Special Relativity and Einstein Equivalence Principle

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Einstein Equivalence Principle is the cornerstone of general theory of relativity. Special relativity is assumed to be verified at any point on the Riemann curved manifold. This leads to a mathematical consistency between Einstein equations and special relativity principles.

INTRODUCTION

In general relativity (GR), the curvature of spacetime is directly related to the stress-energy tensor. This provides a rigorous description for the gravitational phenomena. Yet, the theory is geometrically very complicated. Different attempts for alternatives have been made [1–3], but the consistency with special relativity (SR) is confused. In the present paper, we study the consistency between the notion of the Riemann curvature (in the context of Einstein equivalence principle) and special theory of relativity principles. More specifically, we discuss the cases corresponding to the Schwarzschild and Reissner-Nordstrom metrics. The study simplifies the basic concepts of general relativity, and provides new insights into the relation between classical gravity and the quantitative description of the theory (Regge discretization of space).

This paper is organized as follows: in section 2, we briefly summarize the main concepts in Riemann geometry, in the context of the present study. In section 3, the discuss the Riemann curvature and its relation to gravity. In section 4, some special cases (the spherical symmetric and Reissner-Nordstrom fields) are discussed. Finally,we draw our conclusion.

RIEMANN GEOMETRY

Riemann manifold is the global space on which Einstein equations solutions are represented. Each point, say p, in it corresponds to the center, say $O_l(p)$, of a local frame. Each local frame has its own local basis, with respect to the gravity center. For a global observer at the gravity center, say O, this basis is the coordinates basis. The location in the global space is defined by curved coordinates, say $\{x^\mu\}$, whereas in the local frames the flat coordinates are used instead, say $\{X^\mu\}$. At a point M, the coordinates basis can be defined using the partial derivative of the global position with respect to the curved coordinates as: $e_\mu = \partial_\mu \mathbf{OM}$. Of course, this basis is tangential to the lines of curved coordinates.

Einstein realized that local frames correspond to the case of SR, whereas the general motion in the global space (which corresponds to a continuous jumping between infinite Minkowski spaces) corresponds to the general case of the theory, he called general relativity. The idea is that: for a local observer in free-fall (moving along a given geodesic), the space with respect to him is Minkowskian. This called Einstein equivalence principle (EEP). Einstein equations are the constraints that define the geodesics, as such, define EEP.

The result of the theory is that:*matter distorts spacetime, and beings living in spacetime follow distorted paths.*

CURVATURE

SR is the local description of spacetime. For the global observer, the local frame changes at each new point on the geodesic. This leads to the general case of the theory: global relativity.

Let us make the argument more clear. In a local frame $O_l(p)$, the local observer measures the infinitesimal interval as: $ds^2 = g_{\mu\nu}(X)dX^\mu dX^\nu$, where $g_{\mu\nu}(X)$ is the Minkowskian metric. With respect to the global observer, in the general case, each infinitesimal element in the coordinates is split as: $dx^\mu = f^\mu_\nu(x)dX^\nu$ where $f^\mu_\nu(x) = \frac{\partial x^\mu}{\partial X^\nu}$. In the global space, the coordinates are curved; therefore, the relations between the coordinates are not linear. Moreover, each coordinate is parametrized with the parameter of the embedded curve (geodesic) as: $x^\mu(\tau)$. Furthermore, each coordinate may (generally) construct three planes, e.g. for x^1, we have: $\{(x^1,x^2),(x^1,x^3),(x^1,x^4)\}$; therefore, each coordinate may generally construct three curves by eliminating the parameter τ between the couples, e.g. for $x^1(\tau)$, we have: $\{(x^1(\tau),x^2(\tau)),(x^1(\tau),x^3(\tau)),(x^1(\tau),x^4(\tau))\}$; each couple corresponds to a curve. We use the polar coordinates for each couple, with the choice that the coordinate that construct the planes plays the role of the rho-coordinate, e.g. for $\{(x^1,x^2),(x^1,x^3),(x^1,x^4)\}$, the coordinate x^1 plays the role of the rho-coordinate. These planes are used to parametrize the general form of the curve in the global space; hence, they determine the explicit form of the curvature.

