Local Realism Explains Bell Violations

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Local realism reduces to the proposition that local determinate reality is the necessary and sufficient cause of every physical effect. By any standard account, quantum theory requires that local realism is false, because it embeds the famously spooky premise that some physical effects require causes that are instantly effective from arbitrarily far away. Nonetheless, quantum theory has replaced local realism as the foundation of science because, whereas local realism allegedly cannot violate the Bell inequality, quantum theory is taken to do so in accord with experiments. Here we prove, however, that an epistemic contextual model of local realism solves a puzzle about independent random measurements of local hidden variables in a way that causally explains observed Bell violations. We also reveal exactly how the Bell inequality fails to represent the local realistic prediction. Finally, we show that any theory that denies local realism comprises an unfalsifiable causal claim that is freely adjustable to make arbitrary predictions, which thus provide no validation of its causal claim. Results revitalize the hypothesis that local realism prevails.

Einstein, Podolsky, and Rosen[1] (EPR) proved by thought experiment that if quantum theory provides a complete description of physical reality, then local realism is false. They concluded that quantum theory must be incomplete, because "no reasonable definition of reality" (p. 780) could deny local realism. Bell[2] later formulated an inequality to represent the local realistic prediction for EPR experiments, and he concluded that quantum theory violates it. In the consensus view, physical reality violates Bell-type inequalities accordingly[3–10]. In any case, reasonable or not, if the Bell inequality correctly formulates local realism, and so-called Bell tests are otherwise valid, then observed Bell violations suffice proof that local realism is false.

Given that advocates of quantum theory have widely claimed victory over local realism on precisely this line of inference[11–15], however, it is disturbing that a number of researchers have concluded that the Bell inequality does not correctly formulate local realism[16–22]. In fact, it has been argued that correcting mistakes in the standard quantum account of EPR experiments and their relation to Bell inequalities reveals that both quantum theory and physical reality ultimately obey local realism[23–25]. Claims that quantum theory succeeds where local realism fails are further undermined by findings that several photon-based EPR experiments[4,26–28] defy certain predictions of quantum theory by over four standard deviations[29]. Meanwhile, apart from Bell violations, the only serious claims that local realism fails derive from its alleged failure to explain wave-particle duality. Yet, both the program of event-based simulations[20] and research on pilot-wave hydrodynamics[30] generate wave-particle duality phenomena from strictly mechanical principles, which obey local realism by definition.

We thus consider evidence that local realism might be true despite its widespread denial. The essential bit is to show that local realism explains observed Bell violations. Thus, the first section below formulates the local realistic solution to a puzzle representing an EPR experiment, and the second section graphs resulting Bell violations against those predicted by quantum theory. The third section then specifically examines how the Bell inequality fails its purpose to represent the local realistic prediction. The fourth section presents a general analysis of findings. And the final section raises concerns about the broader implications of denying local realism.

Puzzle

Figure 1 describes a puzzle that obeys local realism. Figures 2 and 3 represent logically distinct contexts for analyzing the probability implications of the two independent random measurements associated with the puzzle.

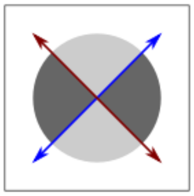

Figure 1 | The puzzle. A circular puzzle object is divided into four equal quadrants by orthogonal axes to form two different measurable factors. Measure the object by drawing two fully independent random lines through the center of the circle. Ignoring the axes, one effect of such measurements is that each line measures either the lighter or the darker factor of the object. A second effect is that the two lines intersect. If an arc of *T* degrees spans the smallest angle of that intersection, then how does p_{FF}, the probability that both lines measure the same factor, relate to *T*?

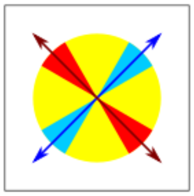

Figure 2 | Non-overlapping *T*-boundaries context ($\mathbf{0 \leq T \leq 45}$). Red and blue sectors represent an interval of *T* marked out in each direction from respective axes corresponding to those of the puzzle object. Given the size of *T*, red and blue sectors cannot overlap each other. Thus, yellow sectors are neither red nor blue. The proportion of each quadrant that is red is $T/90$. The same proportion is blue.

