AN INTRODUCTION TO THE n-FORMAL SEQUENTS AND THE FORMAL NUMBERS

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Abstract. In this work, we introduce the n-formal sequents and the formal numbers defined with the help of the second order logic. We give many concrete examples of formal numbers and n-formal sequents with the Peano's axioms and the axioms of the real numbers. Shortly, a sequent is n-formal iff the sequent is composed by some closed hypotheses and a n-formal formula (a close formula with one internal variable such that the formula is only true when we set that variable to the unique natural number n), and it does not exist some strict sub-sequent which are composed by some closed sub-hypotheses and some sub-m-formal formula with $m > 1$. The definition is motivated by the intuition that the "Nature's hypotheses"do not carry natural numbers or "hidden natural numbers" except for the numbers 0 and 1, i.e., they can be used in a n-formal sequent. Moreover, we postulate at second order of logic that the "Nature's hypotheses"are not chosen randomly: the "Nature's hypotheses"are the only hypotheses which give the largest formal number $N_Z \in \left[10^{3.2\times10^3}, 10^{7.3\times10^5}\right] \cong \left[2^{1.0\times10^4} - 2^{2.4\times10^6}\right]$. The Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture, the generalized Fermat's conjecture and the Schinzel's hypothesis H are reviewed with this new (second order logic) formal axiom. Finally, three open questions remain: Can we prove that a natural number is not formal? If a formal number n is found with a function symbol f where its outputs values are only 0 and 1, can we always replace the function symbol f by a another function symbol $\tilde{f}$ such that $\tilde{f} = 1 - f$ and the new sequent is still n-formal? Does a sequent exist to make a difference between the definition of the n-formal sequents and the following weaker variant of that definition: we look at the explicit sub-formulas of ϕ which induce the m-formal formulas instead of looking at the explicit sub-formulas of $\phi_{n-formal}$ which are m-formal formulas?

Contents

1. Introduction

The present paper is motivated by the consequences (in number theory and in fundamental physics) of the definition of a formal number, the definition of a n-formal sequent, the (second order logic) formal axiom which states the existence of the largest formal number N_Z and the (second order logic) formal hypothesis on the "Nature's hypotheses"(the required hypotheses to explain the physical measurements). The first set of consequences are: the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture, the generalized Fermat's conjecture (which requires computational resources which are not reached today even for checking the simplest case: $a = 2$ and $b = 1$) and the Schinzel's hypothesis H (which requires computational resources which are not reached today for checking about $\left(\pi\left(N_Z\right) 2^{N_Z}\right)^{N_Z}$ cases) are solved by the (second order logic) formal axiom. The second set of consequences are the "Nature's hypotheses"generated by the formal hypothesis on the "Nature's hypotheses".

From researches in fundamental physics, the formal numbers and the n-formal sequents definitions arise from the intuition that the "Nature's hypotheses"do not carry natural numbers or "hidden natural numbers" except for 0 and 1, i.e. the "Nature's hypotheses"can be used in a n-formal sequent. Shortly, a sequent is n-formal iff the sequent is composed by some closed hypotheses and a n-formal formula (a close formula with one internal variable such that the formula is only true when we set that variable to the unique natural number n), and it does not exist some strict sub-sequent which are composed by some closed sub-hypotheses and some sub-m-formal formula with $m > 1$. Moreover, we postulate (at second order of logic) that the "Nature's hypotheses"are not chosen randomly: the "Nature's hypotheses"are the only hypotheses which give the largest formal number $N_Z \in \left[10^{3.2\times10^3}, 10^{7.3\times10^5}\right] \cong \left[2^{1.0\times10^4} - 2^{2.4\times10^6}\right]$.

The paper is organized as follow: firstly, we present the notations used throughout this paper. Secondly, we define what is an explicit sub-formula in order to define what is a formal number and a n-formal sequent. Thirdly, we present some formal number examples. Fourthly, we present the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture, the generalized Fermat's conjecture and the Schinzel's hypothesis H as n-formal sequents. Fifthly, we present the (second order logic) formal axiom, the (second order logic) hypothesis on the "Nature's hypotheses"and their consequences. Sixthly, we ask ourselves three open questions about the formal numbers. Seventhly, we present some larger formal number examples. Eighthly, we present some formal number examples with the axioms of the real numbers and ninethly, the conclusion and the acknowledgment.

2. Notations

In the present paper:

1- We omit some parentheses and parenthesis labels to improve the readability but they are necessary for writing the related explicit formulas and explicit sub-formulas.

2- We use the formula $\phi \to \psi$ instead of the formula $\neg\phi \vee \psi$ to improve the readability. However the strict explicit sub-formulas of the formula $\phi \to \psi$ are ϕ, ψ, $\neg\phi$, and $\phi \vee \psi$ instead of only ϕ and ψ.

3- We use the formula $t_1 = t_2$ instead of the formula $\mathcal{R}_= (t_1, t_2)$ to improve the readability. However the strict explicit sub-formulas of the formula $t_1 = t_2$ are only t_1 and t_2 instead of t_1, t_2, $t_1 =$ and $= t_2$.

4- If the formula ϕ is previously defined, the formula $\phi_{[y/x]}$ is a shortcut for the formula ϕ written with the variable y instead of the variable x with respect to the explicit sub-formulas of $\phi_{[y/x]}$.

3. Definitions

Let consider a language $\mathcal{L} \cup \mathcal{L}_{Peano}$ of first order logic which contains the language needed for the Peano hypotheses.

Let introduce the necessary preliminary definitions and lemmas:

1- Preliminary definitions and lemmas about the explicit sub-formulas of a formula ϕ:

   a- A formula ϕ containing l pair of parentheses is an explicit formula iff the i^{th} opening parenthesis and the corresponding i^{th} closing parenthesis are labeled unambiguously with respect to the other parentheses with an injection $f : \{1, ..., l\} \subset \mathbb{N} \longrightarrow \mathbb{N}$ such that: $... \underset{f(i)}{\Bigg(} .... \underset{f(i)}{\Bigg)} ...$.

   b- Preliminary lemma:

   Every formula ϕ can be written as an explicit formula.

   c- An explicit formula ψ is an explicit sub-formula of a formula ϕ iff the formula ψ is an explicit formula and ψ is a sub-sequence of the symbol sequence of the formula ϕ written as an explicit formula.

   d- Preliminary lemma about the explicit sub-formulas of a formula ϕ:

   An explicit sub-formula of an explicit sub-formula of a formula ϕ is an explicit sub-formula of the formula ϕ.

