Conjecture on Poulet numbers of the form (q+2^n)*2^n+1 where q prime

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Abstract. In this paper I state the following conjecture: let P be a Poulet number and n the integer for which the number (P – 1)/2^n is odd; then there exist an infinity of Poulet numbers for which the number q = (P – 1)/2^n – 2^n is prime.

Conjecture:

Let P be a Poulet number and n the integer for which the number (P – 1)/2^n is odd; then there exist an infinity of Poulet numbers for which the number q = (P – 1)/2^n – 2^n is prime.

The first eighteen such Poulet numbers:

: 561 = (19 + 2^4)*2^4 + 1;
: 645 = (157 + 2^2)*2^2 + 1;
: 1105 = (53 + 2^4)*2^4 + 1;
: 1387 = (691 + 2^1)*2^1 + 1;
: 1905 = (103 + 2^4)*2^4 + 1;
: 2047 = (1021 + 2^1)*2^1 + 1;
: 2821 = (701 + 2^2)*2^2 + 1;
: 4369 = (257 + 2^4)*2^4 + 1;
: 4681 = (577 + 2^3)*2^3 + 1;
: 5461 = (1361 + 2^2)*2^2 + 1;
: 8481 = (233 + 2^5)*2^5 + 1;
: 13747 = (6871 + 2^1)*2^1 + 1;
: 14491 = (7243 + 2^1)*2^1 + 1;
: 15709 = (3923 + 2^2)*2^2 + 1;
: 15841 = (463 + 2^5)*2^5 + 1;
: 16705 = (197 + 2^6)*2^6 + 1;
: 18705 = (1153 + 2^4)*2^4 + 1;
: 19951 = (9973 + 2^1)*2^1 + 1.

Note that, if we consider q the absolute value of the integer (P – 1)/2^n – 2^n, q is prime also in the case of P = 1729 and P = 12801:

: (1729 – 1)/2^6 – 2^6 = - 37;
: (12801 – 1)/2^9 – 2^9 = - 487.

Few larger such Poulet numbers:

: 999253360153 = (124906670011 + 2^3)*2^3 + 1;
: 999431614501 = (249857903621 + 2^2)*2^2 + 1;
: 999607982113 = (31237749409 + 2^5)*2^5 + 1;
: 999814392501 = (249953598121 + 2^2)*2^2 + 1;
: 999855751441 = (62490984449 + 2^4)*2^4 + 1.