta_{\mu\nu}$ which asymptotic value C is piecewise constant, being periodically discontinuously updated to $a(t)$ in such a way that it closely follows the evolution of $a(t)$ through a series of fast discrete transitions on a regular basis. Eventually, clocks and rods coupling to local gravity only but never coupling directly to $a(t)$, can still feel the effects of the continuous global expansion indirectly thanks to this mechanism. At the same time, clocks and rods must remain insensitive to discrete transitions of the scale factor itself such as the one responsible for the cosmological transition to global acceleration which is possible if our mechanism does not roll up those transitions to the local field asymptotic value C, the latter being only locked to the continuous variations of the scale factor. We shall soon understand better how relevant is this asymptotic value within DG which has no obvious peer within GR. Here as in GR for the isotropic static case, C is a mere integration constant, and as such cannot depend on time, however it can take different values in successive time slots, the differential equations being only valid piecewise. We shall soon understand better how relevant is this asymptotic value within DG which has no obvious peer within GR.

Another issue is that gravity in the inner part of the solar system as we know it from thorough studies during the last decades exclude that global gravity applied to clocks and rods without being strongly attenuated. Indeed, it would otherwise lead to strongly excluded expansion effects of orbital planetary periods relative to atomic periods: the gravitational constant G would seem to vary at a rate similar to H_0 which is not the case. GR solves this problem because it predicts that significant expansion effects only take place on scales beyond those of galaxy clusters. At the contrary, the theory involving the physics of the global action above would produce expansion effects with the same magnitude at all scales if the asymptotic value C of local fields was following everywhere the scale factor $a(t)$ evolution as we explained above. Therefore this driving mechanism did not apply to local gravity in the inner part of the solar system at least during the last decades. This is the only possible solution not to conflict with observational constraints: no evidence of expanding planet trajectories so far. This implies the existence of frontiers between space-time domains where the local field $g_{\mu\nu}$ asymptotic value does not change (for instance in the inner part of the solar system during the last decades) and others where the $g_{\mu\nu}$ asymptotic value C is step by step discontinuously driven by the scale factor from the global $\Phi\eta_{\mu\nu}$ according our postulated above mechanism.

4.2. ***Space-time domains and the Pioneer effect***

The following question therefore arises: suppose we have two identical clocks exchanging electromagnetic signals between one domain submitted to the expanding $a(t)$ in $\Phi\eta_{\mu\nu}$ (still through our indirect mechanism) and another without such effect. Electromagnetic periods and wavelengths are not affected in any way during the propagation of electromagnetic waves in the conformal coordinate system where we wrote our cosmological equation even when crossing the inter-domain frontier. Through the exchange of electromagnetic signals, the period of the clock decreasing as $a(t)$ can then directly be tracked and compared to the static clock period and should be seen accelerated with respect to it at a rate equal to the Hubble rate H_0. Such clock acceleration effect indeed suddenly appeared in the radio-wave signal received from the Pioneer space-crafts but with the wrong magnitude by a factor two: $\frac{\dot{f}_P}{f_E} = 2H_0$ where f_P and f_E stand for Pioneer and earth clocks frequencies respectively. This is the so called Pioneer anomaly [10][11]. The interpretation of the sudden onset of the Pioneer anomaly just after Saturn encounter would be straightforward if this is where the spacecraft crossed the frontier between the two regions. The region not submitted to global expansion (at least temporarily) would therefore be the inner part of the solar system where we find our earth clocks and where indeed various precision tests have shown that expansion or contraction effects on orbital periods are excluded during the last decades. Only the origin of the factor 2 discrepancy between theory and observation remains to be elucidated in the following sections as well as a PLL issue we need to clarify first.

4.2.1. *Back to PLL issues*

As we started to explain in our previous article [13] in principle a Pioneer spacecraft should behave as a mere mirror for radio waves even though it includes a frequency multiplier. This is because its re-emitted radio wave is phase locked to the received wave so one should not be sensitive to the own free speed of the Pioneer clock.

Our interpretation of the Pioneer effect thus requires that there was a failure of on board PLLs (Phase Lock Loop) to specifically "follow" a Pioneer like drift in time. We already pointed out that nobody knows how the scale factor actually varies on short time scales: in [13] we already imagined that it might only vary on very rare and short time slots but with a much bigger instantaneous Hubble factor than the average Hubble rate. This behaviour would produce high frequency components in the spectrum which might have not passed a low pass filter in the on board PLL system, resulting in the on board clocks not being able to follow those sudden drifts. The on board clocks would only efficiently follow the slow frequency variations allowing Doppler tracking of the spacecrafts. Only when the integrated total drift of the phase due to the cumulative effect of many successive clock fast accelerations would reach a too high level for the system, this system would "notice" that something went wrong, perhaps resulting in instabilities and loss of lock at regular intervals [13]. This view is now even better supported since our clocks and rods are understood not to be anymore directly sensitive to the scale factor, but rather indirectly, only through the local field asymptotic value C closely following by a succession of discontinuous steps rather than continuously the evolution of $a(t)$ as the latter is implied by our cosmological differential equation. The failure of the PLL system is then even better understood for discontinuous variations of the Pioneer clock frequency with respect to the earth clock frequency. As a result, the frequency of the re-emitted wave is affected by the Pioneer clock successive drifts and the earth system could detect this as a Pioneer anomaly.