2- Preliminary definition about the n-formal formulas:

A formula $\phi_{n-formal}$ is a n-formal formula iff $\phi_{n-formal}$ is a closed formula and a formula ϕ exists such that:

$$\phi_{n-formal} \equiv \phi_{[\underbrace{f_s(...f_s(}_{n \text{ times}} c_0 \underbrace{)...)}_{n \text{ times}} /x]} \wedge \exists! y \left(\phi_{[y/x]}\right) \quad . \tag{3.1}$$

We rewrite the previous equation without the shortcut symbol $\exists!$:

$$\phi_{n-formal} \equiv \phi_{[\underbrace{f_s(...f_s(}_{n \text{ times}} c_0 \underbrace{)...)}_{n \text{ times}} /x]} \wedge \neg\exists y \exists y' \left(\neg y = y' \wedge \phi_{[y/x]} \wedge \phi_{[y'/x]}\right) \quad . \tag{3.2}$$

The main definition: a sequent (see the previous equation),

$$\Gamma, \Gamma_{Peano} \vdash \phi_{n-formal} \tag{3.3}$$

is a n-formal sequent and n is a formal number iff:

1- the hypotheses Γ, Γ_{Peano} are closed formulas and the formula $\phi_{n-formal}$ is a n-formal formula,

2- and for every closed and explicit sub-formula Δ, Δ_{Peano} of the hypotheses Γ, Γ_{Peano} and for every m-formal and explicit sub-formula $\psi_{m-formal}$ of the formula $\phi_{n-formal}$ such that:

$$\Delta, \Delta_{Peano} \vdash \psi_{m-formal} \tag{3.4}$$

then, $m = 0$ or $m = 1$ or ($\Delta \equiv \Gamma$ and $\Delta_{Peano} \equiv \Gamma_{Peano}$ and $\psi_{m-formal} \equiv \phi_{n-formal}$).

4. Some formal number examples

We give in this section some examples of formal numbers. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the prime function symbol f_{Prime}:

$$\begin{aligned}&\forall x \left(\exists y \exists z \left(\neg y = f_s\left(c_0\right) \wedge \neg z = f_s\left(c_0\right) \wedge x = f_\times\left(y, z\right)\right) \rightarrow f_{Prime}\left(x\right) = c_0\right)\\&\forall x \left(\neg\exists y \exists z \left(\neg y = f_s\left(c_0\right) \wedge \neg z = f_s\left(c_0\right) \wedge x = f_\times\left(y, z\right)\right) \rightarrow f_{Prime}\left(x\right) = f_s\left(c_0\right)\right) \ .\end{aligned} \tag{4.1}$$

2- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the sub-function symbol $g_{\sigma_1 - id}$ and the function symbol $f_{\sigma_1 - id}$ which gives the sum of proper divisors:

$$\begin{aligned}&\forall x \left(g_{\sigma_1 - id}(x, f_s\left(c_0\right)) = c_0\right)\\&\forall x \forall y \left(\exists z \left(x = f_\times\left(y, z\right)\right) \rightarrow g_{\sigma_1 - id}\left(x, f_s\left(y\right)\right) = f_+\left(g_{\sigma_1 - id}(x, y), y\right)\right)\\&\forall x \forall y \left(\neg\exists z \left(x = f_\times\left(y, z\right)\right) \rightarrow g_{\sigma_1 - id}\left(x, f_s\left(y\right)\right) = g_{\sigma_1 - id}(x, y)\right)\\&\forall x \left(f_{\sigma_1 - id}(x) = g_{\sigma_1 - id}(x, x)\right) \ .\end{aligned} \tag{4.2}$$

Trivially, 0 and 1 are formal numbers with the following formulas ϕ:

$$\phi \equiv x = c_0 \text{ and } \phi \equiv x = f_s\left(c_0\right) \ . \tag{4.3}$$

2 is a formal number with the following formula ϕ:

$$\phi \equiv x = f_+\left(f_s\left(c_0\right), f_s\left(c_0\right)\right) \tag{4.4}$$

or for instance, the following formula ϕ:

$$\phi \equiv \forall y \left(x < y \vee y = x \vee y = c_0 \vee y = f_s\left(c_0\right)\right) \ . \tag{4.5}$$

In order to prove that some other natural numbers are formal, we use the prime function f_{Prime} (see 4.1):

3 is a formal number with the following formula ϕ (see 4.1):

$$\begin{aligned}\phi \equiv &\forall y \left(\neg y = c_0 \wedge \neg y = f_s\left(c_0\right) \wedge f_{Prime}\left(y\right) = c_0 \rightarrow x < y\right) \wedge\\&\neg\exists x' \left(x < x' \wedge \forall y \left(\neg y = c_0 \wedge \neg y = f_s\left(c_0\right) \wedge f_{Prime}\left(y\right) = c_0 \rightarrow x' < y\right)\right) \ .\end{aligned} \tag{4.6}$$

4 is a formal number with the following formula ϕ (see 4.1):

$$\begin{aligned}\phi \equiv& \neg x = c_0 \wedge \neg x = f_s\left(c_0\right) \wedge f_{Prime}\left(x\right) = c_0 \wedge \\ & \neg \exists x' \left(x' < x \wedge \neg x' = c_0 \wedge \neg x' = f_s\left(c_0\right) \wedge f_{Prime}\left(x'\right) = c_0\right) \ . \end{aligned} \tag{4.7}$$

In order to prove that some other natural numbers are also formal, we use the function $f_{\sigma_1 - id}$ (see 4.2):

6 is a formal number with the following formula ϕ (see 4.2):

$$\phi \equiv \neg x = c_0 \wedge f_{\sigma_1 - id}(x) = x \wedge \neg \exists x' \left(x' < x \wedge f_{\sigma_1 - id}(x') = x'\right) \ . \tag{4.8}$$

5. Conjectures which induce monster formal numbers if counterexamples exist

In the previous section, we introduced some formal numbers that are small and easy to find. In this section, we examine how some monster formal numbers can be produced if some conjectures are false. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the unary inverse function symbol f_s^{-1}:

$$\begin{aligned} & \forall x \left(\neg x = c_0 \rightarrow f_s\left(f_s^{-1}\left(x\right)\right) = x\right) \\ & \forall x \left(x = c_0 \rightarrow f_s^{-1}\left(x\right) = c_0\right) \ . \end{aligned} \tag{5.1}$$

2- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the subtraction function symbol f_-:

$$\begin{aligned} & \forall x \forall y \left(y < x \rightarrow f_+\left(f_-\left(x, y\right), y\right) = x\right) \\ & \forall x \forall y \left(\neg y < x \rightarrow f_-\left(x, y\right) = c_0\right) \ . \end{aligned} \tag{5.2}$$

3- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the twin prime function symbol f_{Twin} (see 4.1 and 5.1):

$$\begin{aligned} & \forall x \left(\left(f_{Prime}\left(f_s^{-1}\left(x\right)\right) = f_s\left(c_0\right) \wedge f_{Prime}\left(f_s\left(x\right)\right) = f_s\left(c_0\right)\right) \rightarrow f_{Twin}\left(x\right) = f_s\left(c_0\right)\right) \\ & \forall x \left(\neg \left(f_{Prime}\left(f_s^{-1}\left(x\right)\right) = f_s\left(c_0\right) \wedge f_{Prime}\left(f_s\left(x\right)\right) = f_s\left(c_0\right)\right) \rightarrow f_{Twin}\left(x\right) = c_0\right) \ . \end{aligned} \tag{5.3}$$

4- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the ceiling prime function symbol f_{CP} (see 4.1):

$$\forall x \exists y \left(f_{CP}\left(x\right) = y \wedge f_{Prime}\left(y\right) = f_s\left(c_0\right) \wedge \neg \exists z \left(x < z \wedge z < y \wedge f_{Prime}\left(z\right) = f_s\left(c_0\right)\right)\right) \ . \tag{5.4}$$

5- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the function symbol $f_{p-Prime}$ which give the n^{th} prime number (see 4.1)

$$\begin{aligned} & f_{p-Prime}\left(c_0\right) = c_0 \\ & \forall x \left(f_{p-Prime}\left(f_s\left(x\right)\right) = f_{CP}\left(f_{p-Prime}\left(x\right)\right)\right) \ . \end{aligned} \tag{5.5}$$

6- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the coprime function symbol $f_{Coprime}$:

$$\forall x \forall x' \left(\exists y \exists z \exists z' \left(\neg y = 1 \wedge x = f_\times (y,z) \wedge x' = f_\times \left(y, z'\right)\right) \to f_{Coprime}\left(x, x'\right) = c_0\right)$$
$$\forall x \forall x' \left(\neg \exists y \exists z \exists z' \left(\neg y = 1 \wedge x = f_\times (y,z) \wedge x' = f_\times \left(y, z'\right)\right) \to f_{Coprime}\left(x, x'\right) = f_s (c_0)\right) \ . \tag{5.6}$$

7- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the power function symbol $f_\wedge$:

$$\forall x \forall y \left(y = c_0 \to f_\wedge (x,y) = f_s (c_0)\right)$$
$$\forall x \forall y \left(\neg y = c_0 \to f_\wedge (x, f_s (y)) = f_\times (f_\wedge (x,y), x)\right) \ . \tag{5.7}$$

5.1. *Goldbach's conjecture.* If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the sub-function symbol $g_{Goldbach-1}$ and the function symbol $f_{Goldbach-1}$ which gives the minimal number of prime numbers necessary to express a natural number as a sum of prime number minus one (see 4.1 and 5.2):

$$\forall x \forall y \left(f_{Prime}(y) = f_s (c_0) \to g_{Goldbach-1}(x,y) = c_0\right)$$
$$\forall x \forall y \Big(\left(\neg y = c_0 \wedge \neg y = f_s (c_0) \wedge f_{Prime}(y) = c_0\right) \to \exists z$$
$$\left(z < y \wedge f_{Prime}(z) = f_s (c_0) \wedge g_{Goldbach-1}(x,y) = f_s \left(g_{Goldbach-1}(x, f_- (y,z))\right) \wedge \neg \exists z'\right.$$
$$\left.\left(z' < y \wedge f_{Prime}\left(z'\right) = f_s (c_0) \wedge g_{Goldbach-1}(x, f_- \left(y, z'\right)) < g_{Goldbach-1}(x, f_- (y,z))\right)\right) \Big)$$
$$\forall x \left(f_{Goldbach-1}(x) = g_{Goldbach-1}(x,x)\right) \ . \tag{5.8}$$

If a first counterexample m_Z exists for the Goldbach's conjecture [Hel13], we can show that m_Z is a formal number with the following formula ϕ (see the previous equation):

$$\phi \equiv \neg x = c_0 \wedge \neg x = f_s (c_0) \wedge \forall y \left(y < x \to f_{Goldbach-1}(y) < f_{Goldbach-1}(x)\right) \wedge$$
$$\forall y \left(x < y \to \neg f_{Goldbach-1}(x) < f_{Goldbach-1}(y)\right) \ . \tag{5.9}$$

5.2. *Polignac's conjecture.* If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the Polignac function symbol $f_{Polignac}$ which gives the difference between the two next prime numbers of a natural number (see 5.2 and 5.4):

$$\forall x \left(f_{Polignac}(x) = f_- \left(f_{CP}\left(f_{CP}(x)\right), f_{CP}(x)\right)\right) \ . \tag{5.10}$$

If a first counterexample m_Z exists for the Polignac's conjecture [dP51], we can show that m_Z is a formal number with following formula ϕ (see the previous equation):

$$\phi \equiv \neg x = c_0 \wedge \neg x = f_s (c_0) \wedge$$
$$\exists y \left(f_{Polignac}(x) = y \wedge \neg \exists z \left(x < z \wedge f_{Polignac}(x) = f_{Polignac}(z)\right)\right) \wedge$$
$$\neg \exists x' \left(x' < x \wedge f_{Polignac}\left(x'\right) = y \wedge \neg \exists z \left(x' < z \wedge f_{Polignac}\left(x'\right) = f_{Polignac}(z)\right)\right) \ . \tag{5.11}$$

Since the set of prime numbers is infinite, the following explicit sub-formula will not work (see 5.2 and 5.4):

$$\forall x\left(f_{Polignac}\left(x\right)=f_{-}\left(f_{CP}\left(x\right),x\right)\right)\ . \tag{5.12}$$

5.3. *Firoozbakht's conjecture.* If a first counterexample m_Z exists for the Firoozbakht's conjecture [20004], we can show that m_Z is a formal number with the following formula ϕ (see 5.5):

$$\begin{aligned}&\phi\equiv\neg x=c_0\wedge\neg f_{\wedge}\left(f_{p-Prime}\left(f_s\left(x\right)\right),x\right)<f_{\wedge}\left(f_{p-Prime}\left(x\right),f_s\left(x\right)\right)\wedge\\&\forall x'\left(x'<x\rightarrow f_{\wedge}\left(f_{p-Prime}\left(f_s\left(x'\right)\right),x'\right)<f_{\wedge}\left(f_{p-Prime}\left(x'\right),f_s\left(x'\right)\right)\right)\ .\end{aligned}\tag{5.13}$$

5.4. *Oppermann's conjecture.* We define the first variant of the Oppermann's conjecture [vsOFS83]:

For all natural numbers x such that $x>1$, there is at least one prime number between $x(x-1)$ and x^2.

