Selfinteraction of Adiabatic Systems

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Das Wesen der Mathematik liegt in ihrer Freiheit.

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Abstract. Given an adiabatic system of particles as defined in [4], the problem is whether and to what degree one can break it into its constituents and describe their mutual interaction.

1. Introduction

An adiabatic system has been defined in [4] as a systems of charged or neutral particles of rest mass $m > 0$ which maintains its total rest mass. Such a system is the relativistic analog of an energy conserving non-relativistic mechanical system. It can be subjected to external forces, still. In the absence of external forces, adiabatic systems become closed dynamical systems. Taking over from classical electrodynamics, I describe these systems mathematically by 4-vector fluxes $j := (j_0, \ldots, j_3)$, where $j_0 : \mathbb{R}^4 \ni x \mapsto \rho(x)$ is a smooth function of mass or charge density within space-time and $j_k(x) := (\rho v_k)(x)$, $k = 1, 2, 3$ denote the components of the flux, with (v_1, v_2, v_3) being the velocity.

Now I want to break j into pieces $j = j^{(1)} + j^{(2)}$ and describe their mutual interaction. However, in order to do so, I must first have a clean definition of the underlying space of coordinates.

2. Space-Time Coordinates

The 4-vector j is a function of Euclidean space and time $\mathbb{R}^4$. If that was to be taken as the "normal" Minkowski space-time with $c \equiv 1$, then we include particles at space-time that an observer at the origin could never see, namely those to which he is space-like displaced with from his origin $x = 0$. These particles do exist at these space-like distances, and they are part of the system in past an future, although yet they do not interact with the system the observer sees at that instance.

There is however a different frame of coordinates which shows just what the observer sees an saw; I already defined it in [4], albeit in terms of abstract mathematics: Let's define it again, let's see then in which coordinate system the laws become simpler, and stick with the simpler one:

An observer identifies spatial locations with two eyes looking at the points, each, and he maps them with his eigentime. So, an observer looks at space at the speed of light. And on the forward and backward light cone the eigentime stands still. So, from him at the vertex, all points on the forward and backward light cone are instantaneous to him. Now, assuming that the observer is situated at the origin, for each location (x_1, x_2, x_3) there are two points on the light cones, $(\pm c\|x\|, x_1, x_2, x_3)$, on the forward an backward light cone, each. Undoubtly, those two points are different, but the observer won't see a difference:
There are two ways to show this, a superficial and a more profound one: To begin with the superficial one, it suffices to remind that light is to be phase invariant. Because time inversion followed by energy inversion is the identity, $\mathcal{T}$-invariance follows (see: [5]), which proves the assertion. The more profound way again goes back to [5], namely the product of mass/charge, energy, and space inversion $\mathcal{CTP} = \mathbb{I}$ being identity. Since light has rest mass zero, it is $\mathcal{C}$-invariant, and $\mathcal{P}$ is a quirk inversion that lives in Euclidean geometry: whenever I have n linearly independent vectors over a field, $\mathbb{R}$, say, I can make these the an orthonormal base of an n-dimensional vector space over that field. And, if in there one base vector is inverted, the signature of that base inverts: right handedness inverts into left handedness and vice versa, and the mathematical and physical way out of the dilemma of telling the handedness is to assume parity invariance. So, $\mathcal{T}$-invariance (modulo parity quirk) results for light, and there is no way of telling for the observer whether the location he spots is on the forward and backward light cone; hence, he will identify the two points as one location (x_1, x_2, x_3) at his eigentime $\tau = 0$. So, the observer sees a flat space $\mathbb{R}^3$ with time perpendicular to it. And yes, he sees, or better memorizes the past time as opposed to the future: If he is smart enough, he will record what is happening at an instance of eigentime and compare these recordings to what he is experiencing at present. Hence, the observer is experiencing $\mathbb{R}^4$ again in this coordinate system. And the problem now is what is simpler to discuss the physical equations in, is it the Minkowki space, or this local set of coordinates.
What will have to be done, to get from the 2^{nd} base of coordinates to the Minkowski base? If a particle jet $j : x \mapsto j(x)$ is described w.r.t. the 2^{nd}, local coordinate system, then $j : x \mapsto \sum_\mu j_\mu(\gamma_0 x_0, \ldots, \gamma_3 x_3)\gamma_\mu$ will transform to the Minkowski coordinates, and $dx = \sum_{0 \le \mu \le 3} dx_\mu$ becomes $\sum_\mu \gamma_\mu dx_\mu$, where the γ_μ are the Dirac matrices. This looks complicated. But, if we look at the integrability of j, then $\sum_\mu j_\mu(x) dx_\mu$ is not a total differential according to the laws of electrodynamics, because j has curls. However, $dA := \sum_\mu j_\mu(\gamma_0 x_0 + \cdots + \gamma_3 x_3)$ is integrable w.r.t. $\sum_\mu \gamma_\mu dx_\mu$ as shown in [4], and therefore I'll stick with that 2^{nd}, local coordinate frame, just because it is simpler to deal with, and I'll drop the Dirac matrices, whenever applicable.

