Strings and Loops in the Language of Geometric Clifford Algebra

Peter Cameron and Michaele Suisse*
Strongarm Studios
PO Box 1030
Mattituck, NY USA 11952

(Dated: March 31, 2017)

Understanding quantum gravity motivates string and loop theorists. Both employ geometric wavefunction models. Gravity enters strings by taking the one fundamental length permitted by quantum field theory to be not the high energy cutoff, rather the Planck length. This comes at a price - string theory cannot be renormalized, and the solutions landscape is effectively infinite. Loop theory seeks only to quantize gravity, with hope that insights gained might inform particle physics. It does so via interactions of two-dimensional loops in three-dimensional space. While both approaches offer possibilities not available to Standard Model theorists, it is not unreasonable to suggest that geometric wavefunctions comprised of fundamental geometric objects of three-dimensional space are required for successful models, written in the language of geometric Clifford algebra.

*michaele.suisse@gmail.com

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INTRODUCTION

Historical absence of two essential tools from the dialog of the physics community is most remarkable[1, 2].

First and foremost is the background independent[3] algebra of spacetime[4–7], the geometric interpretation of Clifford algebra. The algebra of Euclid's point, line, plane, and volume elements. The interaction algebra of geometric primitives of physical space[8, 9]. Geometric algebra is essential in exploration beyond leptons and quarks to the geometric wavefunction model that must be present in a theory of quantum gravity[10, 11].

More obscure but no less essential is that which governs amplitude and phase of these geometric wavefunction interactions - the background independent[3] *exact quantization* of impedances beyond photon and quantum Hall[12] to those associated with all potentials[13–16].

Geometric algebra and the topological symmetry breaking inherent in geometric products permits one to define a geometric vacuum wavefunction comprised of fundamental geometric objects of the three-dimensional Pauli algebra of space.

Wavefunction interactions generate the 4D Dirac algebra of flat Minkowski spacetime, gaining the attribute of quantized impedances when endowed with quantized electromagnetic fields. The resulting model is naturally gauge invariant, finite, and confined, and reveals the pivotal role of impedance matching in energy flow to and from the elementary particle spectrum[17].

Extending the model to Planck scale and examining the mismatch to the massive particle spectrum exposes an exact identity between gravity and impedance mismatched electromagnetism[18, 19]. From this emerges a quantized gauge theory gravity[10, 11] equivalent of general relativity[20–24]. In what follows we present details, and examine string and loop geometries in the context of our results.

GEOMETRIC ALGEBRA

Figure 1 illustrates an important point - geometric algebra (and its extension into geometric calculus) claims to encompass the better part of the particle physicist's mathematical toolkit[25, 26].

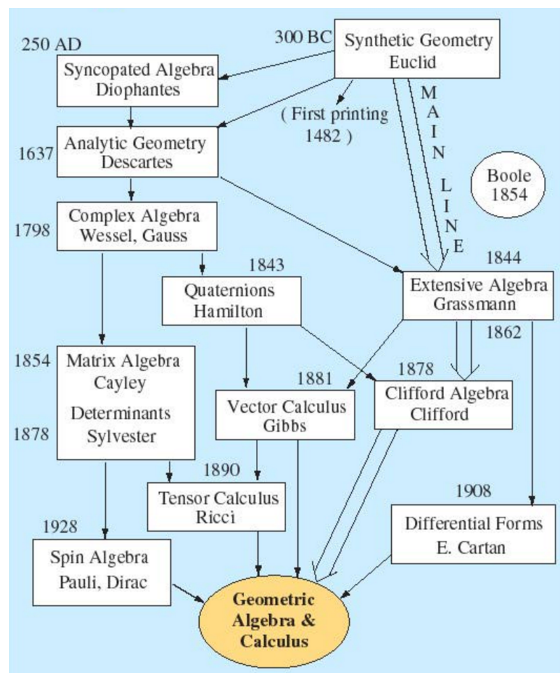

FIG. 1. Evolution of Geometric Algebra [27]

It would seem that there is a certain profundity to this, that the physicist's essential set of mathematical tools are a subset of the interaction algebra of the fundamental geometric objects, of the point, line, plane, and volume elements of our physical space[28].