If a first counterexample m_Z exists for the Oppermann's conjecture [vsOFS83], we can show that m_Z is a formal number with the following formula ϕ (see 4.1 and 5.1):

$$\begin{aligned}&\phi\equiv\neg x=c_0\wedge\neg x=f_s\left(c_0\right)\wedge\neg\exists y\left(f_{\times}\left(x,f_s^{-1}\left(x\right)\right)<y\wedge y<f_{\times}\left(x,x\right)\wedge f_{Prime}\left(y\right)=f_s\left(c_0\right)\right)\\&\wedge\neg\exists x'\neg\exists y\left(x'<x\wedge f_{\times}\left(x',f_s^{-1}\left(x'\right)\right)<y\wedge y<f_{\times}\left(x',x'\right)\wedge f_{Prime}\left(y\right)=f_s\left(c_0\right)\right)\ .\end{aligned}\tag{5.14}$$

If a first counterexample m_Z exists for the Oppermann's conjecture [vsOFS83] and the first variant of the Oppermann's conjecture [vsOFS83] is true, we can show that m_Z is a formal number with the following formula ϕ (see 4.1 and 5.1):

$$\begin{aligned}&\phi\equiv\neg x=c_0\wedge\neg x=f_s\left(c_0\right)\wedge\\&\neg\exists y\exists y'\left(f_{\times}\left(x,f_s^{-1}\left(x\right)\right)<y\wedge y<f_{\times}\left(x,x\right)\wedge f_{\times}\left(x,x\right)<y'\wedge y'<f_{\times}\left(x,f_s\left(x\right)\right)\wedge\right.\\&f_{Prime}\left(y\right)=f_s\left(c_0\right)\right)\wedge\neg\exists x'\neg\exists y\exists y'\\&\left(x'<x\wedge f_{\times}\left(x',f_s^{-1}\left(x'\right)\right)<y\wedge y<f_{\times}\left(x',x'\right)\wedge f_{\times}\left(x',x'\right)<y'\wedge y'<f_{\times}\left(x',f_s\left(x'\right)\wedge\right.\right.\\&f_{Prime}\left(y\right)=f_s\left(c_0\right)\right)\right)\ .\end{aligned}\tag{5.15}$$

5.5. *Agoh-Giuga conjecture.* If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the Giuga sub-function symbol g_{Giuga} and the Giuga function symbol f_{Giuga}(see 4.1, 5.1 and 5.7):

$$\begin{aligned}&\forall x\left(g_{Giuga}\left(x,c_0\right)=f_s\left(c_0\right)\right)\\&\forall x\forall y\left(g_{Giuga}\left(x,f_s\left(y\right)\right)=f_{+}\left(g_{Giuga}\left(x,y\right),f_{\wedge}\left(y,f_s^{-1}\left(x\right)\right)\right)\right)\\&\forall x\left(\left(f_{Prime}\left(x\right)=f_s\left(c_0\right)\rightarrow\exists y\left(f_s\left(g_{Giuga}\left(x,x\right)\right)=f_{\times}\left(x,y\right)\right)\right)\rightarrow f_{Giuga}=f_s\left(c_0\right)\right)\\&\forall x\left(\neg\left(f_{Prime}\left(x\right)=f_s\left(c_0\right)\rightarrow\exists y\left(f_s\left(g_{Giuga}\left(x,x\right)\right)=f_{\times}\left(x,y\right)\right)\right)\rightarrow f_{Giuga}=c_0\right)\ .\end{aligned}\tag{5.16}$$

If m_Z is the last natural number where the Agoh-Giuga conjecture [Giu51] is true, we can show that m_Z is a formal number with the following formula ϕ (see the previous equation):

$$\phi \equiv f_{Giuga}\left(x\right) = c_0 \wedge \neg \exists x' \left(x' < x \wedge f_{Giuga}\left(x'\right) = c_0\right) \ . \tag{5.17}$$

5.6. *Generalized Fermat's conjecture.* We define the generalized Fermat's conjecture [Rie11]:

Let be some natural number a and c, there is an infinite number of natural numbers b such that $a^b + c^b$ is a prime number.

If m_Z is the last number where the generalized Fermat's conjecture [Rie11] for some fixed natural number a and c is true and every explicit sub-formulas which are equivalent to the Generalized Fermat's conjecture [Rie11] with the fixed natural number a' and c' are true , we can show that m_Z is a monster formal number with the following formula ϕ (see 4.1 and 5.7):

$$\begin{aligned}\phi \equiv & f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x\right), f_\wedge\left(n_c, x\right)\right)\right) = f_s\left(c_0\right) \wedge \\ & \neg\exists x'\left(x < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x'\right), f_\wedge\left(n_c, x'\right)\right)\right) = f_s\left(c_0\right)\right) \wedge \\ & \neg\exists x''\left(x'' < x \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x''\right), f_\wedge\left(n_c, x''\right)\right)\right) = f_s\left(c_0\right) \wedge\right. \\ & \left.\neg\exists x'\left(x'' < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, x''\right), f_\wedge\left(n_c, x''\right)\right)\right) = f_s\left(c_0\right)\right)\right) ,\end{aligned} \tag{5.18}$$

where $n_a = \underbrace{f_s(...f_s(}_{a \text{ times}} c_0 \underbrace{)...)}_{a \text{ times}}$ and $n_c = \underbrace{f_s(...f_s(}_{c \text{ times}} c_0 \underbrace{)...)}_{c \text{ times}}$.