3. The Static Closed, Neutral System

Let's consider a system S made of N identical, purely neutral particles of free rest mass m_0 of which all particles stay at rest at all time (w.r.t. the observer in an inertial frame of reference). What can be said of this system in terms of classical mechanics?
First off, the forces between the particles must all sum up to zero. But if we can find two particles (then necessarily at different locations) with an opposite net force w.r.t. eachother, then the potential energy between them must be unequal zero. Assuming the force fields between the particles to be all central and static fields (as is common in mechanics), the field of total, internal forces can be integrated to a potential function $U : \vec{x} \mapsto U(\vec{x})$ of location $\vec{x} \in \mathbb{R}$, (where the constant of integration is determined by demanding U to vanish at long spatial distances from the system). To calculate the rest mass of this system, I now add up $E := \sum_{1\leq k\leq N}(m_0^2 + \left|U(\vec{x}_k)\right|^2)^{1/2}$, where the $\vec{x}_k$ are the locations of the particles in the system, and I claim that this value is the correct rest mass of the system.
To its proof, we first notice that in order to stabilize a static system, given $U(\vec{x}) \neq 0$, adhesive forces must exist at $\vec{x}$. These are very short-ranged forces between the particles, which - because of their short range - do not contribute (appreciably) in the potential function. So, we can remove them without affecting the energy, and the particle will start moving due to the force $\nabla U(\vec{x})$, in fact the whole potential energy $U(x)$ can be converted by the particle into kinetical energy in a completely reversible manner due to the conservation of energy. In this state, when the potential energy has all been converted into kinetical energy, the particle is free, and follows the equation $E^2 = m_0^2 + |\vec{p}|^2 = m_0^2 + \left|U(\vec{x})\right|^2$. So, $m(\vec{x}) = \sqrt{m_0^2 + \left|U(\vec{x})\right|^2}$ is the total rest mass of the particle in the potential field U, and summing up the total rest masses for all the N particles, the formula follows.
Let's summarize:

Proposition 3.1. *The total rest mass of a static system of N identical neutral, elementary particles of (free) rest mass m_0, each, is given by $M := \sum_{1\leq k\leq N}(m_0^2(\vec{x}_k) + \left|U(\vec{x}_k)\right|^2)^{1/2}$, where U is the potential field of internal, central field forces between the particles.*

Corollary 3.2. *An internal potential field always increases the total rest mass. To an outside observer, a static, neutral field with internal potential U is identical to that with the inverted potential $-U$. An internal motion of the particles in that system is equivalent to an internal potential field and is therefore unobservable to the outside.*

Corollary 3.3. *The total rest mass of a static neutral system of particles depends on the square of potential energies (or equivalently the square of momenta) of the, but not on first order energy (or first order momentum), and it's the (square of) potential energy of the particles, it's not the potential field itself.*

In particular, the non-relativistic limit of the total rest mass is given by $E = N|m_0| + \sum_{1\leq k\leq N} \frac{1}{|2m_0|}\left|U(\vec{x}_k)\right|^2$, *and* $N|m_0|$ *is constant, hence an unimportant constant of integration.*

Remark 3.4. Now, defining pressure as the differential of net force by surface element, one may associate the subsystem of pairwise repelling particles with a positive pressure, the attractive one as having negative pressure. That definition, however is not a mathematical stringent one, since it relies on the "proper" choice of parity as to the orientation of the surface elements. And that is spoilt in a time-symmetric theory, because inversion of time inverts parity of space-time. As evident as it might appear that a heated gas in a volume will produce a positive pressure, as an observable quantity, pressure already relies on $\mathcal{T}$ being broken.