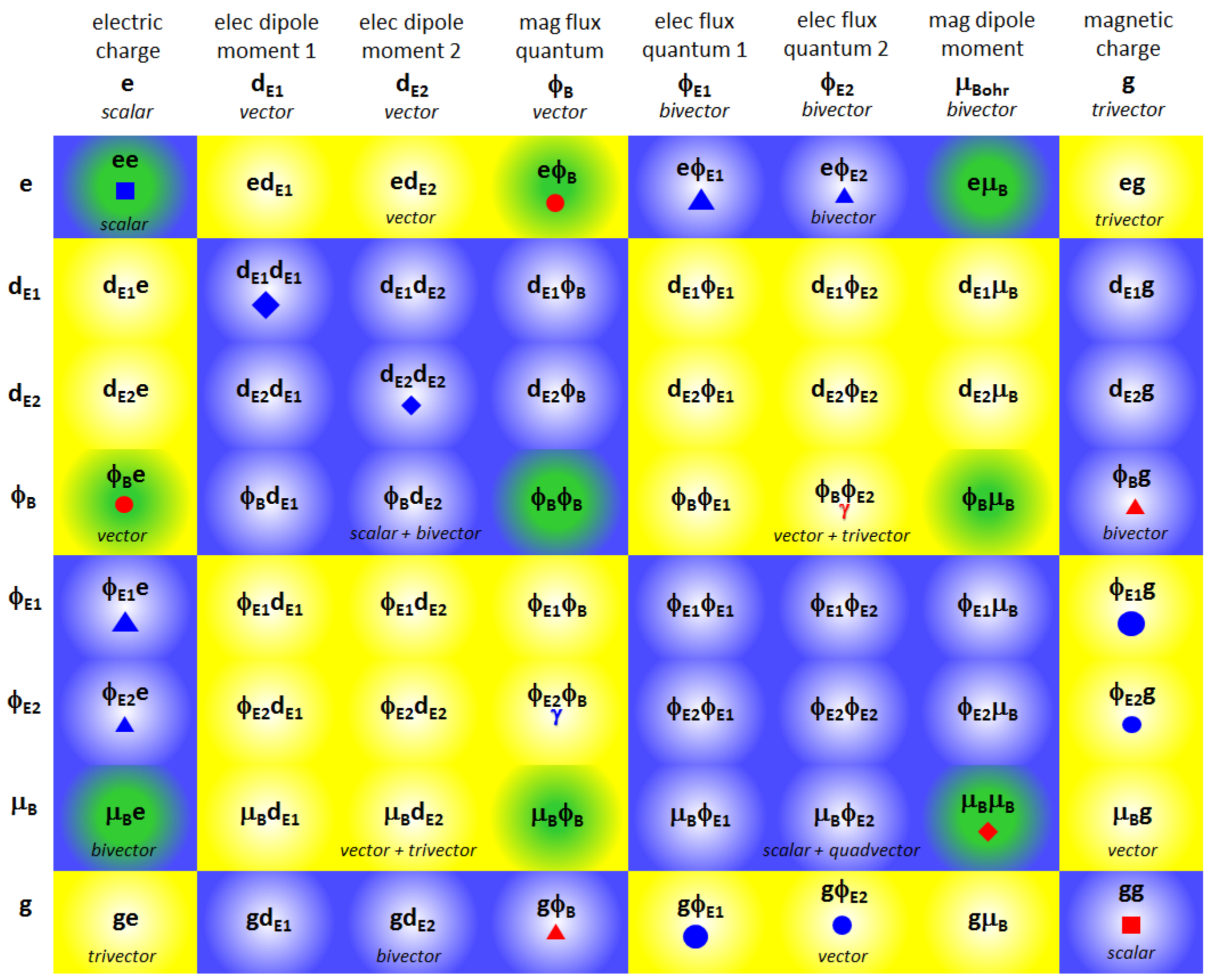

FIG. 2. **The S-matrix:** Pauli algebra of three-dimensional space is comprised of one scalar, three each vectors and bivectors, and one trivector. All are orientable, with sign of the scalar giving time direction, the opposing phase evolutions of particle and anti-particle. Attributing quantized electric and magnetic fields to these fundamental geometric objects yields the wavefunction model. Taking those at top as the electron wavefunction suggests those at left correspond to the positron. Their geometric product generates the background independent four-dimensional Dirac algebra of flat Minkowski spacetime, arranged in odd transition (yellow) and even eigenmodes (blue) by grade (dimension). Time emerges from the interactions. Modes of the stable proton are highlighted in green[29, 30]. Modes indicated by symbols (circle, square,...) are plotted in figure 3.

Topological symmetry breaking is implicit in geometric algebra. Given two vectors a and b, the geometric product ab mixes products of different dimension, or *grade*. In the product $ab = a \cdot b + a \wedge b$, two 1D vectors are transformed into point scalar and 2D bivector.