If we can show that the generalized Fermat's conjecture is true for many fixed natural numbers a and c, we can show that m_Z is a formal number with the following formula ϕ (see 4.1 and 5.7):

$$\begin{aligned}\phi \equiv & \exists y\left(f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x, y\right)\right)\right) = f_s\left(c_0\right) \wedge\right. \\ & \left.\neg\exists x'\left(x < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x', y\right)\right)\right) = f_s\left(c_0\right)\right)\right) \wedge \\ & \neg\exists x''\exists y\left(x'' < x \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x'', y\right)\right)\right) = f_s\left(c_0\right) \wedge\right. \\ & \left.\neg\exists x'\left(x'' < x' \wedge f_{Prime}\left(f_+\left(f_\wedge\left(n_a, y\right), f_\wedge\left(x', y\right)\right)\right) = f_s\left(c_0\right)\right)\right) .\end{aligned} \tag{5.19}$$

If we can show that the generalized Fermat's conjecture is true for many fixed natural numbers a, we can show that m_Z is a formal number with the following formula ϕ (see 4.1 and 5.7):

$$\begin{aligned}\phi \equiv &\exists y \exists z \Bigg(f_{Prime}\left(f_{+}\left(f_{\wedge}(x,y), f_{\wedge}(z,y)\right)\right) = f_s(c_0) \wedge \\ &\neg\exists x' \left(x < x' \wedge f_{Prime}\left(f_{+}\left(f_{\wedge}(x',y), f_{\wedge}(z,y)\right)\right) = f_s(c_0)\right) \Bigg) \wedge \\ &\neg\exists x'' \exists y \exists z \Bigg(x'' < x \wedge f_{Prime}\left(f_{+}\left(f_{\wedge}(x'',y), f_{\wedge}(z,y)\right)\right) = f_s(c_0) \wedge \\ &\neg\exists x' \left(x'' < x' \wedge f_{Prime}\left(f_{+}\left(f_{\wedge}(x',y), f_{\wedge}(z,y)\right)\right) = f_s(c_0)\right) \Bigg) .\end{aligned} \tag{5.20}$$

5.7. *Schinzel's hypothesis H.* If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the r polynomials function symbol $f_{i,Schinzel}$ of maximal degree d (see 5.7):

$$\begin{aligned}&\forall x (\\ &f_{1,Schinzel}(x) = f_{+}\left(f_{+}\left(...f_{+}\left(f_{\times}\left(a_{10}, f_{\wedge}(x,b_0)\right), f_{\times}\left(a_{11}, f_{\wedge}(x,b_1)\right)\right)...\right), f_{\times}\left(a_{1d}, f_{\wedge}(x,b_d)\right)\right) \\ &\qquad\vdots \\ &f_{r,Schinzel}(x) = f_{+}\left(f_{+}\left(...f_{+}\left(f_{\times}\left(a_{r0}, f_{\wedge}(x,b_0)\right), f_{\times}\left(a_{r1}, f_{\wedge}(x,b_1)\right)\right)...\right), f_{\times}\left(a_{rd}, f_{\wedge}(x,b_d)\right)\right) \\ &)\end{aligned} \tag{5.21}$$

where $a_{ij} = \underbrace{f_s(...f_s(}_{a(i,j) \text{ times}}\ c_0\ \underbrace{)...)}_{a(i,j) \text{ times}}$ and $b_i = \underbrace{f_s(...f_s(}_{i \text{ times}}\ c_0\ \underbrace{)...)}_{i \text{ times}}$.

Since the r polynomials $f_{i,Schinzel}$ are irreducible, the polynomial coefficients a_{ij} satisfy the first following constraint (see the previous equation):

$$\left(\nexists x f_{1,Schinzel}(x) = c_0\right) \wedge ... \wedge \left(\nexists x f_{r,Schinzel}(x) = c_0\right) . \tag{5.22}$$

Since the product of the r polynomials $f_{i,Schinzel}$ has not a fixed prime divisor, the polynomial coefficients a_{ij} satisfy the second following constraint (see 5.22):

$$\begin{aligned}&\nexists x \forall y \exists z \Bigg(f_{Prime}(x) = f_s(c_0) \wedge \\ &f_{\times}\left(f_{\times}\left(...f_{\times}\left(f_{1,Schinzel}(y), f_{2,Schinzel}(y)\right)...\right), f_{r,Schinzel}(y)\right) = f_{\times}(x,z) \Bigg) .\end{aligned} \tag{5.23}$$

If m_Z is the last number where the Schinzel's hypothesis H [Guy04] for some fixed polynomial is true and every explicit sub-formulas which are equivalent to the Schinzel's hypothesis H [Guy04] for some fixed polynomials are true, we can show that m_Z is a monster formal number with the following formula ϕ (see 4.1 and 5.22):

$$\begin{aligned}\phi \equiv &f_{Prime}\left(f_{1,Schinzel}(x)\right) = f_s(c_0) \wedge ... \wedge f_{Prime}\left(f_{r,Schinzel}(x)\right) = f_s(c_0) \wedge \\ &\nexists x' \left(x < x' \wedge f_{Prime}\left(f_{1,Schinzel}(x')\right) = f_s(c_0) \wedge ... \wedge f_{Prime}\left(f_{r,Schinzel}(x')\right) = f_s(c_0)\right) .\end{aligned} \tag{5.24}$$

We build new formulas for new monster formal numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for a fixed number of polynomial r, a fixed maximal degree d, many fixed polynomial coefficients a_{ij} and some running polynomial coefficients a_{ij}, we can show that m_Z is a monster formal number with a formula ϕ.

We build new formulas for new monster formal numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for a fixed number of polynomial r, many maximal degrees d and the running polynomial coefficients a_{ij}, we can show that m_Z is a monster formal number with a formula ϕ.

Finally, we build new formulas for new monster formal numbers like in the generalized Fermat's conjecture:

If we can show that the Schinzel's hypothesis H [Guy04] is true for many numbers of polynomial r, the running maximal degree d and the running polynomial coefficients a_{ij}, we can show that m_Z is a monster formal number with a formula ϕ.

6. The second order logic formal axiom and the second order logic hypothesis on the "Nature's hypotheses"

We introduce one important axiom on formal numbers and one important hypothesis on the "Nature's hypotheses" at second order logic for both of them:

The (second order logic) formal axiom:

Any formal number is smaller or equal to:

$$N_Z \in \left[10^{3.2\times 10^3}, 10^{7.3\times 10^5}\right] \cong \left[2^{1.0\times 10^4} - 2^{2.4\times 10^6}\right] \tag{6.1}$$

. The (second order logic) hypothesis on the "Nature's hypotheses":

The hypotheses of any N_Z-formal sequent are the "Nature's hypotheses" which explain the physical measurements.

Some consequences:

1- The physical measurements confirm but do not prove that the "Nature's hypotheses", the mathematical explorations over the formal numbers confirm but do not prove that N_Z is the largest formal number and they do not prove but confirm that the hypotheses of any N_Z-formal sequent are the "Nature's hypotheses" which explain the physical measurements.

2- The Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht conjecture's, the Oppermann's conjecture, the Agoh-Giuga conjecture can be checked with quantum computers with $1.0 \times 10^4 - 2.5 \times 10^6$ qubits since the computation can be fully parallel [LS14], the generalized Fermat's conjecture (which requires computational resources which are far from what we can imagine technically even for the simplest case: $a = 2$ and $b = 1$) and the Schinzel's hypothesis H (which requires monster computational resources for checking about $\left(\pi\left(N_Z\right) 2^{N_Z}\right)^{N_Z}$ cases). . Moreover, 27 is a formal number if the Goldbach's conjecture is true.