Remark 3.5. Because in a purely neutral, closed system, the flux part $\vec{j} = (\rho v_1(x), \ldots, \rho v_3(x))$ is unobservable, the whole observable information of such a system as seen from the ouside is captured within its rest mass density $\rho : x \mapsto \rho(x)$.
So, unless we can find forces, which explicitly break the symmetry of $\mathcal{C}$, $\mathcal{T}$, or $\mathcal{P}$ for neutral particles, we will end with a scalar field theory for gravitation. And, in view of the Poisson equation $\Delta\Phi = -4\pi G\rho$, where G is the gravitational constant, one would be pointed to $\Box\Phi = -(Const)\rho$ as a relativistic candidate for that.

4. Charged Static Closed Systems

Consider a closed system S made of N particles of free rest mass m_0, each, as above, let no particle be moving, and let now the system be such that a partition into $N = N_1 + N_2$ particles exists, where the first N_1 and last N_2 particles repel eachother amongst themselves, but each particle of the first N_1 is being attracted by each of the N_2 other particles (and vice versa).
Obviously, this is a special case of a neutral, static closed system: I still have to calculate the net force from each of the particles to the others, from which I deduce a pairwise repulsion or attraction over a distance, calculate the potential from this, and add its absolute value to m_0. The only innovation will be terminology, when I define the first N_1 particles to have a positive charge, and the other N_2 particles to be negatively charged. (Equivalently, I could have allowed the energies to be positive and negative, because the square of energies enter the equations, which is positive. And the same goes to the rest masses.)

The difference between a neutral and and electromagnetic system comes however in one $\mathcal{T}$ and $\mathcal{C}$ breaking, additional law: Matter consists of negatively charged electrons, surrounding positively charged protons in either orientation, where the quotient m_e/m_p of electronic rest mass m_e by protonic rest mass m_p is about $1/1836$, whereas the quotient q_e/q_p is equal to -1.

(In addition, proton and electron maintain their net charge under all proper Lorentz transformations.)

Evidently, this breaks charge inversion symmetry, because negatively charged nucleons and positively charged electrons are not existing (under normal conditions). But then, the electrons must have an angular momentum, and their average velocity must exceed that of the protons. Plus, collisions of atoms will lead to repulsion by the electrons and will accelerate the light electrons to a much larger speed than the heavy protons. The (magnetic) flux $q_e\vec{v}$ of the electrons will therefore exceed in absolute value the flux of the relatively slow protons $q_p\vec{v}$. That allows us to detect the internal speed and hence its kinetic energy inside the matter, and now $j = (\rho, j_1, \ldots, j_3)$ becomes an observable 4-vector.

Let's look at the mutual electromagnetic interaction, given a closed system $\tilde{j}$ of charged fluxes 4-vector fluxes. I have to assume that at all times I can cut space time in two parts from $t = -\infty$ to $t = \infty$, such that the number of electrons and protons are maintained in each part at any instant of time. Then I get two fluxes $j : x \mapsto j(x)$ and $j' : x' \mapsto j'(x')$ and want to calculate the force density beween both fluxes as a function of x and x'.

First off, if I can calculate the interaction field of two particles, then I'll be able to sum up the field (by superposition) for any finite numbers of particles. That means, that the interaction of j and j' would be independent from the inner interactions within j and j' each. So, let's assume for a moment that both j and j' represent single particles.