4.3. *Cyclic expanding and static regimes*

We are now ready to address the factor two discrepancy between our prediction and the observed Pioneer clock acceleration rate. We know from cosmology that, still in the same coordinate system, earth clocks must have been accelerating at a rate H_0 with respect to still standing electromagnetic periods of photons reaching us after travelling across cosmological distances: this is just the description of the so called cosmological redshift in conformal time rather than usual standard time coordinate. However, according our above analysis this was not locally the case at least during the last decades which did not manifest any cosmological effect (G did not vary) in the inner part of the solar system.

This necessarily implies that earth clocks must have been submitted to alternating static and expanding regimes. It just remains to assume (further justification will be provided in a forthcoming section) that through cosmological times, not only earth clocks but also all other clocks in the universe, spent exactly half of the time in the expanding regime and half of the time in the static regime, in a cyclic way. It follows that the instantaneous expansion rate $H_0 = 2\bar{H}_0$ of our global field as deduced from the Pioneer effect is twice bigger than the average expansion rate (the average of $2\bar{H}_0$ and zero respectively in the expanding and static halves of the cycle) as measured through a cumulative redshift over billions of years.

In our previous article we presented a very different more complicated and less natural explanation on how we could get the needed factor two which we do not support anymore. This article also discussed the possibility of field discontinuities at the frontier between regions with different expansion regimes, and likely related effects. Those discontinuities do not necessarily imply huge potential barriers even though the scale factors have varied by many orders of magnitude between BBN and now. At the contrary they could be so small to have remained unnoticed as far as our cycle is short enough to prevent some regions to accumulate a too much C drift relative to others.

5. Frontier dynamics

Our next purpose is to understand the physics that governs the location of frontier surfaces between regions identified in the previous sections.

Consider the gravitational field total action in a space-time domain where our driving mechanism from global to local gravity is switched off :

$$\int_{Global} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})+ \tag{11}$$

$$\int_{Local} d^4x(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R}) \tag{12}$$

where in the global (resp local) actions the gravitational field is $\Phi\eta_{\mu\nu}$ (resp asymptotically static $g_{\mu\nu}$). We would like to determine the frontier surface of this domain at the time t the local field asymptotic value C is reset to the scale factor beyond this surface (not in our domain). Considering the frontier to be stationary between two such successive updates, the frontier position is determined at any time. If such surface is moving because of successive updates it will of course scan a space-time volume as time is running out. To determine this hypersurface we extend the extremum action principle. Not only the total action should be extremum under any infinitesimal field variations which as we all know allows to get the field equations but also the total action is required to be extremum i.e. stationary under any infinitesimal displacement of this hypersurface which is nothing but the frontier of the action validity domain. But the displaced hypersurface might only differ from the original one near some arbitrary point, so that requiring the action variation to vanish actually implies that the total integrand should vanish at this point and therefore anywhere on the hypersurface. Eventually, anywhere and at any time at the domain boundary we have:

$$(\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{global} + (\sqrt{g}R + \sqrt{\tilde{g}}\tilde{R})_{local} = 0 \tag{13}$$

This equation is merely a constraint relating local gravity (terms 3 and 4) to global gravity (terms 1 and 2) at the hyper surface and it can be further simplified remembering that at the present time we could neglect term 1 because our side scale factor is negligible compared to the dark side scale factor. We assume g is also negligible relative to $\tilde{g}$ for local gravity. Though this is not expected in the weak field approximation for C=1, this is the case for $C << 1$ and we will further justify this crucial point in the forthcoming section. Considering that we are in vacuum on our side, the dark side fluid source term is dominant in the local field equation which can therefore be approximated and contracted by $\tilde{g}_{\mu\nu}$ to get $\tilde{R}_{local} = 8\pi G\tilde{T}_{local} = 8\pi G(\tilde{\rho}-3\tilde{p})_{local}$ which is nothing but a GR equation, the Einstein equation for the dark side gravity.