3- A paper is under preparation in order to present the theory of everything where its hypotheses are the hypotheses of a N_Z-formal sequent (N_Z would be the number of Lagrangian terms but experimentally, we can only access to a very small fraction of that terms) and to show that any obvious variant of the theory of everything requires some hypotheses which give a formal number strictly smaller than N_Z.

7. Some open questions about the n-formal sequents and the formal numbers

1- Can we show that a natural number n is not formal? The difficulty is to prove that there is no n-formal sequent among an infinite set of possible sequents which give the formal number n.

2- If a formal number n is found with the help of a function symbol f where its output values are only 0 and 1, can we replace the function symbol f by a function symbol $\tilde{f}$ such that $\tilde{f} = 1 - f$ and the new sequent is still n-formal?

3- Does a sequent exist to make a difference between the definition of the n-formal sequents and the following weaker variant of that definition: we look at the explicit sub-formulas of ϕ which induce the m-formal formulas instead of looking at the explicit sub-formulas of $\phi_{n-formal}$ which are m-formulas?

8. Extra: some larger formal number examples

In this section, with the help of the formulas satisfied by the symbol function f_{Prime} joined to the hypotheses of some n formal sequents, we try to reach the closest formal number (1024 in the present section) to the largest one $N_Z \in \left[10^{3.2\times10^3}, 10^{7.3\times10^5}\right] \cong \left[2^{1.0\times10^4} - 2^{2.4\times10^6}\right]$. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the sub-function symbol g_{σ_0-1} and the function symbol f_{σ_0-1} which gives the number of proper divisor of a natural number:

$$\begin{aligned}
&\forall x \left(g_{\sigma_0-1}(x, f_s\left(c_0\right)) = c_0\right)\\
&\forall x \forall y \left(\exists z \left(x = f_\times\left(y, z\right)\right) \rightarrow g_{\sigma_0-1}\left(x, f_s\left(y\right)\right) = f_s\left(g_{\sigma_0-1}(x, y)\right)\right)\\
&\forall x \forall y \left(\neg\exists z \left(x = f_\times\left(y, z\right)\right) \rightarrow g_{\sigma_0-1}\left(x, f_s\left(y\right)\right) = g_{\sigma_0-1}(x, y)\right)\\
&\forall x \left(f_{\sigma_0-1}(x) = g_{\sigma_0-1}(x, x)\right)\ .
\end{aligned} \tag{8.1}$$

2- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the highly composite function symbol f_{HC} (see the previous equation):

$$\begin{aligned}
&\forall x \left(\forall y \left(\left(\neg y = c_0 \wedge y < x\right) \rightarrow f_{\sigma_0-1}\left(y\right) < f_{\sigma_0-1}\left(x\right)\right) \rightarrow f_{HC}(x) = f_s\left(c_0\right)\right)\\
&\forall x \left(\neg \left(\forall y \left(\neg y = c_0 \wedge y < x\right) \rightarrow f_{\sigma_0-1}\left(y\right) < f_{\sigma_0-1}\left(x\right)\right) \rightarrow f_{HC}(x) = c_0\right)\ .
\end{aligned} \tag{8.2}$$

3- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the Euler's totient sub-function symbol g_Φ and the Euler's totient function symbol f_Φ which gives the number of coprime numbers below it (see 5.6):

$$\begin{aligned}
&\forall x\left(g_\Phi(x, f_s\left(c_0\right)) = c_0\right)\\
&\forall x \forall y\left(f_{Coprime}\left(x, y\right) = f_s\left(c_0\right) \rightarrow g_\Phi\left(x, f_s\left(y\right)\right) = f_s\left(g_\Phi(x, y)\right)\right)\\
&\forall x \forall y\left(f_{Coprime}\left(x, y\right) = c_0 \rightarrow g_\Phi\left(x, f_s\left(y\right)\right) = g_\Phi(x, y)\right)\\
&\forall x\left(f_\Phi(x) = g_\Phi(x, x)\right)
\end{aligned} \tag{8.3}$$

4- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the highly coprime function symbol f_{HCP}(see the previous equation):

$$\begin{aligned}
&\forall x\left(\forall y\left(\left(\neg y = c_0 \wedge y < x\right) \rightarrow f_\Phi\left(y\right) < f_\Phi\left(x\right)\right) \rightarrow f_{HCP}(x) = f_s\left(c_0\right)\right)\\
&\forall x\left(\neg \forall y\left(\left(\neg y = c_0 \wedge y < x\right) \rightarrow f_\Phi\left(y\right) < f_\Phi\left(x\right)\right) \rightarrow f_{HCP}(x) = c_0\right) \ .
\end{aligned} \tag{8.4}$$

5- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the sub-function symbol g_{σ_1-1} and the function symbol f_{σ_1-1} which gives the sum of divisors minus one of a natural number (see 5.1):

$$\begin{aligned}
&\forall x\left(g_{\sigma_1-1}(x, f_s\left(c_0\right)) = c_0\right)\\
&\forall x \forall y\left(\left(\neg y = f_s\left(c_0\right) \wedge \exists z\left(x = f_\times\left(y, z\right)\right)\right) \rightarrow g_{\sigma_1-1}\left(x, y\right) = f_+\left(g_{\sigma_1-1}(x, f_s^{-1}\left(y\right)), y\right)\right)\\
&\forall x \forall y\left(\neg\left(\neg y = f_s\left(c_0\right) \wedge \exists z\left(x = f_\times\left(y, z\right)\right)\right) \rightarrow g_{\sigma_1-1}\left(x, y\right) = g_{\sigma_1-1}(x, f_s^{-1}\left(y\right))\right)\\
&\forall x\left(f_{\sigma_1-1}(x) = g_{\sigma_1-1}(x, x)\right) \ .
\end{aligned} \tag{8.5}$$

With the concept of complement, we can show that 10 is a formal number with the following formula ϕ (see 8.2 and 8.4):

$$\phi \equiv f_{HC}\left(x\right) = c_0 \wedge f_{HCP}\left(x\right) = f_s\left(c_0\right) \wedge \neg \exists y\left(y < x \wedge f_{HC}\left(x\right) = c_0 \wedge f_{HCP}\left(x\right) = f_s\left(c_0\right)\right) \tag{8.6}$$

and we can show that 24 is a formal number with the following formula ϕ (see 8.2 and 8.4):

$$\phi \equiv f_{HC}\left(x\right) = f_s\left(c_0\right) \wedge f_{HCP}\left(x\right) = c_0 \wedge \neg \exists y\left(y < x \wedge f_{HC}\left(x\right) = f_s\left(c_0\right) \wedge f_{HCP}\left(x\right) = c_0\right) \ . \tag{8.7}$$