Next, because the mutual interaction is transmitted by light, interaction of particles is confined to the light cones, which means that $j'(x')$ and $j(x)$ can exchange a force only when x and x' are light-like separated, i.e.: if their Minkowski distance $d(x', x) = 0$ vanishes.
Since the light cones with vertexes x and x' differ for $x \neq x'$, we have to choose either x or x' as frame of reference. Picking x as frame of reference, I may displace the coordinates such that the point under consideration becomes the origin, and with the local coordinates introduced in the beginning, the light cone degenerates into the hyperplane $\tau = 0$. This is where the interaction happens, and this hyperplane should intersect the path $t' \mapsto j'(t, \vec{x})$ in a single point $(t = 0, \vec{y})$ in terms of x, where I assume that j' is in a single point of location $\vec{x'}$ at each instant of time t'. The potential energy of j' at $(t = 0, \vec{y})$ then is given as 4-(co-)vector $(q'_e(0, \vec{y})A_0(0, \vec{y}), \ldots, q'_e v_3 A_3(0, \vec{y}))$, where $(q'_e, q'_e\vec{v})$ is the integration of the flux j' at $(t = 0, \vec{y})$, and A is the 4-vector of electromagnetic potential, which obeys $\Box A_\mu = j_\mu$, $(\mu = 0, \ldots, 3)$. As shown in [1, Vol.II, Ch.21], the components A_μ decompose into a sum of an advanced spherical wave from the target $(0, \vec{y})$ to the source $(0, \vec{0})$ (backwards in time) and a retarded waves from the source $(0, \vec{0})$ to $(0, \vec{y})$ (in the forward time direction). Then, instead of taking half the distance $\|\vec{y}\|/2$ to calculate the potential for the advanced and the retarded wave and adding it, the potential is calculated for the retarded wave over the whole distance. That gives (in our local system)

$$q_e A_\mu(0,\vec{y}) = \int \frac{j_\mu(x)\delta(x)}{4\pi\epsilon_\mu\|\vec{y}\|} d^4x, \qquad (0 \le \mu \le 3),$$

where q_e is the charge (due to the definition of A_μ being potential energy per unit charge), $\epsilon = \epsilon_0(1, c^{-2}, c^{-2}, c^{-2})$ and $c \equiv 1$ is the speed of light.

There is however a subtle error in this calculation, as pointed out in [3]: $j'(x')$ is supposed to be constant in an environment around $(0,\vec{y})$! Let's retransform the coordinate system back to the ordinary Minkowki space to see what happens: the path of $j'(t',\vec{x'})$ crosses the backward light cone at the eigentime $\tau - 0 = 0$ of the coordinate system of j, and at the next moment $\tau + 0 = 0$, j' appears on the forward light cone. What appears to be instantaneous w.r.t. j, is an appreciable time span $\Delta t' > 0$ w.r.t. j', in which j' will generally change speed and location. And when this happens, the situation gets unpleasantly non-differentiable for j at the eigentime $\tau = 0$! We'll even end up with two locations for single particle j' around the eigentime $\tau = 0$: one is the location where j' really was, when it was the source of interaction with j at the origin $x = 0$, and the other one would be that location, where it was supposed to be a moment of time later and where it becomes the target of the source j at $\tau = 0$!

Remark 4.1. To clarify things a bit more, let's derive the above equations anew: The space $L^2(\mathbb{R}^4)$ of square integrable (complex) functions is a Hilbert space with the scalar product $< \cdot,\cdot >$: $(f,g) \mapsto < f,g > := \int \overline{f(x)}g(x)d^4x$. The wave operator $\Box := (\partial_0^2 - \partial_1^2 - \partial_2^2 - \partial_3^2)$ is a selfadjoint operator "on" $L^2(\mathbb{R}^4)$, and the Fourier transform $\mathcal{F} : f \mapsto \frac{1}{(2\pi)^2}\int f(\lambda)e^{-ix\cdot\lambda}d^4\lambda$ a unitary operator on $L^2(\mathbb{R}^4)$, such that $< f, \Box g > = < \mathcal{F}f, \mathcal{F}\Box\mathcal{F}^{-1}\mathcal{F}g >$. And $\mathcal{F}\Box\mathcal{F}^{-1}$ is the multiplication operator by the function $x \mapsto (2\pi)^{-2}(x_0^2 - \|\vec{x}\|^2)$, so that the inverse of that map must be the multiplication operator $x \mapsto (2\pi)^2(x_0^2 - \cdots - x_3^2)^{-1}$.
We get, formally:

$$\Box^{-1}\delta(x) = \frac{1}{(2\pi)^2}\int e^{i\lambda\cdot x}\frac{1}{(\lambda_0 + \|\vec{\lambda}\|)}\frac{1}{(\lambda_0 - \|\vec{\lambda}\|)}d^4\lambda,$$