Replacing $\tilde{R}_{local}$ by this expression in the equation relating local to global gravity, we get:

$$(\sqrt{\tilde{g}}\tilde{R})_{global} = -8\pi G\sqrt{\tilde{g}_{local}}(\tilde{\rho} - 3\tilde{p})_{local} \tag{14}$$

By the way $(\tilde{\rho} - 3\tilde{p})_{local}$ does not vanish exactly as long as there are massive particles in the fluid. This expression varies like the densities and pressures themselves which are here constant because we are dealing with the pressure and density in the local gravitational field alone so it is static (remember C also remains unchanged because the periodic re-actualization of C is switched off in the domain we are considering). But the lhs is $\tilde{a}^2\frac{\ddot{\tilde{a}}}{\tilde{a}}$ which according to our cosmological equation is constant in the exponential acceleration scenario and varies as $a = \frac{1}{\tilde{a}}$ in the Big Rip scenario. Therefore, in the external gravity of a massive spherical body, planet or star on our side, which radial a-dimensional potential is $\Phi(r) = -GM/rc^2$ we are led to:

$$a^{\gamma}(t) \propto e^{\frac{2MG}{rc^2}} \tag{15}$$

with $\gamma = 1$ for the Big Rip and 0 for the exponential acceleration.

This equation obtained here in the conformal time t coordinate system is also valid in standard time t' coordinate since the standard scale factor and the "conformal scale factor" are related by $a(t) = a'(t')$. It is valid to PN order being understood that the exponential metric is here used for simplicity as a weak field PN approximation of a GR Schwarzschild solution rather than really a DG Schwarzschild solution as we shall show in the next section. This equation I=J implies $\dot{I}/I = \dot{J}/J$ so that:

$$\gamma 2\bar{H}_0 = -2\frac{d\Phi}{dr}\frac{dr}{dt} \tag{16}$$

here taking into account that the instantaneous Hubble factor H_0 is actually $2\bar{H}_0$, e.g twice the average cosmological Hubble parameter that we know from cosmological observables as we explained earlier.

The latter equation tells us that the frontier between the two domains is drifting at speed $\frac{dr}{dt} = -\frac{\bar{H}_0}{\frac{d\Phi(r)}{dr}}$ in the Big Rip Scenario whereas it is fixed in the exponentially accelerated scenario. The Big Rip option is therefore our favorite because it could involve a characteristic period, the time needed for the scale factor to scan $e^{\frac{2MG}{r}}$ from the asymptotic value to the deepest level of the potential at which point a new scan cycle is started except that this time the two regions will need to exchange their roles about the moving frontier. In other words if for a given cycle the expanding region is the outer one and the static region the inner one, the next cycle will be with the inner part expanding and the outer part static. After two such complete cycles any area will have spent exactly the same total time static and expanding at $2\bar{H}_0$ resulting in the promised average $\bar{H}_0$. A Geogebra animation in [14] helps visualizing the evolution of the local potential over one complete cycle. Notice that the scale factor, as shown in the animation, also needs periodical resets because it's mean evolution rate is then twice the evolution rate of C.

It is worthy of special mention that then the total time to scan the potential well of our sun which is the deepest at the sun surface is about the same as the equinoxes precession period. Betting on a driving mechanism that might along many cycles lead to synchronize the two phenomena, we can estimate $\bar{H}_0$ from the precession of the equinoxes cycle and get $\bar{H}_0 = 80,56 \pm 0.01 (km/sec)/Mpc$ to be compared with the best precision "recent" cosmological measurement of $\bar{H}_0 = 73.03 \pm 1.79 (km/sec)/Mpc$ [15][16]. Therefore, according this interpretation, the present value would be greater by four standard deviations than the cosmological one over the two last billion years (300 SNe Ia at $z < 0.15$ having a Cepheid-calibrated distance) which itself exceeds by three standard deviations the one predicted by LCDM from Planck data. This is noteworthy because an unexpectedly high recent acceleration could of course be the signature of our Big Rip scenario vs LCDM expectations.

6. Unconventional asymptotic values

After many cycles of successive static and expanding phases, the local field asymptotic value is everywhere going to be very different from it's initial C=1 value. This also implies that the new asymptotic values of the local field and its conjugate will be very different. This is also going to be our justification for having neglected g relative to $\tilde{g}$ even for weak fields, in the previous section.

Given that $g^{C\eta}_{\mu\nu} = C g^{\eta}_{\mu\nu}$ and $\tilde{g}^{\eta/C}_{\mu\nu} = \frac{1}{C}\tilde{g}^{\eta}_{\mu\nu}$, where the $< g^{\eta}, \tilde{g}^{\eta} >$ Janus field is asymptotically η, it is straightforward to rewrite the local DG Janus Field equation now satisfied by this asymptotically Minkowskian Janus field after those replacements. Hereafter, we omit all labels specifying the asymptotic behaviour for better readability and only write the time-time equation satisfied by the asymptotically $\eta_{\mu\nu}$ Janus field.

$$C\sqrt{g}\frac{G_{tt}}{g_{tt}} - \frac{1}{C}\sqrt{\tilde{g}}\frac{\tilde{G}_{tt}}{\tilde{g}_{tt}} = -8\pi G(C^2\sqrt{g}(\rho - <\rho>) - \frac{1}{C^2}\sqrt{\tilde{g}}(\tilde{\rho} - <\tilde{\rho}>)) \qquad (17)$$

Where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\rho - <\rho>$ is as usual the energy density for matter and radiation density fluctuations. The tilde terms again refer to the same tensors except that they are built from the corresponding tilde (dark side) fields. The stability issues related to partial cancellation between positive and negative energy gravitational waves will be investigated in a forthcoming section. Notice that for no fluctuations, the solutions are Minkowskian as needed, being understood that the background plays it's dynamics in the global Janus field equation rather than in this local Janus field equation.

