An Elementary Proof of Goldbach's Conjecture

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years. Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Human is very easily cheated by 2 words, infinite and anything, because we never see infinite and anything, so we always make simple thing complex. Goldbach's conjecture is about all very simple numbers, the pattern of prime numbers likes a "kaleidoscope" of numbers, we divided any even numbers into 5 groups and primes into 4 groups, Goldbach's conjecture becomes much simpler. Here we give a clear proof for Goldbach's conjecture based on the fundamental theorem of arithmetic, the prime number theorem, and Euclid's proof that the set of prime numbers is endless.

Key words: Goldbach's conjecture, fundamental theorem of arithmetic, Euclid's proof of infinite primes, the prime number theorem

Introduction

Prime numbers[1] are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. Two of the basic rules are the "fundamental theorem of arithmetic" and Euclid's proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, [1] which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.[2] Primes can thus be considered the “basic building blocks” of the natural numbers.

Euclid's proof[2] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled $\mathbf{P_n}$. Now we can form the number Q by multiplying together all these primes and adding 1, so

$$\mathbf{Q = (P_1 \times P_2 \times P_3 \times P_4 ... \times P_n) + 1}$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n.

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and in all of mathematics. It states:

> Every even integer greater than 2 can be expressed as the sum of two primes.

A kaleidoscope can produce an endless variety of colorful patterns and it looks like a magic, but when you open it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Goldbach’s conjecture is about all numbers, the pattern of prime numbers likes a “kaleidoscope” of numbers, if we divide all even numbers into 5 groups and primes into 4 groups, Goldbach’s conjecture will be much simpler.

If a number ($N>13$) is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. Any number is divisible by 7, it have 1/3 chance is divisible by 3, any number is divisible by 11, it have 1/3 chance is divisible by 3 and 1/7 chance is divisible by 7, any number is divisible by 13, it has 1/3 chance to be divisible 3 and 1/7 chance to be divisible by 7, and 1/11 chance to be divisible by 11, so on, so we have terms: 1/3, 1/7x2/3, 1/11x2/3x6/7, 1/13x2/3x6/7x10/11…,

Let N_o represent any odd number, the chance of N_o to be a non-prime is: [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + …]} ----------Formula 1

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5.

Let ∑ represent the sum of the infinite terms and Δ=1-∑, according to Euclid's proof[3] that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. ∑ may be very close to 1 when N is growing to ∞, but always less than 1. Let Δ=1-∑, when N is growing to ∞, Δ may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If Δ is 0, then there is no prime, that is not true.

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) =[0.333333 + 0.095238 + 0.051948 + 0.039960 +0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009069 + 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003989] =0.688872

For the first 20 term: ∑ = 0.688872, Δ=1-∑ =0.311128

The chance of N_o to be a prime is: Δ=1-[[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/19x2/3x6/7x10/11x12/13x16/17) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43 + …]} ------------Formula 2

Let $1 represents a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; $3 represents a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193….; $7 represents a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197…; and $9 represents a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….

Let O1 represents an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; O3 represents an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,….; O7 represents an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77…; and O9 represents an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

Fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number (O1) with 1 as its last digit is a product of \$1 x \$1, \$3 x \$7, and \$9 x \$9. The first \$9 is 19, but the odd number 9 = 3 x 3, so 3 is the smallest prime for \$9.

The chance of any odd number O1 to be a prime is: Δ_1=1-$\sum_1$=1-[(1/3) + (1/11x2/3) + (1/13x2/3x10/11)+ (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) + (1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31)] + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+…]------------ Formula 3.

The sum of first 20 terms of $\sum_1$ = [(1/3) + (1/11x2/3) + (1/13x2/3x10/11) + (1/19x2/3x10/11x12/13) +(1/23x2/3x10/11x12/13x18/19) + (1/29x2/3x10/11x12/13x18/19x22/23) + (1/31x2/3x10/11x12/13x18/19x22/23x28/29) +
(1/41x2/3x10/11x12/13x18/19x22/23x28/29x30/31) + (1/43x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41)+
(1/53x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43) +
(1/59x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53) +
(1/61x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59) +
(1/71x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61) +
(1/73x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71)+
(1/79x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73) +
(1/83x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79) +
(1/89x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83) +
(1/101x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89) +
(1/103x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89x100/101) +
(1/109x2/3x10/11x12/13x18/19x22/23x28/29x30/31x40/41x42/43x52/53x58/59x60/61x70/71x72/73x78/79x82/83x88/89 x100/101x102/103)] =0.333333 +0.060606 + 0.046620 + 0.029444 +0.046086 + 0.01748 + 0.01568 + 0.011966 + 0.010747 + 0.008517 + 0.007506 + 0.006484 + 0.006032 + 0.005783 + 0.00529 + 0.004953 + 0.004564 + 0.003976 + 0.003861 + 0.003612 = 0.63254

