Grand Unification from a Classical Interpretation of "Spin" Angular Momentum

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Abstract: It is shown from the laws of conservation of energy and angular momentum that the gravitational energy of a bound state particle is dependent upon its geometric mean angular frequency. A classical interpretation for the intrinsic "spin" angular momentum of a particle is submitted that can be tested macroscopically with the Rarita−Schwinger equation for the orbit of Mercury. Librations in the relative obliquity of planetary bodies are shown to be analogous to the Larmor precession of atomic particles. Schrodinger's concept of a bound state electron's zitterbewegung is hypothesized to be the librations in the electron's relative obliquity.

Keywords: Gravitational Energy, Spin, Obliquity, Larmor Precession, Zitterbewegung

INTRODUCTION

It is well known that the angular momentum **L** of a bound state particle in an eccentric orbit is equivalent at the extreme points of its orbit:

$$(1)\quad \mathbf{L} = m\boldsymbol{v}_{\max} \times \boldsymbol{r}_{min} = m\boldsymbol{v}_{\min} \times \boldsymbol{r}_{max}\,,$$

where m is a particle's mass, $\boldsymbol{v}$ is its tangential velocity, and $\boldsymbol{r}$ is its orbital radius. We also know from the law of conservation of energy that

$$(2)\quad 0 = (½ m\boldsymbol{v}^2_{\max} - \frac{\mathrm{G}Mm}{\boldsymbol{r}_{\min}}) - (½ m\boldsymbol{v}^2_{\min} - \frac{\mathrm{G}Mm}{\boldsymbol{r}_{\max}}),$$

where G is Newton's gravitational constant and M is the mass of a primary body.

A PARTICLE'S NON−RELATIVISTIC GRAVITATIONAL ENERGY

Rearranging Eq. 2 and removing the common factor m,

$$(3)\quad ½(\boldsymbol{v}^2_{\max} - \boldsymbol{v}^2_{\min}) = \mathrm{G}M\left(\frac{1}{\boldsymbol{r}_{\min}} - \frac{1}{\boldsymbol{r}_{\max}}\right).$$

Substituting the values of $\boldsymbol{v}_{\min}$ and $\boldsymbol{v}_{\max}$ from Eq. (1) in Eq. (3),

$$(4)\quad \frac{\mathbf{L}^2}{2m^2}\left(\frac{1}{\boldsymbol{r}_{\min}} + \frac{1}{\boldsymbol{r}_{\max}}\right)\left(\frac{1}{\boldsymbol{r}_{\min}} - \frac{1}{\boldsymbol{r}_{\max}}\right) = \mathrm{G}M\left(\frac{1}{\boldsymbol{r}_{\min}} - \frac{1}{\boldsymbol{r}_{\max}}\right),$$

and upon rearrangement of the radii,

$$(5)\quad \frac{\mathbf{L}^2}{2m^2} = \frac{GM}{\left(\frac{1}{r_{\min}} + \frac{1}{r_{\max}}\right)}.$$

From orbital mechanics we know the semi−major axis a and semi−minor axis b of a particle's orbit are equivalent to its arithmetic and geometric mean distances from the focus respectively. The sum of the inverse extreme radii in Eq. (5) is therefore,

$$(6)\quad \left(\frac{1}{r_{\min}} + \frac{1}{r_{\max}}\right) = \frac{r_{\min} + r_{\max}}{r_{\min} r_{\max}} = \frac{2a}{b^2}.$$

Combining Eqs. (5 & 6),

$$(7)\quad \frac{\mathbf{L}^2}{2m^2} = \frac{GMb^2}{2a},$$

from which a particle's angular momentum **L** is,

$$(8)\quad \mathbf{L} = mb\sqrt{\frac{GM}{a}}.$$

Since **L** is a conserved quantity and a particle's geometric mean distance $b = \sqrt{r_{min} r_{\max}}$, a particle's geometric mean tangential velocity v_b is,

$$(9)\quad v_b = \sqrt{v_{\min} v_{max}} = \frac{\mathbf{L}}{mb} = \sqrt{\frac{GM}{a}}.$$

A particle's non−relativistic gravitational energy U(b, t) can then be defined from Eqs. (7 & 9) as,

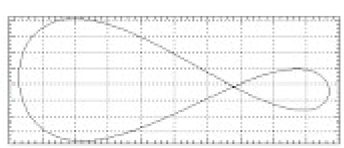
$$(10)\quad \mathrm{U}(b, t) = -\frac{GMm}{a} = -\frac{\mathbf{L}^2}{mb^2} = -mv_b^2 = -\mathbf{L}\cdot\frac{v_b}{b} = -\mathbf{L}\cdot\omega_b,$$

where ω_b is a particle's geometric mean angular frequency.