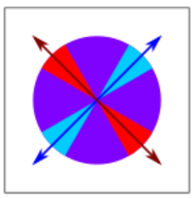

Figure 3 | Overlapping *T*-boundaries context ($\mathbf{45 < T \leq 90}$). Red and blue sectors represent an interval of *T* marked out in each direction from respective axes corresponding to those of the puzzle object. Here, given the size of *T*, red and blue sectors must overlap each other. Thus, purple sectors are both red and blue. The proportion of each quadrant that is non-purple red is $(90 - T)/90$. The same proportion is non-purple blue.

To solve the puzzle presented in Figure 1, we make proxy measurements of Figures 2 and 3 to obtain the solutions for their respective separate contexts. We use the three premises listed below. We justify (A) immediately. We prove (B) and (C) in a subsequent paragraph.

(A) The puzzle asks for the probability of a certain joint effect that is causally sufficed by two fully independent random measurements that are conceptually identical. Its solution must therefore involve a fully symmetric causal account reflected by a correspondingly squared term. By contrast, any approach using an arc of *T* degrees to measure a random rotational position on the puzzle object reduces to a single measurement that requires Arc *T* to have a determinate length before it has the two determinate endpoints that are the necessary cause of that length. Thus, such an approach represents a false causal model that will obtain a falsely linear solution.

(B) Two independent random measurements of the puzzle object in Figure 1 measure *different* factors if and only if when simulated using the appropriate context of Figure 2 or Figure 3:

(B1) *both* proxy measurements measure red (which includes purple in Fig. 3), OR

(B2) *both* proxy measurements measure blue (which includes purple in Fig. 3).

(C) Two independent random measurements of the puzzle object in Figure 1 measure the *same* factor if and only if when simulated using the appropriate context of Figure 2 or Figure 3:

(C1) *not both* proxy measurements measure red (which includes purple in Fig. 3), AND

(C2) *not both* proxy measurements measure blue (which includes purple in Fig. 3).

Prove (B) and (C) as follows. Simulate the two puzzle measurements of Figure 1 using two independent random proxy measurements of either Figure 2 or Figure 3 depending on the assumed size of T. For example, consider a simulation using Figure 2 in which both proxy measurements measure red. Both consequently imply measurements of Figure 1 that fall within T degrees of its red axis. To join these two measurements of Figure 1 by the required outcome arc of T degrees, however, Arc T must cross the red axis in Figure 1. This geometry means that our two implied measurements of Figure 1 must measure different factors of the puzzle object. Remembering to treat the purple in Figure 3 as part of its red sectors, we verify that this same logic holds for the context of Figure 3, which proves (B1). Prove (B2) by the symmetry of blue to red. Conversely, if neither (B1) nor (B2) obtain, then our implied Arc T cannot cross either axis of the puzzle object in Figure 1, which means that the two measurements it connects there must measure the same factor of the puzzle object. This proves (C) in its entirety.

We can now calculate the solution to our puzzle. For the non-overlapping context depicted in Figure 2, the probability that a given random proxy measurement measures red is $T/90$. The probability that both proxy measurements measure red is therefore $(\frac{T}{90})^2$. Thus, the probability that either both measure red or both measure blue is $2(\frac{T}{90})^2$. Premise (B) tells us that this is the probability of measuring different factors of the puzzle object. Because the only alternative is to measure the same factor, we can calculate that probability by subtraction from unity. Thus:

$$p_{FF}(0 \leq T \leq 45) = 1 - 2\left(\frac{T}{90}\right)^2. \tag{1}$$

For the overlapping context depicted in Figure 3, begin by verifying that the only way to meet both conditions (C1) and (C2), and thus the only case corresponding to two measurements of the same factor of the puzzle object, is for one proxy measurement to measure non-purple red and the other to measure non-purple blue. The probability in each case is $(90 - T)/90$, but either color might be measured first. Thus:

$$p_{FF}(45 \leq T \leq 90) = 2\left(\frac{(90-T)}{90}\right)^2. \tag{2}$$

If we suppose that Figure 1 represents the measurable cross-section of a photon (or electromagnetic wave), then we can further suppose that an identical photon with the same rotational position is stacked directly behind it (through the page) at some arbitrary distance. If our two puzzle measurements correspond to one each of these two respective photons, then the result is analogous to Bell-test linear polarization measurements of correlated (Type 1) polarization-entangled photons. Because our puzzle fully obeys local realism even after these transformations, equations (1) and (2) constitute the local realistic predictions for such Bell tests.