For the first 20 term: $\sum_1$ = 0.63254, Δ_1=1-$\sum_1$ =0.36746, for any number N, Δ_1 >0.

Every odd number with 3 as its last digit is a product of \$3x\$1 or \$7x\$9; \$1 is decided by \$3 or \$3 is decided by \$1and \$9 is decided by \$7 or \$7 is decided by 9, so we need to consider only \$3 and \$7, \$1 and \$9, \$3 and \$9, or \$1 and \$7.

The chance of any odd number O3 ($3 x $1 or $7 x $9) to be a prime is: Δ_3=1-$\sum_3$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x12/13x16/17x22/23x36/37)+ (1/47x2/3x6/7x12/13x16/17x22/23x36/37x42/43) +…]-----------Formula 4

The chance of any odd number O7 ($7 x $1, or $3 x $9) to be a prime is: Δ_7=1-$\sum_7$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) +…]-----------Formula 5

The formula 4 is same as formula 5, so there is almost same number of primes with 3 or 7 as their last digit.

The sum of first 20 terms of $\sum_3$ = [(1/3) + (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47) + (1/67x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53) + (1/73x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67) +(1/83x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73) +(1/97x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83) +(1/103x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97) +(1/107x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103) +(1/113x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107) +(1/127x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113) + (1/137x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127) + (1/157x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137) + (1/163x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47x52/53x66/67x72/73x82/83x96/97x102/103x106/107x112/113x126/127x136/137x156/157) =N/10[1/3+1/10.5+1/22.75+1/32.23+1/46.33+1/77.92+1/93.07+1/104.15+1/120+1/164.61+1/171.01+1/197.14+1/233.2+1/250.2+1/262.47+1/279.80+1/317.27+1/344.97+1/398.24+1/416.11+=[0.333333+0.095238+0.043956+0.031028+0.021585+0.012834+0.010745+0.009602+0.008333+0.006075+0.005848+0.0050773+0.004288+0.003997+0.003810+0.003574+0.003152+0.002899+0.002511+0.002403+0.002331] =0.6102883

For the first 20 term: $\sum_3 = \sum_7$= 0.6103, $\Delta_3=\Delta_7=1-\sum_3$ =0.3897, $\Delta_3= \Delta_7$, For a large number N, Δ_3 should be very close to Δ_7. For or any number N, Δ_3 or Δ_7 >0

Every odd number (O9) with 9 as its last digit is a product terms of \$1 x\$9, \$3x\$3 or \$7x\$7. We need to consider only \$1, \$3, and \$7.

The chance of any odd number O9 to be a prime is: Δ_9=1-$\sum_9$=1-[(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11)+ (1/17x2/3x6/7x10/11x12/13) +(1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) +…] -------Formula 6

The sum of first 20 terms of $\sum_9$ = [(1/3) + (1/7x2/3)+ (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13)] +[1/2(1/23x2/3x6/7x10/11x12/13x16/17) + (1/31x2/3x6/7x10/11x12/13x16/17x22/23) +(1/37x2/3x6/7x10/11x12/13x16/17x22/23x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47) + (1/61x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53) + (1/67x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71) + (1/83x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73) + (1/97x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83) + (1/101x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97) + 1/103x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101)] = 0.333333 + 0.095238 + 0.051948 + 0.039960+ 0.028208 + 0.019622 + 0.013926 + 0.01129 + 0.009914 + 0.009222 + 0.008242 + 0.007152 + 0.006097 + 0.005462 + 0.005004 + 0.004868 + 0.004222 + 0.003424 + 0.003393 + 0.003294 = 0.660395

For the first 20 term: $\sum_9$ =$\sum_9$= 0.660395, Δ_9=1-$\sum_9$ =0.339605, for any number N, Δ_9 >0

Euclid's proof that the set of prime numbers is endless, from these formulas, the number of primes \$1, \$3, \$7, \$9 are very close and they are endless.