Then for $C >> 1$ we are back to $G_{tt} = -8\pi G C g_{tt}(\rho - <\rho>)$, a GR like equation for local gravity from sources on our side because all terms depending on the conjugate field become negligible on the left hand side of the equation while the local gravity from sources on the dark side is attenuated by the huge $1/C^4$ factor (in the weak field approximation, $G_{tt} = 8\pi G \frac{\tilde{\rho} - <\tilde{\rho}>}{C^3}$). From $g^{\eta}_{\mu\nu}$ we can get back $g^{C\eta}_{\mu\nu}$ and then of course absorb the C constant by the adoption of a new coordinate system and redefinition of G, so for $C >> 1$ we are back to GR (with its Horizon in the Schwarzschild solution and it's gravitational waves) except that on the dark side everything will feel the effect of the anti-gravitational field from bodies on our side amplified by the same huge factor relative to the gravity produced by bodies on their own side.

Of course the roles are exchanged in case $C << 1$. Then the GR equation $\tilde{G}_{tt} = -\frac{8\pi}{C} G \tilde{g}_{tt}(\tilde{\rho} - <\tilde{\rho}>)$ is valid on the dark side while the anti-gravity we should feel from the dark side is enhanced by the huge $1/C^4$ factor relative to our own gravity (given in the weak field approximation by solving $\tilde{G}_{tt} = 8\pi G C^3(\rho - <\rho>)$ for $\tilde{g}_{\mu\nu}$ from which we derive immediately our side $g_{\mu\nu}$ of the Janus field). Here is our promised justification for having assumed that the local gravitational field was the weak field PN approximation of the GR Schwarzschild solution rather than a DG Schwarzschild solution in the previous section.

Only in case C=1 do we recover our local Dark gravity, with no significant GW radiations and no Black Hole horizon and also a strength of gravity ($G_{tt} = -4\pi G(\rho - <\rho>)$) reduced by a factor 2C relative to the above GR gravity ($G_{tt} = -8\pi G C(\rho - <\rho>)$) as a consequence of two geometrical terms adding up on the lhs of the equations.

It's important to stress that the phenomenology following from different asymptotic behaviours of the two faces of the Janus field here has no peer within GR in which a mere coordinate transformation is always enough to put the gravitational field in an asymptotically Minkowskian form in which a redefinition of the gravitational constant G gives back the usual gravitational potentials. This would still be possible in DG for one face of the Janus field but not for both at the same time. The new physics emerges from their relative asymptotic behaviour which can't be absorbed by any choice of coordinate system.

Eventually, depending on the local C value in a given space-time domain, a departure from GR predictions could be expected or not both for the gravitational waves radiated power and the local static gravitational field e.g. depending on the context, we could get either exponential elements or the GR Schwarzschild solution for the static isotropic gravity; and get either no gravitational waves at all or the same radiated power as in General Relativity.

Because clocks and rods submitted to local gravity also indirectly felt the effects of global expansion through our quantized (discontinuous step by step) evolution of C, if we could test gravity over the past cycles we would necessarily detect that it's strength was different and has changed in the same proportion as the scale factor itself. Current tests in the solar system and in some strong field binary systems constrain relative variations of G at levels much lower than H_0 however what we need in the inner part of the solar system is either an instantaneous test in the expanding regime (so far inaccessible because we are apparently currently in the stationary half cycle) or a test for multi-millennial variations hence necessarily over much longer time scales than the cycle period to exclude or not a mean variation at the Hubble rate. A recent publication [22] claiming that galaxies 10 billion years ago were less dark matter dominated might support a long term variation of the strength of gravity all the more so if those effects are enhanced beyond a MOND radius as we shall argue now.