By contrast, for these same Bell tests, quantum theory predicts:

$$p_{FF(QT)}(0 \leq T \leq 90) = \cos^2 T. \tag{3}$$

Note, to obtain the local realistic predictions of p_{FF} for anticorrelated (Type 2) polarization-entangled photons, we simply subtract each of equations (1) and (2) from unity.

Violation

Remarkably, our newly derived local realistic predictions for Bell-tests, equations (1) and (2), violate the Bell inequality. To visualize this along with the violations predicted by quantum theory, we follow Aspect[31] in computing the Clauser-Horne-Shimony-Holt[32] (CHSH) test statistic, S_{max}. We begin by noting that

$$S_{\max}(T) = 3E(T) - E(3T), \tag{4}$$

where

$$E(T) = 2p_{FF}(T) - 1. \tag{5}$$

Thus, we input our $p_{FF}(T)$ values from equations (1), (2), and (3) into (4) via (5) to obtain the graphs in Figure 4 below.

The one caveat is that the rules of our puzzle do not permit T values to exceed 90. Thus, for inputs (1) and (2), whenever "$3T$" in (4) exceeds 90, interpret that value using the rules of our puzzle. Do this by taking two measurements of the puzzle object such that one is rotated by $3T$ from the other. The smallest resulting angle between these two measurements then lawfully replaces "$3T$" in (4).

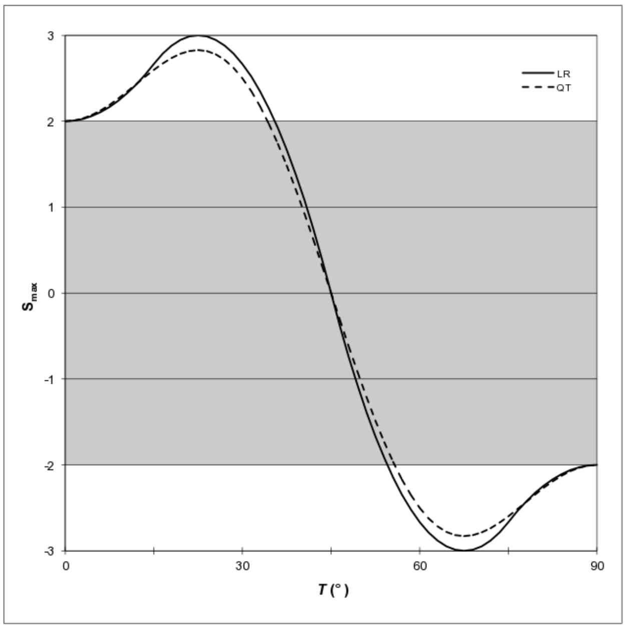

Figure 4 | Bell violations. Quantity S_{max} is the maximized value of the test statistic for the CHSH version of the Bell inequality. Quantity T is the angle of offset between two independent measurements corresponding to a Bell-test of correlated (Type I) polarization-entangled photons. The shaded region represents all predictions that obey the Bell inequality. Thus, the two unshaded regions represent Bell violations. Predictions from our new formulation of local realism (LR) via the puzzle presented in Figure 1 correspond to the solid line. Predictions from quantum theory (QT) correspond to the dotted line.

Per Figure 4, both local realism and quantum theory predict their largest Bell violations at the Bell test angles of 22.5° and 67.5°. Local realism, however, predicts larger Bell violations ($|S_{\max}(T)| = 3$) than quantum theory predicts ($|S_{\max}(T)| = 2\sqrt{2} \cong 2.83$).