"The problem is that even though we can transform the line continuously into a point, we cannot undo this transformation and have a function from the point back onto the line..." [31].

Geometric wavefunction interactions are represented by geometric products, break topological symmetry in grade increasing operations (origin of parity violation?). Topological duality[32–35] is evident in the differing geometric grades of electric and magnetic charges of figure 2. Electric charge is a scalar, magnetic its topological dual, the Pauli algebra pseudoscalar. Their ratio is the electromagnetic fine structure constant, $\alpha = e/g$.

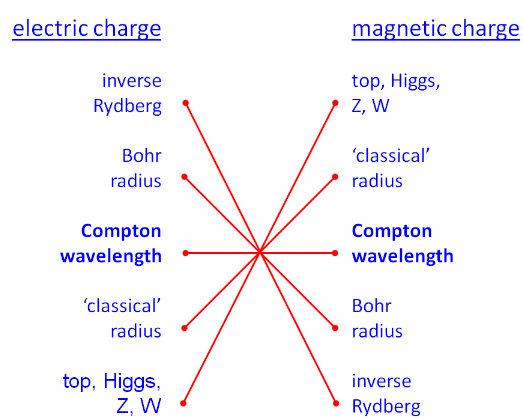

FIG. 3. Inversion of fundamental lengths by magnetic charge. The product $eg = \hbar$ is the dyon[36, 37], a pseudovector in the Dirac algebra. Compton wavelength depends only on mass. Importance of Grassman's contribution[38], the unique invertibility of geometric algebra, is evident here.