7. The MOND phenomenology

We derived in a former section the speed $\frac{dr}{dt} = -\frac{\bar{H}_0}{d\Phi(r)/dr}$ at which our local vs global frontier sitting at an isopotential between internal and external regions should radially propagate in the potential well of a given body. From this formula the speed of light $\frac{dr}{dt} = c$ is reached anywhere the acceleration of gravity equals $c\bar{H}_0$. This appears to be nothing but the MOND acceleration and the corresponding radius nothing but the MOND radius beyond which gravity starts to be anomalous in galaxies [17][25]. We are therefore tempted to postulate that to prevent the frontier discontinuities from propagating faster than the speed of light something must be happening at the MOND radius. Our best guess is that the local Janus field asymptotic C and $\frac{1}{C}$ exchange their roles there, which, as we explained in the previous section would result in the gravitational field from the Dark side in the region beyond the MOND radius to be enhanced by a huge factor C^4. Then because a galaxy on our side implies a slightly depleted region on the dark side by it's anti-gravitational effects, even such slightly under-dense fluctuation of the highly homogeneous radiative fluid on the dark side would result in an anti-anti-gravitational effect on our side, significantly enhanced beyond the MOND radius, and it would be difficult to discriminate from the effect of a Dark Matter hallow! Also the most spectacular features of Dark Matter and MOND Phenomenology in galaxies such as galaxies that seem to be dominated at more than 99 percent by Dark Matter [18] or unexpectedly high acceleration effects in the flyby of galaxies [21] are more naturally interpreted in a framework where the gravitational effects from the hidden side can be enhanced by huge factors beyond the MOND radius.

8. Back to Black-Holes and gravitational waves

Let's consider the collapse of a massive star which according to GR should lead to the formation of a Black Hole. As the radius of the star approaches the Schwarzschild radius the metric becomes singular there so the process lasts an infinite time according to the exterior observer. If the local fields both outside and inside the star have huge asymptotic C values, we already demonstrated that the gravitational equations are GR like. We postulate however that the metric actually never becomes singular at the Schwarzschild radius but instead when the metric reaches some threshold, the inner region (the volume defined by the star itself) global and local fields are respectively reset to Minkowski and C=1. Therefore this is where and when the DG solution is triggered avoiding thereby the GR black hole Horizon but producing in place a huge discontinuity in the vicinity of the Schwarzschild Radius which phenomenological signature might already have been detected [20]. At the center of the star, the two faces of the Janus field will get very close to each other just because C=1 and because this is where the star potential vanishes. The crossing of the metrics is the required condition to allow the transfer of the star matter to the conjugate side there all the more since the pressure is huge. This effect along with the strength of gravity being reduced by a factor 2C for DG relative to GR might eventually stop the collapse whenever the conditions are again reached for the stability of a star.

The resulting object having no horizon is in principle still able to radiate light. It must also have lost a significant part of its initial mass content transferred to the dark side and also much of its gravific mass because of the 2C reduction factor. Something new however is that the discontinuity itself might have a contribution to the total effective gravific mass and this might lead to pseudo BHs much more massive than we believed them to be.

Although we have seen that DG does not allow significant Gravitational Waves radiation from such BH inner region, considering that the discontinuity itself is gravific and can radiate as any accelerating body according the GR laws in the outer region, we are sure to avoid any conflict with all the observational evidence from GWs emitted by "black holes". Shocks and matter anti-matter annihilation at the discontinuity (an excess of gamma radiation from our Milky Way giant black hole has indeed been reported [19]) which we remember is also a bridge toward the Dark side and it's presumably anti-matter dominated fluid, could also produce further GWs radiation which would be much less natural from a regular GR Black Hole [20].

9. The Wave function Collapse

We earlier explained that in a theory with discrete symmetries having a genuine dynamical role to play, here time reversal relating the two faces of a Janus field, discontinuities are expected at the frontier of space-time domains. All along this article we started to postulate various possible new discrete physical laws assumed to apply there: we can have discontinuous transitions in time when the conjugate scale factors exchange their roles, other kind of discontinuities in space at the frontier between static (driving mechanism from global to local gravity switched off) and expanding (driving mechanism from global to local gravity switched on) spatial regions, and in the expanding regions we also postulated a succession of step by step discontinuous and fast periodic re-actualization of the local field piecewise constant asymptotic value allowing it to follow the evolution of the scale factor. We also already drew the reader attention to the harmlessness of discontinuous potentials as for the resolution of wave function equations in the presence of discontinuities. Of course the exploration of this new physics of discontinuities in relation to discrete symmetries is probably still at a very early and fragile stage and requires an open minded effort because it obviously questions habits and concepts we used to value highly as physicists.

Discontinuous and global fields as our scalar-tensor field also put into question the validity of the Noether theorem implying that local conservation laws might be violated and may be hopefully replaced by new global and discontinuous conservation laws wherever the new physics rules apply. However, we should remind ourselves that the most fundamental postulates of quantum physics remain today as enigmatic as they appeared to physicists one century ago: with the Planck-Einstein quantization rules discontinuous processes came on to the scene of physics as well as the collapse of a wave function taken at face value obviously implies a violation of almost all local conservation laws. Based on these facts, a new theoretical framework involving a new set of discrete and non local rules which, being implied by symmetry principles are no more arbitrary at the contrary to the as well discontinuous and non-local quantum mechanics postulates might actually be a chance. A real chance indeed as they open for the first time a concrete way to hopefully derive the so arbitrary looking quantum rules from symmetry principles and may be eventually relate the value of the Planck constant to the electric charge, in other words compute the fine structure constant. We are certain that only our ability to compute the fine structure constant would demonstrate that at last we understand where quantum physics comes from rather than being only able to use it's rules like a toolbox.