For N=600, the smallest prime $1 is 11, it decided the possible largest prime $3 is 53, the smallest prime $9 is 19, but 3 x 3 is 9, so the possible largest prime $7 is 47, 47 x 3 x3=423 (the next will 67 x 3 x3=603>600), so we have: Prime number with 3 as its last digit=600/10 –{600/10[(1/3)+ (1/7x2/3) + (1/13x2/3x6/7) + (1/17x2/3x6/7x12/13) + (1/23x2/3x12/13x6/7x16/17) + (1/37x2/3x6/7x12/13x16/17x22/23) + (1/43x2/3x6/7x16/17x36/37x12/13x22/23)+ (1/47x2/3x6/7x12/13x16/17x36/37x22/23x42/43) + (1/53x2/3x6/7x12/13x16/17x36/37x22/23x42/43x46/47)] = 600/10-600/10[0.333333 + 0.095238 + 0.043956 + 0.031028 + 0.021585 + 0.012834 + 0.010745 + 0.009602 + 0.008333 = 60-60x0.566654 = 60-34 = 26, it is same as the real number. For N=600, we have Δ3=1-∑3=1-0.566654 = 0.433346, every odd number with 3 as its last digit has almost 43% chance to be bigger than 600, every odd number with 3 as its last digit has less than 43% chance to be a prime. a prime number smaller than 600, every odd number with 3 as its last digit has more than 43% chance to be a prime.

1. When any even integer (N) has 0 as its last digit, such as 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120,…,

N-$3= O7, if we can prove at least one of these O7 is $7, then N is the sum of 2 primes; or N-$7= O3, if we can prove at least one of these O3 is $3, then N is the sum of 2 primes as table 1.

Table 1.

$7	…	137	127	107	97	67	47	37	17	7
N-$7	…	N-137	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7
$3	…	113	103	83	73	53	43	23	13	3
N-$3	…	N-113	N-103	N-83	N-73	N-53	N-43	N-23	N-13	N-3

From Formula 3, 4, 5, and 6, we know there are similar numbers of primes with 1, 3, 7, or 9 as their last digit.

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 7 as their last digit(the first line of table 1, 4 x is due to there are only ¼ of total primes with 7 as its last digit), correspondingly, we have N/[4xln(N)] of O3(N-$7, the second line of table 1) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}. For any number N>100, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N>200, there are more than 4 pairs of primes in which one prime ($7) has 7 as its last digit and another prime (N-$7) has 3 as its last digit. When N with 0 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $3 and N-$3 pairs (the bottom 2 lines of table 1) is also true.

Table 2.

$1	…	151	131	101	71	61	41	31	11
N-$1	…	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11
$9	…	149	139	109	89	79	59	29	19
N-$9	…	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 1 as their last digit (the first line of table 2), correspondingly, we have

N/[4xln(N)] of O9(N-$1, the second line of table 2) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}. For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($1) has 1 as its last digit and another prime (N-$1) has 9 as its last digit. When N with 0 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $9 and N-$9 pairs (the bottom 2 lines of table 2) is also true.

For N = 600 (see table 3), {N/[4xln(N)]}/ln{N/[4xln(N)]}={600/[4xln(600)]}/ln{600/[4xln(600)]} =23.449/ln(23.449) =23.449/3.155 =7.43, this is for only $7 and N-$7 pairs, plus $3 and N-$3 pairs, there are 7.43x2=14.86, in fact, 600 can be expressed as the sum of 15 pairs of primes in which one prime with 3 as its last digit and another prime with 7 as its last digit. That should be true for $1 and N-$1 pairs and $9 and N-$9 pairs, in fact, 600 can be expressed as the sum of 16 pairs of primes in which one prime with 1 as its last digit and another prime 9 as its last digit.

Table 3.