where the integrand is analytic in λ_0 outside the set of the λ, for which $\lambda_0 = \pm\|\vec{\lambda}\|$. In these points, the integral has a pole of order 1, each, with one exception: at the origin, $\lambda = 0$ the poles degenerate into a pole of order 2. We can now apply Cauchy theorem to get rid of the integration over λ_0: Displacing the pole at the origin infinesimally to the left and right of $\lambda_0 = 0$ on the λ_0-axis allows path integration in the positive imaginary of the poles for negative λ_0 and bypassing the poles on the positive λ_0-axis in the negative imaginary plane. Closing the path in the upper imaginary plane for positive time x_0, where the integral vanishes due to the factor $e^{i\lambda_0 x_0}$, then gives:

$$\Box^{-1}\delta(x) = \frac{1}{4\pi i(2\pi)^2}\int e^{i\vec{\lambda}\cdot\vec{x}}\frac{1}{\|\vec{\lambda}\|}d^3\lambda,$$

which - up to scaling factors - is the same as each of the four components of the above equations.

And now we see that the integration over the negative poles has been left out in the calculation. But what, if we would have added it? By encircling the poles for negative λ_0 in the lower imaginary half plane and encircling the positive ones in the positive half planes, we would include the negative ones and exclude the positive ones. This λ_0-integral would be just the negative of the first, and the terms would cancel, where they should add instead!
(That appears to be a different problem as before, but - as we'll see - the cause is the very same: the second path integration must be in the opposite λ_0-direction, closing the path in an arc in the lower imaginary λ_0 half plane. And to ensure that the integrand vanishes along the lower arc, time must now be negative: what now comes into play is that time inversion followed by energy inversion is the identity. In other words: The mutual interaction between two particles j, j' is to be symmetrically composable into the sum of two parts: from source to time-inverted target and from the time inverted target back in time with inverted energy to the source.)

So, in order to fix the above problem, we have to rely on $j'(\tau, \vec{x})$ at eigentime $\tau = 0 - \epsilon$, where the particle really had been, when it caused interaction with $j(\tau = 0, \vec{x})$, and then the A must become the sum of two retarded waves, one from the source $j'(\tau = 0 - \epsilon, \vec{y})$ to the target $j(0, \vec{0})$, and one from $j(0, \vec{0})$ targeting $j'(\tau = 0 + \epsilon, \vec{y})$.

That means, the time inversion $\mathcal{T}j$ of the target flux j will have to be included as a factor to $q_e A$ into the integrand and, also including q_e into A (because of 3.3), we need to re-dimension j and A, such that jA dimensionaly becomes energy square by spacetime volume. So, j and A have to be re-dimensioned to energy by square root density of spacetime.

Next, recall that $\mathcal{T} = \mathcal{CP}$ (see: [5]), where $\mathcal{P}j'(x) = (j_0'(x), -\vec{j}'(x))$, and $\mathcal{C}j'(x_0, \vec{x}) = -j'(x_0, \vec{x})$.

Also, with $O^* := \overline{O^t}$ denoting the adjoint of an operator O (which is the complex conjugate of its transpose), the operators ∂_μ are anti-hermititian, i.e. satisfy: $\partial_\mu^* = -\partial_\mu$; and because γ_0 is hermititian, whilst $\gamma_1, \gamma_2, \gamma_3$ are anti-hermititian, $(\sum_\mu \gamma_\mu \partial_\mu)^* = -\gamma_0\partial_0 + \cdots + \gamma_3\partial_3$. That is: $(\sum_\mu \gamma_\mu \partial_\mu)^* = \mathcal{T}(\sum_\mu \gamma_\mu \partial_\mu)\mathcal{T}^{-1}$, so: $(\sum_\mu \gamma_\mu \partial_\mu)^* \mathcal{T}j = \mathcal{T}((\sum_\mu \gamma_\mu \partial_\mu)j)$.
Next, by transforming $j = (j_0, \ldots, j_3) \mapsto \sum_\mu \mathrm{j}_\mu \gamma_\mu$, we note that the complex conjugation maps to $(\overline{j_0}\gamma_0 - \cdots - \overline{j_3}\gamma_3)$, such that for $j_0, \ldots, j_3 \in L^2(\mathbb{R}^4)$:

$$< j_0\gamma_0 - \cdots - j_3\gamma_3, j_0\gamma_0 + \cdots + j_3\gamma_3 >= \int (\left|j_0(x)\right|^2 - \cdots - \left|j_3(x)\right|^2) d^4x,$$

which is the square total rest mass of j.