In this perspective, it's may be meaningful to notice that the Black Hole postulated discontinuity of the previous section, which would lie at the frontier between GR and DG domains, behaves as a wave annihilator for incoming GW waves and a wave creator for outgoing waves. In the DG domain the waves if any, carry no energy at least to second order while in the GR domain they carry energy and momentum as usual. This is a fascinating remark because this would make it the only known concrete mechanism for creating or annihilating waves à la QFT or even a step toward a real understanding of the wave function collapse e.g in line with a realistic view of quantum mechanics.

Such collapse is indeed known to be completely irreducible to classical wave physics because it is non local, and in fact just as non local as would be a transition from GR $C >> 1$ to DG, C=1 in the inside domain. The latter transition is indeed non local because it is first of all driven by a transition of our global scalar-tensor field which by definition ignores distances.

10. Stability issues

Generic instability issues arise again when C is not anymore strictly equal to one. This is because the positive and negative energy terms do not anymore cancel each other as in the DG C=1 solution. Gravitational waves are emitted either of positive or negative (depending on C being less or greater than 1) energy whereas on the source side of the equation we have both positive and negative energy source terms. Whenever two interacting fields (here the gravitational field and some of the matter and radiation fields) carry energies with opposite sign, the vacuum instability is unavoidable even at the classical level (see [23] section IV for a simple description of the problem and [24] for a much more complicated one) and the problem is even worsen by the massless property of the gravitational field. More generally, although the most obvious kind of instability, the runaway of a couple of particles with opposite sign of the energy, is trivially avoided in DG theories where particles from the two side of the Janus field just repel each other, the models proposed by Hossenfelder [4] and Petit [3] are unstable for this reason because they have non zero energy gravitational waves coupling to matter sources with both positive and negative energies.

We are therefore led to understand that whenever C becomes different from 1, the Local Janus field $< g^C, \tilde{g}^{1/C} >$ needs to split in two independent Janus fields $< f^C, \tilde{f}^{1/C} >$ and $< h^C, \tilde{h}^{1/C} >$ (superscripts C and 1/C still denote asymptotic values). It is considered a well established result that a theory with two interacting massless spin 2 fields is not viable. However, the no-go demonstration was carried on only in case the theory is derived from a single action. Here we shall consider two separate actions where the two spin 2 fields respectively play their dynamics separately in the sense that in one action where one field plays it's dynamics the other field is non dynamical (not varied). The equations following from the extremization process will then be quite different from the equations we would have derived from a single total action, sum of the two actions. As a starting point we are tempted to consider the following actions running in parallel and decoupled in which we omit asymptotic behaviour superscripts for better readability.

$$\int_{Local} d^4x \sqrt{f} R_f + \sqrt{f} L_f \tag{18}$$

$$\int_{Local} d^4x \sqrt{\tilde{h}} R_{\tilde{h}} + \sqrt{\tilde{h}} \tilde{L}_{\tilde{h}} \tag{19}$$

to avoid $\tilde{L}_{\tilde{f}}$ and L_h terms in the first and second action respectively which ensures that we will not end up with source terms carrying an energy opposite to the energy of gravitational waves in any of the two actions. The permutation symmetry is now between f and $\tilde{h}$. This is a bit silly however because we lost $\tilde{f}$ and h and anti-gravity in that new game. But this is just an intermediary step because we can actually recover easily the conjugates of the Janus fields along with anti-gravity if matter and radiation are actually coupled to a combination of f and h instead of f alone in the first action, and equivalently to a combination of $\tilde{f}$ and $\tilde{h}$ rather than $\tilde{h}$ alone in the second action.

The composite metrics being denoted and defined by $[fh]_{\mu\nu} = \eta^{\rho\sigma} f_{\mu\rho} h_{\nu\sigma}$ and $[\tilde{f}\tilde{h}]_{\mu\nu} = \eta^{\rho\sigma} \tilde{f}_{\mu\rho} \tilde{h}_{\nu\sigma}$ let's now consider the two actions.

$$\int_{Local} d^4x \sqrt{f} R_f + \sqrt{[fh]} L_{[fh]} \tag{20}$$

$$\int_{Local} d^4x \sqrt{\tilde{h}} R_{\tilde{h}} + \sqrt{[\tilde{f}\tilde{h}]} \tilde{L}_{[\tilde{f}\tilde{h}]} \tag{21}$$

Being understood that the Janus field $f, \tilde{f}$ is only dynamical in the first action and $\tilde{h}$,h only dynamical in the second action, stability is still granted because even though our side matter and radiation fields in L feel the anti gravitational effect of matter and radiation fields from $\tilde{L}$ through h and reciprocally through $\tilde{f}$, the gravitational field f is only sourced by matter and radiation fields coupled to f (and not $\tilde{f}$) and spectator h in the first action and equivalently the gravitational field $\tilde{h}$ is only sourced by matter and radiation fields coupled to $\tilde{h}$ (and not h) and spectator $\tilde{f}$ in the second action.