A CLASSICAL INTERPRETATION OF "SPIN"

We can see from the equation of time graph to the left that the frequency of Earth's obliquity (mauve dashed curve) is roughly twice the frequency of its eccentricity (blue dash−dot curve). The superposition of the orthogonal temporal waves (red curve) can be can be graphed parametrically as a Lissajous curve that can be observed from the Earth's surface as the solar analemma:

$$(11)\quad x(t) = A\sin(st + \delta),\ \ y(t) = B\sin t =$$



where A and B are the amplitudes of the eccentric and oblique temporal waves respectively, δ is the observational angle of the Lissajous figure, and the "spin" ratio $s = f_E : f_O \approx 1{:}2$ (for Earth's orbit), where f_E and f_O are the frequencies of the Earth's eccentricity and obliquity respectively.

The visual appearance of a Lissajous figure is dependent upon the "height−to−width" ratio $A : B$ and the spin ratio s (a circle resulting when $A = B$, $s = 1{:}1$, and $\delta = \pi{:}2$). We can see from the visual appearance of the solar analemmas of other planets in our system,

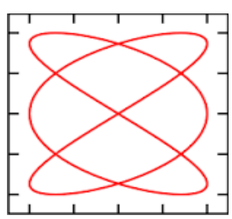
that a planet's spin ratio s can vary between $\approx 1{:}1$ to $1{:}2$, analogous to the spin of bosons and fermions in electrodynamics. It can also be deduced from Eq. (11) that $s = 2{:}1$, $\delta = \pi{:}2$ results in,

$$(12)\quad (A\cos(2\,t), \sin t) = (A\,(1 - 2\sin^2 t), \sin t) = (A - 2A\sin^2 t, \sin t)\,,$$

which is a horizontally offset version of the parametric equation for a parabola. This could explain why particle's with $s \geq 2$ have not been observed in atomic nuclei. The planet Mercury, with $s \approx 3{:}2$, would be analogous to a composite gravitino; theoretically governed by the Rarita−Schwinger equation[1],

$$(13)\quad (\epsilon^{\mu\kappa\rho\nu}\gamma_5\gamma_\kappa\partial_\rho - m\frac{i^2}{2}[\gamma^\mu, \gamma^\nu])\psi_\nu = 0,$$

where $\epsilon^{\mu\kappa\rho\nu}$ is the alternating symbol, γ_5 and γ_κ are Dirac matrices, m is the gravitino's mass, and ψ_ν is a vector−valued spinor that has additional components compared to the Dirac equation spinor[2]. With the values in Eq. (11) set to $A = B$, $s = 3{:}2$, $\delta = \pi{:}2$, the Lissajous figure for a gravitino would be:



THE ORIGIN OF A PLANET'S OBLIQUE ANGULAR FREQUENCY

It is proposed that a planet's oblique angular frequency $\tilde{\boldsymbol{\omega}} = 2\pi f_O$ (Larmor precession)[3] is caused by the torque $\boldsymbol{\tau}$ induced on its magnetic moment $\boldsymbol{m}_P$ by the total magnetic field $\mathbf{B}$ of the system[4,5,6,7],

$$(14)\quad \boldsymbol{\tau} = \boldsymbol{m}_P \times \sum_{i=1}^{n}\mathbf{B}_i = \gamma\mathbf{J} \times \sum_{i=1}^{n}\mathbf{B}_i = \mathbf{J} \times \tilde{\boldsymbol{\omega}} = 1 + \frac{\boldsymbol{M}_{orb}}{(\boldsymbol{M}_{spin} + \boldsymbol{M}_{orb})}\mathbf{J} \times \sum_{i=1}^{n}\mathbf{B}_i\,,$$

where the last term describes the Einstein−de Haas[4] and Barnett[5] effects, γ is a planet's gyromagnetic ratio, $\mathbf{J}$ is its total (gyromagnetic) angular momentum[3], $\boldsymbol{M}_{orb}$ is its orbital magnetization[6,7] and $\boldsymbol{M}_{spin}$ is its spin magnetization[5]. Isolating the oblique angular frequency $\tilde{\boldsymbol{\omega}}$[3] to one side in Eq. (14) yields,