In order to find much larger formal number, we use the concept of amicable numbers:

1- 220 is a formal number with the following formula ϕ (see 8.5):

$$\phi \equiv \exists y\left(x < y \wedge f_{\sigma_1-1}\left(x\right) = f_{\sigma_1-1}\left(y\right)\right) \wedge \forall z\left(z < x \rightarrow \neg \exists y\left(z < y \wedge f_{\sigma_1-1}\left(z\right) = f_{\sigma_1-1}\left(y\right)\right)\right) \ . \tag{8.8}$$

2- 284 is a formal number with the following formula ϕ (see 8.5):

$$\phi \equiv \exists y\left(y < x \wedge f_{\sigma_1-1}\left(x\right) = f_{\sigma_1-1}\left(y\right)\right) \wedge \forall z\left(z < x \rightarrow \neg \exists y\left(y < z \wedge f_{\sigma_1-1}\left(z\right) = f_{\sigma_1-1}\left(y\right)\right)\right) \ . \tag{8.9}$$

3- 503 is a formal number with the following formula ϕ (see 8.5):

$$\begin{aligned}
\phi \equiv &\exists y \exists z\\
&\left(x = f_{\sigma_1-1}\left(y\right) \wedge x = f_{\sigma_1-1}\left(z\right)\right) \wedge \forall w\left(w < x \rightarrow \neg \exists y \exists z\left(w = f_{\sigma_1-1}\left(y\right) \wedge w = f_{\sigma_1-1}\left(z\right)\right)\right) \ .
\end{aligned} \tag{8.10}$$

9. Extra bis: some formal number examples with the axioms of the real numbers

In this section, with the help of the formulas satisfied by the axioms of the real numbers joined to the hypotheses of some n-formal sequents, we try to reach the closest formal number (1024 in the present section) to the largest one $N_Z \in \left[10^{3.2\times10^3}, 10^{7.3\times10^5}\right] \cong \left[2^{1.0\times10^4} - 2^{2.4\times10^6}\right]$. Firstly, we write the preliminary formulas satisfied by the following function symbols:

1- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the natural number function symbol $f_{\mathbb{N}}$:

$$\begin{aligned}
&f_{\mathbb{N}}(c_0) = f_s(c_0)\\
&\forall x \left(f_{\mathbb{N}}(x) = f_s(c_0) \to f_{\mathbb{N}}(f_s(x)) = f_s(c_0)\right)\\
&\forall x \left(f_{\mathbb{N}}(x) = c_0 \to f_{\mathbb{N}}(f_s(x)) = c_0\right) \ .
\end{aligned} \tag{9.1}$$

2- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the integer part function symbol f_{IP} (see the previous equation):

$$\forall x \left(\exists n \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge \neg n < x \wedge x < f_s(n) \wedge f_{IP}(x) = n\right)\right) \ . \tag{9.2}$$

3- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the ceiling function symbol $f_{Ceiling}$ (see 9.1):

$$\forall x \left(\exists n \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge x < n \wedge \neg f_s(n) < x \wedge f_{Ceiling}(x) = n\right)\right) \ . \tag{9.3}$$

4- If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the negative function symbol f_-

$$\begin{aligned}
&\forall x \exists y \left(f_+(x, y) = c_0 \wedge f_-(x) = y\right)\\
&\forall x \neg \exists y \exists y' \left(\neg y = y' \wedge f_+(x, y) = c_0 \wedge f_+\left(x, y'\right) = c_0\right) \ .
\end{aligned} \tag{9.4}$$

5 If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the factorial function symbol $f_!$ (see 5.1 and 9.1):

$$\begin{aligned}
&\forall n \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge n = c_0 \to f_!(n) = f_s(c_0)\right)\\
&\forall n \left(f_{\mathbb{N}}(n) = f_s(c_0) \wedge \neg n = c_0 \to f_!(n) = f_\times\left(n, f_!\left(f_s^{-1}(n)\right)\right) \right) \ .
\end{aligned} \tag{9.5}$$

6 If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the exponential series function symbol g_{exp} (see 5.1 and 5.7):

$$\begin{aligned}
&\forall x \left(g_{exp}(x, c_0) = c_0\right)\\
&\forall x \forall y \left(g_{exp}(x, f_s(y)) = f_+\left(g_{exp}(x, y), f_\times\left(f_\wedge(x, y), f_{-1}(f_!(y))\right) \right) \right) \ .
\end{aligned} \tag{9.6}$$

7 If required, we add to the hypotheses of the n-formal sequents the following formulas satisfied by the exponential function symbol f_{exp} (see 9.1 and the previous equation):

$$\forall\epsilon\exists N\forall n\Bigg((0<\epsilon\wedge N<n\wedge f_{\mathbb{N}}(N)=f_s(c_0)\wedge f_{\mathbb{N}}(n)=f_s(c_0)) \rightarrow (g_{exp}(x,n)<f_{exp}(x)\wedge f_{exp}(x)<f_{+}(\epsilon,g_{exp}(x,n)))\Bigg)\ . \tag{9.7}$$

We suppose we can define the Lebesgue integral or the Riemann integral formally in order to define the function $f_{\sqrt{\pi}/2}$ in a formal form (see 5.7, 9.1, 9.4 and 9.7):

$$\forall n\left(f_{\mathbb{N}}(n)=f_s(c_0)\wedge\neg n=c_0\rightarrow f_{\sqrt{\pi}/2}(n)=\int_0^{\infty}f_{exp}(f_{-}(f_{\wedge}(x,n)))\,dx\right)\ . \tag{9.8}$$

We can define the real number $\sqrt{\pi}/2$ formally with the following formula ϕ:

$$\phi\equiv\exists n\left(f_{\mathbb{N}}(n)=f_s(c_0)\wedge x=f_{\sqrt{\pi}/2}(n)\right)\wedge\neg\exists n\left(f_{\mathbb{N}}(n)=f_s(c_0)\wedge f_{\sqrt{\pi}/2}(n)<x\right)\ . \tag{9.9}$$

Sketch to prove that 5 and 7 are formal numbers: The $(n-1)$-sphere of radius R and center $\vec{r}$ can be defined formally by imposing a maximum volume for a fixed surface in $\mathbb{R}^n$ or a minimum surface for a fixed volume. By defining a formal n-cube with vertex coordinates $\underbrace{(\pm1,...,\pm1)}_{n\text{ times}}$ and taking the biggest $(n-1)$-sphere inside it, we can find the n which maximize the volume $V(n)$ or the surface $S(n)$ of the $(n-1)$-sphere: 5 or 7.