7 Prime	17 Prime	27 3x9	37 Prime	47 prime	57 3x19	67 Prime	77 7x11	87 3x29	97 Prime	107 prime	117 3x3x13	127 Prime	137 prime	147 3x7x7	157 prime	167 prime	177 3x59	187 11x17
593 Prime	583 11x53	573 3x191	563 Prime	553 7x79	543 3x181	533 13x41	523 prime	513 3x3x3x19	503 Prime	493 17x29	483 3x7x23	473 11x43	463 prime	453 3x151	443 Prime	433 prime	423 3x3x47	413 7x59
223 prime	233 prime	243 3x3x3x3x3	253 11x23	263 prime	273 3x7x13	283 prime	293 prime	303 3x101	313 Prime	323 17x19	333 3x111	343 7x7x7	353 Prime	363 3x11x11	373 Prime	383 Prime	393 3x131	403 13x31
377 13x29	367 prime	357 3x7x17	347 prime	337 prime	327 3x109	317 Prime	307 prime	297 3x3x33	287 7x41	277 Prime	267 3x89	257 Prime	247 13x19	237 3x79	227 Prime	217 7x31	207 3x3x23	197 Prime
387 3x3x43	397 prime	407 11x37	417 3x139	427 7x61	437 23x19	447 3x149	457 prime	467 prime	477 3x3x53	487 prime	497 7x71	507 3x13x13	517 11x47	527 17x31	537 3x179	547 prime	557 prime	567 3x3x3x3x7

213 3x71	203 7x29	193 prime	183 3x61	173 Prime	163 prime	153 3x3x17	143 11x13	133 7x19	123 3x41	113 3x3x13	103 prime	93 3 x31	83 prime	73 7x11	63 3x3x7	53 prime	43 prime	33 3x11
																3 Prime	13 Prime	23 prime
																597 3x199	587 prime	577 prime
589 19x31	579 3x193	569 Prime	559 13x43	549 3x3x61	539 7x7x11	529 23x23	519 3x178	509 Prime	499 Prime	489 3x163	479 Prime	469 7x67	459 3x3x3x17	449 Prime	439 Prime	429 3x11x13	419 Prime	409 Prime
11 Prime	21 3x7	31 Prime	41 Prime	51 3x17	61 Prime	71 Prime	81 3x3x3	91 7x13	101 Prime	111 3x37	121 11x11	131 Prime	141 3x47	151 Prime	161 7x23	171 3x3x19	181 Prime	191 Prime
381 3x127	371 7x53	361 19x19	351 3x3x3x13	341 11x31	331 Prime	321 3x107	311 Prime	301 7x43	291 3x97	281 Prime	271 Prime	261 3x3x29	251 Prime	241 Prime	231 3x7x11	221 13x17	211 Prime	201 3x67
219 3x73	229 Prime	239 Prime	249 3x83	259 7x37	269 Prime	279 3x3x31	289 17x17	299 13x23	309 3x103	319 11x29	329 7x47	339 3x113	349 Prime	359 Prime	369 3x3x41	379 prime	389 Prime	399 3x7x19
209 11x19	199 Prime	189 3x3x7	179 Prime	169 13x13	159 3x53	149 Prime	139 Prime	129 3x43	119 7x17	109 Prime	99 3x3x11	89 Prime	79 Prime	69 3x23	59 Prime	49 7x7	39 3x13	29 Prime

391 17x23	401 Prime	411 3x137	421 Prime	431 Prime	441 3x3x7x7	451 11x41	461 Prime	471 3x157	481 13x37	491 Prime	501 3x167	511 7x73	521 Prime	531 3x3x59	541 Prime	551 19x29	561 3x11x17	571 Prime
																	591 3x197	581 7x81
																	9 3x3	19 Prime

2. When any even integer (N) has 2 as its last digit, such as 12, 22, 32, 42, 52, 62, 72, 82, 92, 102, 112, 122,…,

Table 4.

$3	…	103	83	73	53	43	23	13	3
N-$3	…	N-103	N-83	N-73	N-53	N-43	N-23	N-13	N-3
$9	…	149	139	109	89	79	59	29	19
N-$9	…	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 3 as their last digit (the first line of table 4), correspondingly, we have N/[4xln(N)] of O9(N-$3, the second line of table 4) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>100, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pair of primes in which one prime ($3) has 3 as its last digit and another prime (N-$3) has 9 as its last digit. When N with 2 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $9 and N-$9 pairs (the bottom 2 lines of table 4) is also true.

Table 5.