$$(15)\quad \tilde{\boldsymbol{\omega}} = \frac{|\boldsymbol{\tau}|}{\mathbf{J}\cdot\sin\theta} = \gamma\sum_{i=1}^{n}\mathbf{B}_i = \left(1+\frac{\boldsymbol{M}_{\mathrm{orb}}}{(\boldsymbol{M}_{\mathrm{spin}}+\boldsymbol{M}_{\mathrm{orb}})}\right)\sum_{i=1}^{n}\frac{\mu_0\boldsymbol{m}_i}{4\pi\boldsymbol{r}_i^3}\sqrt{1+3\cos^2\theta_i}\,,$$

where μ_0 is the magnetic constant and θ_i is the azimuthal angle from the polar axis of each of the magnetic dipole moments $\boldsymbol{m}_i$ within the system.

The oblique angular frequency $\tilde{\boldsymbol{\omega}}$ (Larmor precession) for planets with $s \approx 1{:}2$ (analogous to composite fermions) could theoretically be described with the Bargmann−Michel−Telegdi (BMT) equation[8],

$$(16)\quad \tilde{\boldsymbol{\omega}} = \frac{|\boldsymbol{\tau}|}{\mathbf{J}\cdot\sin\theta} = \gamma\sum_{i=1}^{n}\mathbf{B}_i = 2\boldsymbol{m}_{\mathbf{p}}(F^{\tau\lambda} - u^{\tau}u_{\sigma}F^{\sigma\lambda})a_{\lambda}\,,$$

where $\boldsymbol{m}_{\mathbf{P}}$ is the planet's magnetic moment, u^{τ} is its four−velocity, and $F^{\tau\sigma}$ is the "gravitomagnetic" field−strength tensor. Assuming the coefficient 2 in the BMT equation is a particle's inverse spin ratio $\dot{s} = 2 = 1{:}s$, the coefficient would be $\dot{s} = 1$ for bosons and $\dot{s} = 2/3$ for gravitinos.

CONCLUSION

It was discovered by Schrodinger[9] from analysis of solutions to the Dirac equation[2] that the energy of a bound state electron oscillates with an angular frequency $\tilde{\boldsymbol{\omega}}_{\mathbf{e}}$ [9,10] equivalent to,

$$(17)\quad \tilde{\boldsymbol{\omega}}_{\mathbf{e}} = \frac{2m_e c^2}{\hbar}\,,$$

where c is the speed of light in a vacuum, m_e is an electron's mass, and ħ is Planck's reduced constant. Schrodinger referred to this as an electron's "zitterbewegung" (wiggling motion). It is hypothesized that an electron's zitterbewegung is the angular frequency of its obliquity $\tilde{\boldsymbol{\omega}}$[3,10], and the reason $\tilde{\boldsymbol{\omega}}$ differs by a factor of 2 from the Planck−Einstein relation is because fermions are $s = 1{:}\dot{s} = 1{:}2$ particles.

Since an electron's angular momentum is quantized in units of ħ, and we know from Einstein's special theory of relativity that $E = mc^2$, the energy of a bound state electron's orbit could be interpreted from Eqs. (10 & 17) as being its gravitational (potential) energy,

$$(18)\quad E = \frac{\hbar\cdot\tilde{\boldsymbol{\omega}}_{\mathbf{e}}}{2} = \frac{\mathbf{L}\cdot\tilde{\boldsymbol{\omega}}}{\dot{s}} = \mathbf{L}\cdot\omega_b = \mathrm{U}(b,t)\,.$$

Since a particle's angular momentum **L** is dependent upon its geometric mean distance b (Eq. 8), the oscillations in an electron's energy states could be interpreted as oscillations in ω_b relative to b (the gravitational energy $\mathrm{U}(b, t)$ radius is normalized to b instead of ∞).

With this interpretation of Eq. (18), the relative consistency observed in the stellar orbital speeds of spiral galaxies could be explained if the geometric mean angular frequency ω_b of the stars increase proportionately with their distance from the galactic nucleus. Geological records[11] indicate our Sun oscillates relative to the plane of our galaxy in $\approx 31 \pm 1$ Myr cycles during its $\approx 225 - 250$ Myr orbit, which could provide physical evidence for this hypothesis[12].

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