We can gain more insight about what's new by varying the first action with respect to $f_{\mu\nu}$ to get :

$$\sqrt{f} G_f^{\mu\nu} \delta f_{\mu\nu} + 8\pi G \sqrt{[fh]} T_{[fh]}^{\mu\sigma} \eta^{\nu\rho} h_{\rho\sigma} \delta f_{\mu\nu} = 0 \tag{22}$$

In the perfect fluid case, after some replacements this yields :

$$G_f^{\mu\nu} = -8\pi G \sqrt{h} T_f^{\mu\nu} \tag{23}$$

and equally we could get

$$G_{\tilde{h}}^{\mu\nu} = -8\pi G \sqrt{\tilde{f}} \tilde{T}_{\tilde{h}}^{\mu\nu} \tag{24}$$

By the way, there is not any issue with the Bianchi identities in such equations because for instance $T_f^{\mu\nu}$ is not here a covariantly conserved energy momentum tensor with respect to the f metric. It is rather $T_{fh}^{\mu\nu}$ that is covariantly conserved with respect to metric fh and here we just denoted $T_f^{\mu\nu}$ the tensor obtained through replacing fh by f in $T_{fh}^{\mu\nu}$ which is straightforward in the case of a perfect fluid energy-momentum tensor.

The conservation of $T_{fh}^{\mu\nu}$ with respect to metric fh merely means that our side matter and radiation fields follow the kind of geodesics sourced by a positive mass through f as well as the kind of geodesics sourced by a negative mass through h even though on the other hand these matter and radiation fields can only exchange energy with the f gravitational field almost in the GR way and exactly as in GR to second order in perturbations as could be read in a straightforward way from the perturbative equations. This is why we are confident that this framework is at last completely free of ghosts.

A very striking feature of those equations is that assuming the same cosmological constant source term in both equations as expected e.g. from the same vacuum energy presumably huge contributions, the resulting f and $\tilde{h}$ fields would be the same but the composite field fh that matter and radiation fields would feel would remain Minkowski as the result of a perfect compensation. Unfortunately, such vacuum energy terms are not expected in our fluctuations local equations but rather in our background global equation where the cancellation does not take place. We could of course give up all what we did in previous sections and adopt the new equations to describe both background and perturbations, in a stable theory involving anti-gravity and solving the vacuum energy issue. However we would lose the benefit of being able to explain the recent acceleration of the universe, the Pioneer effect, the MOND radius and enhanced gravity beyond...anyway, further investigation is needed as for the vacuum energy issue in our framework.

Now we can repeat the reasoning of a former section with the time-time equation for C asymptotic f and h fields. The asymptotically Minkowskian part in the weak field approximation is solution of $G_{f\,tt} = -8\pi G C^3(\rho- <\rho>)$ while we find $G_{\tilde{h}\,tt} = -8\pi G C^{-3}(\tilde{\rho}- <\tilde{\rho}>)$ and recover a previous section conclusion that the gravity from the dark side is damped with respect to gravity from our side but now by an even greater factor $1/C^6$. And of course the situation again gets reversed for $C < 1$. Thus, eventually all our results as for the MOND/DM phenomenology remain valid.

11. Conclusion

New developments of DG not only seem to be able to solve the tension between the theory and gravitational waves observations but also provide a renewed and reinforced understanding of the Pioneer as well as a recent cosmological acceleration greater than expected. An amazing unifying explanation of MOND/Dark Matter phenomenology seems also at hand. The outlook for a wave-function collapse new mechanism also appears promising on an unprecedented scale.