Since the coordinates X^μ are flat, in the general case, we write $dX^i = \omega^i dT^i, i = \overline{1,3}$. That is, the theory (generally) is described with three constants. Therefore

$$dx^\mu = f^\mu_\nu(x)dX^\nu = \left(g^\mu_\nu(X) + g^\mu_4(X)\omega_\rho\delta^\rho_\nu\right)dX^\nu \qquad (1)$$

where $\omega_\rho = \frac{1}{\omega^\rho}, \omega_1 = 1$ (Note that $\{x^\nu\}$ locally reduce to $\{X^\nu\}$ and the relations between the coordinates $\{X^\nu\}$ are linear).

An important note to mention is that the differential elements used here are not infinitesimal in the mathematical sense, but in the context of EEP. That is, they correspond (physically) to sufficiently small region of space.

Clearly, the quantities $\{f^\mu_4(x)\}$ correspond to curvatures (of curved lines in 2d spaces), as such, they correspond to accelerations; therefore, multiplying these with the differential element of the time coordinate $d\tau$, we get elements of velocity in the same/opposite direction of the μ-axis.

Of course, $\frac{ds}{d\tau}$ is invariant, as such, the square $g_{\mu\nu}(X)\frac{X^\mu}{d\tau}\frac{dX^\nu}{d\tau} = M$. Since the components of this four-vector are the velocity, thus by adopting constants for these components, we get a geometric stress-energy tensor; its physical interpretation is simple: it represents locally the four-vector of impulsion-energy (caused by gravity) of the test particle under study.

But, how the picture is, for the global observer? The answer is simple; we just apply SR principles. The first SR postulate corresponds to the conservation of momentum, and the second corresponds to the constancy of speed of light (as it is measured by the local and global observers). The result is that we find the generalized geometric stress-energy tensor, say $G_{\mu\nu}$. The gravity theory, therefore, lies in the equation $ds^2 = G_{\mu\nu}dx^\mu dx^\nu$. It is clear that $G_{\mu\nu}$ reduces to $M_{\mu\nu}$ locally. Considering the general case by adding the stress-energy tensor of ordinary matter gives the final form of the gravity equation:

$$ds^2 = \hat{G}_{\mu\nu}dx^\mu dx^\nu \qquad (2)$$

where $\hat{G}_{\mu\nu}$ corresponds to the matter-geometry stress-energy tensor as it is measured by the global observer. Evidently, the last equation defines the metric of spacetime, together with the geodesic equations (the geodesics equations are closely to the local coordinate [4]), the curved paths can be determined explicitly.

THE SPHERICAL SYMMETRIC CASE

For the case of spherical symmetric gravitational field, the orbits are circles. A circular orbit can be represented locally (which depends on the position on the curve; and it takes discrete description as in Regge geometry) and globally (continuously; which we get by taking the lengths of the edges in the discretization to zero; as it looks globally) in a contextual form. In this case, $f^r_4(x) = \frac{-1}{r}$ (curvature of a circular curve), thus $dx^r = \left(1 - \frac{1}{r}\right)dX^r = k(r)dX^r$. for dx^t, we use the spacetime diagram (as in SR) together with the last equation, we get the result: $f^t_4(x) = \frac{\frac{-1}{r}}{k(r)}$, which leads to the formula $dx^t = \frac{1}{k(r)}dX^t$.

Note that this derivation (the last equation) can be extracted straightforwardly using the postulate of the constancy of the speed of light of SR, without using the spacetime diagram, i.e. the last derivation fundamentally reflects the speed of light constancy postulate. This completes the derivation in question.

One may be wondered that: in GR, some solutions naturally involve non-diagonal metrics. Of course, if the curves corresponding to the curvature $f^\mu_4(x)$ depend on the corresponding three curved coordinates, then (when we apply the second SR postulate) new constraints are considered, as such, relations between coordinates are resulted, which lead finally (when we compute the square of infinitesimal coordinates elements) to those terms being discussed.

We can easily find the spherical symmetric with cosmological constant by considering the term of this constant in $\hat{G}_{\mu\nu}$. Also, for charged gravity sources, the Reissner-Nordstrom metric can be found explicitly, and easily.

The findings show the key concept behind GR: EEP is the geometric realization of SR principles.

In summary, we have studied the consistency between Einstein equivalence principle and special relativity principles. The results provide new insights into the study of Regge discretization of space, in the context of the assumption that the local Regge simplexes [5, 6] define the Einsteinium sufficiently small regions of space.

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