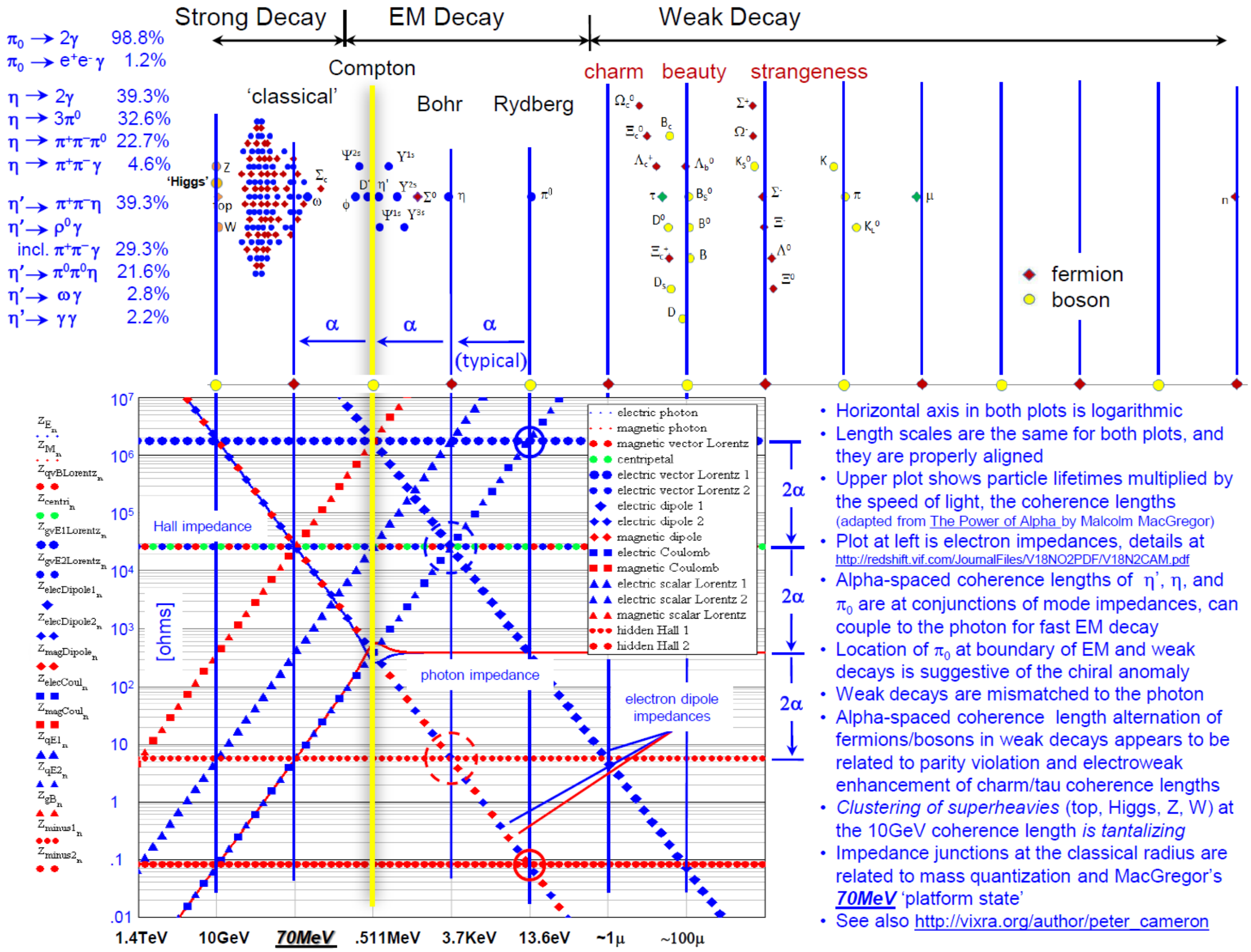

FIG. 4. Correlation between lifetimes/coherence lengths (light cone boundary) of the unstable particle spectrum and nodes of the energy/scale dependent impedance network of a subset of the modes of figure 2 [17], showing the match of a .511MeV photon to the node at the fundamental length of the model, the electron Compton wavelength.

IMPEDANCE QUANTIZATION

Knowing geometries and fields of modes shown in figure 2, one can calculate mode impedances, an equivalent representation[39] of the scattering matrix[40–46]. Absent electric and magnetic fields, the geometric wavefunction model represents the virtual vacuum impedance structure. Excitation of the lowest order mode, the electric Coulomb mode (blue square at upper left of figure 2), yields the 377 ohm vacuum impedance seen by the photon[47], as shown in figure 4.

Strong correlation of network nodes with unstable particle coherence lengths[48–54] follows from the requirement that impedances be matched for energy flow between modes during decoherence. More generally, correlation supports the premise of S-matrix theory, that the matrix of figure 2 governs the flow of energy to and from observables of the unstable particle spectrum. For example, precise calculation of π_0, η, and η' branching ratios shown at upper left of the figure and chiral anomaly resolution follow from impedance matching[55].

Figure 5 shows far-to-near field transition of a 13.6eV photon, permitting impedance matching to the hydrogen atom quantum Hall impedance at the Bohr radius.