References

1. Henry-Couannier, F. 2004, Int.J.Mod.Phys, vol. A20, no. NN, pp. 2341-2346, 2004.
2. Henry-Couannier, F. 2013, Global Journal of Science Frontier Research A. Vol 13, Issue 3, pp 1-53.
3. Petit, J. P. 1995, Astr. And Sp. Sc. Vol. 226, pp 273.
4. Hossenfelder, S. 2008, Phys. Rev. D 78, 044015.
5. Henry-Couannier, F. 2012, Dark Gravity, Global Journal of Science Frontier Research F. Vol 12, Issue 13, pp 39-58.
6. Petit, J.P. 1977, C. R. Acad. Sci. Paris t.285 pp. 1217-1221.
7. Petit, J.P. and D'Agostini, G. 2014, Astr. And Sp. Sc. 354: 611-615.
8. Bondi H., Rev. of Mod. Phys., Vol 29, N3 (1957)12.
9. Bonnor W.B., General Relativity and Gravitation Vol.21, N11 (1989)
10. Anderson, J D., Laing, P A., Lau, E L., Liu, A S., Nieto M M., Turyshev, S G. , 2002 Phys.Rev.D65:082004,2002
11. Anderson, J D., Feldman, M R., 2015 Int.J.Mod.Phys, vol. D24 1550066
12. Henry-couannier, F., 2015, J. Condensed Matter Nucl. Sci. 18 (2016) 1–23
13. Henry-couannier, F., 2016, J. Condensed Matter Nucl. Sci. 21 (2016) 59–80
14. Geogebra animation at https://ggbm.at/EX2DSukC
15. A. G. Riess, L. M. Macri, S. L. Hoffmann, D. Scolnic, S. Casertano, A. V. Filippenko, B. E. Tucker, M. J. Reid, D. O. Jones, J. M. Silverman, R. Chornock, P. Challis, W. Yuan, P. J. Brown, and R. J. Foley, APJ 826, 56 (2016), arXiv:1604.01424.
16. V. Bonvin, F. Courbin, S. H. Suyu, P. J. Marshall, C. E. Rusu, D. Sluse, M. Tewes, K. C. Wong, T. Collett, C. D. Fassnacht, T. Treu, M. W. Auger, S. Hilbert, L. V. E. Koopmans, G. Meylan, N. Rumbaugh, A. Sonnenfeld, C. Spiniello, 2016, arXiv:1607.01790
17. S. McGaugh, F. Lelli, J. Schombert, Physical Review Letters, Volume 117, Issue 20, arxiv:1609.05917
18. M. A. Beasley, A. J. Romanowsky, V. Pota, I. M. Navarro, D. M. Delgado, F. Neyer, A. L. Deich The Astrophysical Journal Letters, Volume 819, Issue 2, article id. L20, 7 pp. (2016) arxiv:1602.04002
19. Daylan, T., Finkbeiner, D. P., Hooper, D., Linden, T., Portillo, S. K. N., Rodd, N. L., Slatyer, T. R. 2016, Physics of the Dark Universe, 12, 1 astro-ph:1402.6703
20. Abedi, Jahed, Dykaar, Hannah and Afshordi, Niayesh, "Echoes from the Abyss: Evidence for Planckscale structure at black hole horizons" (2016). arXiv:1612.00266
21. Indranil Banik, Hongsheng Zhaoa, Anisotropic Distribution of High Velocity Galaxies in the Local Group astro-ph:1701.06559
22. R. Genzel, N.M. Förster Schreiber, H. Übler, P. Lang, T. Naab, R. Bender, L.J. Tacconi, E. Wisnioski, S. Wuyts, T. Alexander, A. Beifiori, S. Belli, G. Brammer, A. Burkert, C.M. Carollo, J. Chan, R. Davies, M. Fossati, A. Galametz, S. Genel, O. Gerhard, D. Lutz, J.T. Mendel, I. Momcheva, E.J. Nelson, A. Renzini, R. Saglia, A. Sternberg, S. Tacchella, K. Tadaki, D. Wilman, Strongly baryon-dominated disk galaxies at the peak of galaxy formation ten billion years ago Accepted for publication in Nature, astro-ph:1703.04310
23. S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68 (2003) 023509 arxiv:0301273
24. R. J. Gleiser, G. Dotti, Class.Quant.Grav. 23 (2006) 5063-5078 arXiv:0604021
25. M. Milgrom, Monthly Notices Roy. Astronomical Soc. 405 (2) (2010) 1129–1139.

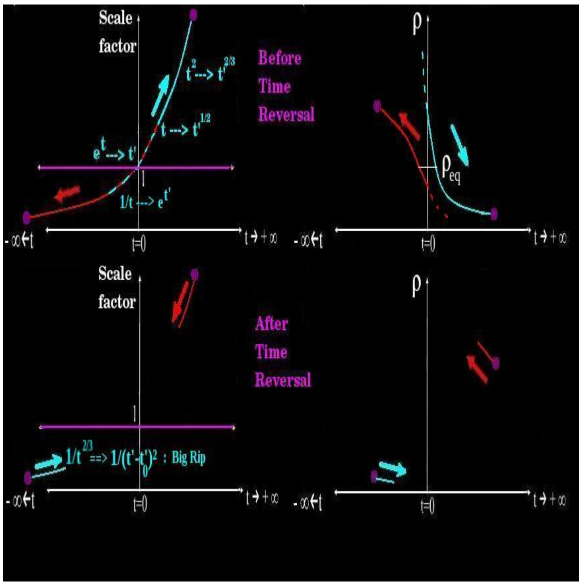

Fig. 1. Evolution laws and time reversal of the conjugate universes, our side in blue