Postmortem

How is something as celebrated as Bell inequality so widely mistaken for what it is not? Paradox, subtle or otherwise, invariably implies reference frame confusion. In this arguably subtle case, we can write the original Bell inequality[2] as

$$1 + X(b,c) \geq |X(a,b) - X(a,c)|. \tag{6}$$

The terms in (6) follow the form $X(s_{L1}, s_{L2})$, where X is the expected outcome for a Bell-test experiment that depends on two settings, s_{L1} and s_{L2}, corresponding to respective measurement devices at separate localities *L1* and *L2*. Thus, the reference frame of (6) requires that its two settings b_{L1} and b_{L2} refer to respective separate localities such that $b_{L1} = b_{L2}$.

Yet, local realism requires that separate localities are separate causalities. We can therefore physically recalibrate the meaning of b_{L1} without causing any such effect on b_{L2}. For example, consider two pencils lying on separate plates. We can use these pencils such that their respective rotational positions represent our respective device settings b_{L1} and b_{L2}. Local realism, like physical reality, permits us to rotate one pencil without rotating the other. Thus, there is no difficulty generating the condition that $b_{L1} \neq b_{L2}$, which contradicts (6). In other words, the reference frame of (6) represents neither local realism nor physical reality.

Analysis

The puzzle presented by Figure 1 represents an ontic model of local realism that corresponds to Bell tests of correlated (Type 1) linear polarization-entangled photons. Figures 2 and 3 represent a corresponding epistemic model because their associated proxy measurements measure probability regimes, which are knowledge constructs rather than physical realities as required by an ontic model.

Critically, our predictive equations, (1) and (2), derive exclusively from Figures 2 and 3. These two figures serve as separate contexts for interpreting identical pairs of proxy measurements. For example, consider two proxy measurements that both fall near the vertical axis of the quadrant pattern shared by Figures 2 and 3. If we assume they measure Figure 2, then we interpret them to measure the same factor of the puzzle object. Yet, if we assume they measure Figure 3, then we interpret them to measure different factors of the puzzle object. We need to make both assumptions to calculate respective predictions. Thus, our predictive equations, (1) and (2), formulate a model of local realism that is not only epistemic but also contextual. We suggest that any "impossibility proof" would have to be imaginative indeed to rule out such a formulation.

In any case, Bell violations are not equivalent to violations of local realism, because the Bell inequality does not correctly formulate local realism. Figure 4 proves this by example. More specifically, Figure 4 reveals that local realism causally explains Bell violations similar to but also even larger than those predicted by quantum theory. These findings contradict all arguments to date that observed Bell violations support quantum theory over local realism. In fact, the reverse is now plausible -- which opens new prospects for empiricism. For example, if equations (1) and (2) aptly predict empirical data from entangled photons as Figure 4 suggests they might, then, together with Figures 1, 2, and 3, they constitute a model of local realism that renders quantum theory causally incomplete in precisely the sense concluded by EPR[1]. Further implications for the nature of physical reality might then be profound.

Motivation

Bell-based arguments against local realism fail as we have shown. Wave-particle duality arguments against local realism fail as we have noted. Meanwhile, local realism explains the results of every chain of dominoes ever toppled and every imaginable analog of such chains. These analogs now include Bell violations and thus arguably all known physical phenomena.

Yet, somehow, the most shocking thing we can say appeals to no new facts at all. The only alternative, now or ever, to the physically continuous causal chains of local realism is the claim that some physically discontinuous causal chains cause (miraculous) effects anyway. As desirable as this might be, such chainless chains explain all phenomena with the same facility that unobservable psychokinetic aliens do. Simply put, they are omnipotent and unfalsifiable. Thus, any theory that denies local realism is freely adjustable to make arbitrary predictions, but any claim that the resulting predictive validity implies validated causal explanation lies somewhere on a continuum between poor logic and foreclosed belief.

Nonetheless, such claims remain widely endorsed from positions of authority. And in that context, hidden physical realities threaten knowledge far less than hidden human motives do. The question thus arises: If we remain motivated to deny local realism, then how does that motivation reflect the mission of science?

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