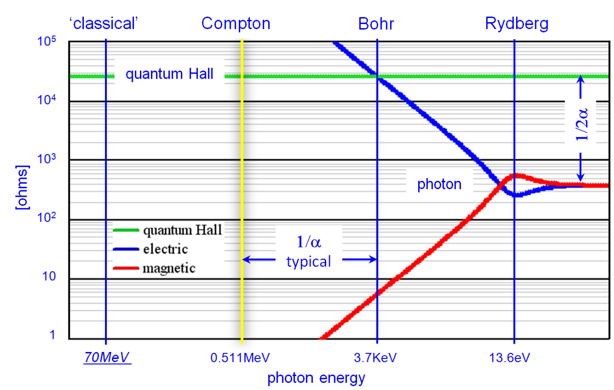

FIG. 5. Photon match to a free electron [52, 56].

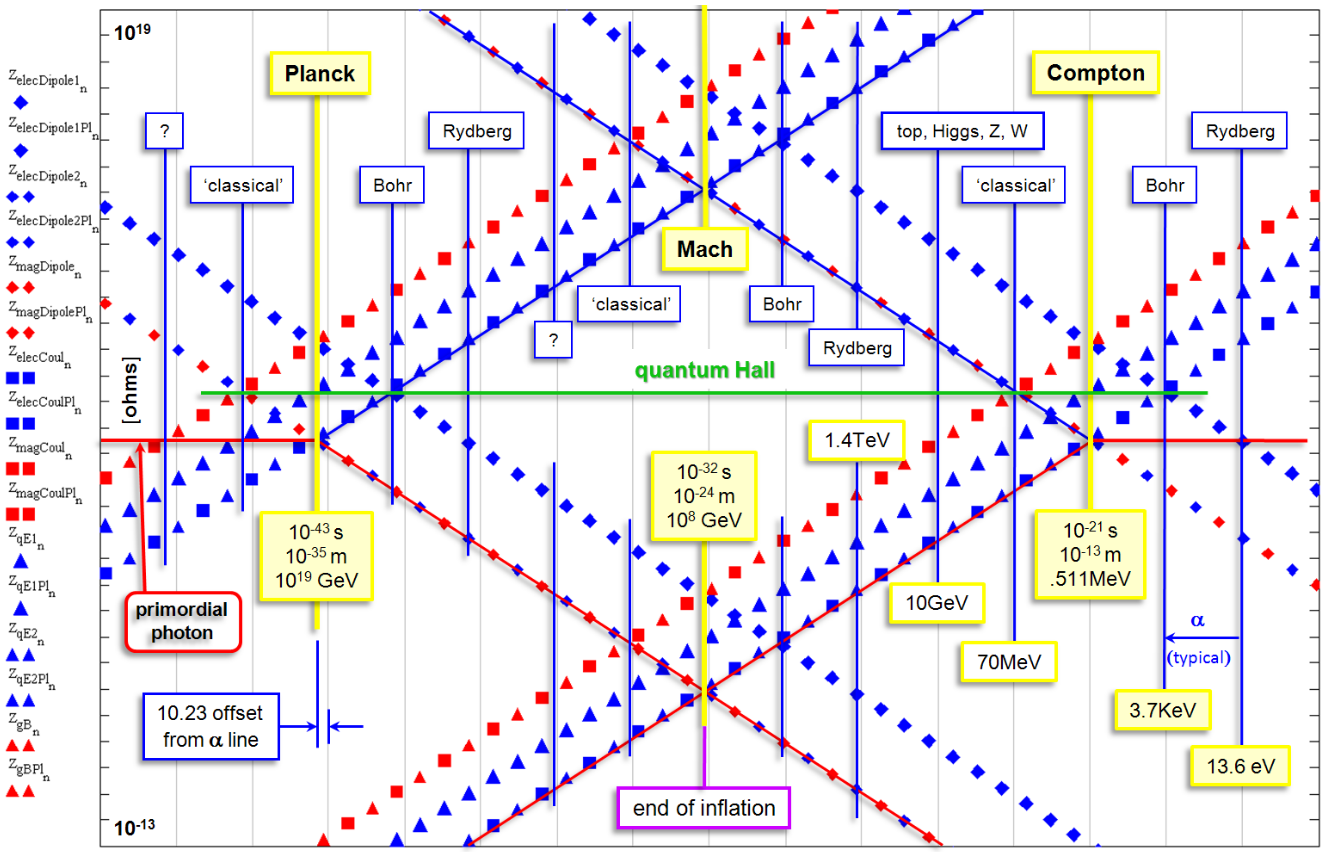

FIG. 6. A subset of impedance networks of the electron and Planck particle, showing both a .511 Mev photon entering from the right and the 'primordial photon' from the left. The end of inflation in the impedance approach (as in the cosmological Standard Model) comes at $\sim 10^{-32}$ seconds, at the intersection of the two networks, the 'Mach scale'.

QUANTIZING GAUGE THEORY GRAVITY

The network of figure 6 results from choosing the quantization scale to be not the electron Compton wavelength but rather the Planck length. The gravitational force between electron and Planck particle exactly equals the product of Coulomb force and impedance mismatch[18, 19, 57], suggesting gravity is mismatched electromagnetism. However, two essential properties of gravity seem to rule out electromagnetic origin [58].

First, unlike electromagnetic forces, it appears that gravity cannot be shielded. However, scale invariant impedances cannot be shielded[54, 59]. Consider for instance centrifugal force, or the Aharonov-Bohm effect.

Second, unlike the bipolarity of electromagnetism, gravity appears to have only one sign. We observe only attractive gravitational forces. Here the distinction between near and far fields plays a pivotal role. Gravity is forty-two orders of magnitude weaker than the Coulomb force, a consequence of the impedance mismatch. Given the $\sim 10^{-12}$ meters wavelength of a .511MeV photon, the mismatched 'gravity photon' wavelength will be about forty-two orders of magnitude greater, or $\sim 10^{30}$ meters. The observable universe is about 10^{26} meters.

Our material existence appears to be in the extreme near field of the 'gravity photons' of almost all of the mass in the universe, where the scale dependent impedances appear scale invariant due to flatness of the phase. One might conjecture that this is what permits scale dependent impedances to have the 'cannot be shielded' property of scale invariant impedances. Hopefully topological character of the algebra will provide a proper formalism.

STRINGS AND LOOPS