$1	…	151	131	101	71	61	41	31	11
N-$1	…	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 1 as their last digit (the first line of table 5), correspondingly, we have N/[4xln(N)] of O1(N-$1, the second line of table 5) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($1) has 1 as its last digit and another prime (N-$3) has 1 as its last digit. When N with 2 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

3. When any even integer (N) has 4 as its last digit, such as 14, 24, 34, 44, 54, 64, 74, 84, 94, 104, 114,….

Table 6.

$7	…	127	107	97	67	47	37	17	7
N-$7	…	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 7 as their last digit (the first line of table 6), correspondingly, we have N/[4xln(N)] of O7(N-$7, the second line of table 6) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($7) has 7 as its last digit and another prime (N-$7) has 7 as its last digit. When N with 4 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 7.

$3	…	153	113	103	83	53	43	23	13	3
N-$3	…	N-153	N-113	N-103	N-83	N-53	N-43	N-23	N-13	N-3
$1	…	181	151	131	101	71	61	41	31	11
N-$1	…	N-181	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 3 as their last digit (the first line of table 7), correspondingly, we have N/[4xln(N)] of O1(N-$3, the second line of table 7) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($3) has 3 as its last digit and another prime (N-$3) has 1 as its last digit. When N with 4 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $1 and N-$1 pairs (the bottom 2 lines of table 7) is also true.

3. When any even integer (N) has 6 as its last digit, such as 6,
4. 16, 26, 36, 46, 56, 66, 76, 86, 96, 106, 116,….

Table 8.

$3	…	153	113	103	83	73	53	43	23	13	3
N-$3	…	N-153	N-113	N-103	N-83	N-73	N-53	N-43	N-23	N-13	N-3

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 3 as their last digit (the first line of table 8), correspondingly, we have N/[4xln(N)] of O3(N-$3, the second line of table 8) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($3) has 3 as its last digit and another prime (N-$3) has 3 as its last digit. When N with 6 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 9.

$9	…	149	139	109	89	79	59	29	19
N-$9	…	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19
$7	…	127	107	97	67	47	37	17	7
N-$7	…	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 9 as their last digit (the first line of table 9), correspondingly, we have N/[4xln(N)] of O7(N-$9, the second line of table 9) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($9) has 9 as its last digit and another prime (N-$9) has 7 as its last digit. When N with 6 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $7 and N-$7 pairs (the bottom 2 lines of table 9) is also true.

5. When any even integer (N) has 8 as its last digit, such as 8, 18, 28, 38, 48, 58, 68, 78, 88, 98,108, 118,….

Table 10.

$9	…	149	139	109	89	79	59	29	19
N-$9	…	N-149	N-139	N-109	N-89	N-79	N-59	N-29	N-19

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 9 as their last digit (the first line of table 10), correspondingly, we have N/[4xln(N)] of O9(N-$9, the second line of table 10) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($9) has 9 as its last digit and another prime (N-$9) has 9 as its last digit. When N with 8 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true.

Table 11.

$7	…	127	107	97	67	47	37	17	7
N-$7	…	N-127	N-107	N-97	N-67	N-47	N-37	N-17	N-7
$1	…	151	131	101	71	61	41	31	11

N-$1	…	N-151	N-131	N-101	N-71	N-61	N-41	N-31	N-11

According to the prime number theorem, the number of prime numbers less than N is approximately given by N/ln(N), so for any number N, there is approximately N/[4xln(N)] of primes with 7 as their last digit (the first line of table 11), correspondingly, we have N/[4xln(N)] of O1(N-$7, the second line of table 11) in which there should be approximately {N/[4xln(N)]}/ln{N/[4xln(N)]}, For any number N>200, {N/[4xln(N)]}/ln{N/[4xln(N)]}>4, so for any number N, there are more than 4 pairs of primes in which one prime ($7) has 7 as its last digit and another prime (N-$7) has 1 as its last digit. When N with 8 as its last digit, we have proved Goldbach's Conjecture is true if the prime number theorem is true. For $1 and N-$1 pairs (the bottom 2 lines of table 11) is also true.

So we have proved that Goldbach's Conjecture is true if the prime number theorem is true for any even number N.

References:

1. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2.
2. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5.
3. Dudley, Underwood (1978), Elementary number theory(2nd ed.), W. H. Freeman and Co., p. 10, section 2.
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