Therefore, we can also define formally the real numbers $16\pi^3/15$ and $8\pi^2/15$ with the following formula ϕ:

$$\phi\equiv\exists n\left(f_{\mathbb{N}}(n)=f_s(c_0)\wedge x=S(n)\right)\wedge\neg\exists n'\left(f_{\mathbb{N}}(n')=f_s(c_0)\wedge x<S(n')\right)\ , \tag{9.10}$$

and the following formula ϕ

$$\phi\equiv\exists n\left(f_{\mathbb{N}}(n)=f_s(c_0)\rightarrow x=V(n)\right)\wedge\neg\exists n'\left(f_{\mathbb{N}}(n')=f_s(c_0)\wedge x<V(n')\right)\ . \tag{9.11}$$

We can also define formally the real number e with the following formula ϕ:

$$\phi\equiv x=f_{exp}(f_s(c_0))\ . \tag{9.12}$$

For some formal real numbers x and x', we can show that n is a formal number with the following formula ϕ:

$$\begin{aligned}\phi\equiv\exists m\exists p\exists q\,(&f_{\mathbb{N}}(n)=f_s(c_0)\wedge f_{\mathbb{N}}(m)=f_s(c_0)\wedge f_{\mathbb{N}}(p)=f_s(c_0)\wedge f_{\mathbb{N}}(q)=f_s(c_0)\wedge\\ &f_{\times}(p,f_{\wedge}(x,n))=f_{\times}\left(q,f_{\wedge}\left(x',m\right)\right)\wedge f_{Coprime}(p,q)=f_s(c_0)\wedge f_{Coprime}(m,n)=f_s(c_0))\ .\end{aligned} \tag{9.13}$$

For some formal real numbers x and x', we can show that p is a formal number with the following formula ϕ:

$$\begin{aligned}\phi\equiv\exists n\exists m\exists q\,(&f_{\mathbb{N}}(n)=f_s(c_0)\wedge f_{\mathbb{N}}(m)=f_s(c_0)\wedge f_{\mathbb{N}}(p)=f_s(c_0)\wedge f_{\mathbb{N}}(q)=f_s(c_0)\wedge\\ &f_{\times}(p,f_{\wedge}(x,n))=f_{\times}\left(q,f_{\wedge}\left(x',m\right)\right)\wedge f_{Coprime}(p,q)=f_s(c_0)\wedge f_{Coprime}(m,n)=f_s(c_0))\ .\end{aligned} \tag{9.14}$$

Therefore, from the formal real numbers, $\sqrt{\pi}/2$, $16\pi^3/15$, $8\pi^2/15$ and the help of the two last formulas, we deduce that 2, 3, 4, 6, 15, 128 and 1024 are formal numbers.

Finally, with the help of the integer part function f_{IP} and the ceiling function $f_{Ceiling}$ on the formal real number $16\pi^3/15$, we deduce that 33 and 34 are formal numbers.

10. Conclusion

This paper may open a new area in second order logic with some important consequences in number theory and in fundamental physics if we do not notice contradictions between the (second order logic) formal axiom and other well known axioms, and we do not observe experimental contradictions between the hypotheses to produce the largest formal number found and the experimental measurements. It is the first paper which gives a hint to solve the Goldbach's conjecture, the Polignac's conjecture, the Firoozbakht's conjecture, the Oppermann's conjecture, the Agoh-Giuga conjecture with a quantum computer of $1.0 \times 10^4 - 2.5 \times 10^6$ qubits and with only one (second order logic) formal axiom. The generalized Fermat's conjecture requires computational resources which are far from what we can imagine technically even for the simplest case: $a = 2$ and $b = 1$ and the Schinzel's hypothesis H requires also monster computational resources for checking about $\left(\pi\left(N_Z\right)2^{N_Z}\right)^{N_Z}$ cases. It is also the first paper which gives a hint to generate the "Nature's hypotheses"with only one (second order logic) hypothesis.

Since I am not a mathematician and I am a lonely human, I may have overseen some mistakes (especially, I could miss an explicit sub-formula in the present paper or do not noticed that a sequent is not formal or my approach to the generalized Fermat's conjecture and the Schinzel's hypothesis H are sensitive to some mistakes since it is one level more of abstraction from the other conjectures). Moreover, N_Z may change after the publication of the next paper about the theory of everything. Please send me an email (see it below the references) for any mistake noticed in the present paper. Every ideas or comments related to the present paper are also very welcome.

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References

[20004] *The Little Book of Bigger Primes.* Springer Nature, 2004.

[CL00] R. Cori and D. Lascar. *Mathematical Logic: A Course with Exercises.* Number ptie. 1 in A devotional commentary, edited by the Rev. C. H. Irwin. Oxford University Press, 2000.

[DNR01] R. David, K. Nour, and C. Raffalli. *Introduction à la logique: théorie de la démonstration : cours et exercises corrigés.* Sciences sup. Dunod, 2001.

[dP51] A. de Polignac. *Recherches nouvelles sur les nombres premiers.* Bachelier, 1851.

[Dub00] Harvey Dubner. Twin prime conjectures. 30:199–205, 2000.

[End01] H.B. Enderton. *A Mathematical Introduction to Logic.* Harcourt/Academic Press, 2001.

[Giu51] Giuseppe Giuga. Su una presumibile proprietà caratteristica dei numeri primi. *Ist. Lombardo Sci. Lett., Rend., Cl. Sci. Mat. Natur., III. Ser.*, 14:511–518, 1951.

[Guy04] Richard Guy. *Unsolved Problems in Number Theory (Problem Books in Mathematics).* Springer, 2004.

[Hel13] H. A. Helfgott. Major arcs for goldbach's problem, 2013.

[LS14] Jose I. Latorre and German Sierra. Quantum Computation of Prime Number Functions. *Quant. Inf. Comput.*, 14:577–588, 2014.

[Rie11] H. Riesel. *Prime Numbers and Computer Methods for Factorization.* Modern Birkhäuser Classics. Birkhäuser Boston, 2011.

[vsOFS83] Kongelige Danske videnskabernes selskab, H.C. Orsted, G. Forchhammer, and J.J.S. Steenstrup. *Oversigt over det Kongelige Danske videnskabernes selskabs forhandlinger ...* 1883.

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