The geometric wavefunction model is naturally gauge invariant, finite, and confined. For input it requires five fundamental constants - speed of light, Planck's constant, magnetic permeability of free space, electric charge quantum, and electron Compton wavelength. There are no adjustable parameters. Given the diversity and simplicity of the model, it would be most helpful to establish connections with the mainstream in particle and gravity theory, with strings and loops.

For **string theory** the problem is not insignificant. The partners of the one-dimensional open strings would seem to be grade-1 vectors of the wavefunction model.

However strings vibrate transversely in ten-dimensional spacetime, whereas vectors are truly one-dimensional and have only orientational and longitudinal degrees of freedom in four-dimensional spacetime. The possibility exists that the open string might be represented by the D-branes at the ends taken to be bivectors. This does not make things more simple, and offers no obvious advantage.

The partners of two-dimensional closed strings would presumably be bivectors of the wavefunction model, like the D-branes. This would seem to exhaust the possible geometric correspondences with strings. Many questions remain, among them - what fields (electric or magnetic) would one assign to the vectors and bivectors, and how would one accomplish the dimensional reduction to four-dimensional spacetime?

For **loop theory** the problem seems much simpler, particularly as it proposes to explain only gravity. The loops are taken to be the bivectors of the wavefunction model, exist in our physical three-dimensional space, and their interactions are modeled by the geometric product. Question remains what fields (electric or magnetic) to assign to the bivectors.

SUMMARY AND CONCLUSION

Keeping in mind that the photon near field has longitudinal electric field, and that we are in the near field of the mismatched 'gravity photons' of almost all the mass in the universe, one might conclude that the phase shifts will be longitudinal. This has implications for attempting to triangulate the origins of gravity waves when a third detector comes on line, and opens the possibility of experimental confirmation of the geometric wavefunction approach, as general relativity requires that the phase shifts be transverse.

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* michaele.suisse@gmail.com

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