Now, note that the functions j_μ are phase symmetric complex functions, because the field components A_μ are, and $\Box A_\mu = j'_\mu$ holds. So we can use this phase symmetry to gain control over time inversion, and we let the complex conjugate of $\sum_\mu \gamma_\mu \partial_\mu$ be its time inverse. Then $< \sum_\mu j_\mu \gamma_\mu, \sum_\mu j_\mu \gamma_\mu >$ becomes the square of the total rest mass of j, and $(\sum_\mu \gamma_\mu \partial_\mu)^*$ becomes $-(\sum_\mu \gamma_\mu \partial_\mu)$, i.e.: it becomes anti-hermititian.

Now, let's review [4]: In there, it was departed from a 4-vector flux $j = (j_0, \ldots, j_3)$ and its derivative Dj, which is a matrix, that in turn can be decomposed into the sum of a symmetric matrix $D_{sym}j$ and a matrix $D_{asym}j$, which is antisymmetric in its off-diagonal elements, and so j becomes decomposable into a sum $j = j_{sym} + j_{asym}$ of two 4-vector fluxes, which have $D_{sym}j$ and $D_{asym}j$ as their derivatives.

Because of Poincaré's lemma, j is integrable w.r.t. the Euclidean metrics - given smoothness - to a scalar function f, if and only if Dj is symmetric, and because j_{sym} won't be Lorentz contravariant, we dropped it. That might appear to be a deliberate action, missing nearer justification: It's not: In the absence of external fields, the system must be Lorentz covariant, and we can find a Lorentz transformation, such that Dj transforms into a sum of a diagonal matrix Dj_{diag} and a completely anti-symmetric matrix Dj_{as} (with 0 elements in its diagonal). Then $Dj_{asym} := Dj_{diag} + Dj_{as}$ is a matrix that is anti-symmetric in its off-diagonal elements, which shows that $Dj_{sym} = 0$ can be assumed without loss of generality.
Then $\left((Dj_{asym})_{\mu\nu}\gamma_\nu\gamma_\nu\right)_{0\le\mu,\nu\le 3}$ is symmetric and can be integrated twice to a function $Sj : x \mapsto Sj(\sum_\mu \gamma_\mu x_\mu) \in \mathbb{C} \subset Cl(\mathbb{C})$, where $Cl(\mathbb{C})$ denotes the complex algebra generated by the Dirac matrices $\gamma_0, \ldots, \gamma_3$.
Now, let $\mathcal{L}^2$ be the space of all functions $f : \sum_\mu \gamma_\mu x_\mu \mapsto f(\sum \gamma_\mu x_\mu) \in \mathbb{C}$, for which $\tilde{f} : x \mapsto f(\sum_\mu \gamma_\mu x_\mu)$ is in $L^2(\mathbb{R}^4)$, and, by defining $< f, g > := < \tilde{f}, \tilde{g} >$ for all $f, g \in \mathcal{L}^2$, this space becomes a Hilbert space.

So, preliminating $j'_\mu, j_\mu, A_\mu j_\mu \in \mathcal{L}^2$ for $\mu = 0, \ldots, 3$ and $Sj', Sj \in \mathcal{L}^2$, we have:

$$< j'_0, A_0 j_0 > - \cdots - < j'_3, A_3 j_3 > = - < Sj', Sj > .$$

Sj then is the (space-time square root) density of rest energy of field interaction, which j takes on any other part of the system, and $\|Sj\| := < Sj, Sj >^{1/2}$ will be the total rest energy of field interaction with itself. But j itself already defines a (square root) density of energy. So, as in classical mechanics, where we have two kinds of masses, a gravitational mass and an equivalent inert mass, we conclude that Sj should be the "weight density", equivalent to the "inert energy density" j.

Remark 4.2. I chose the above Hilbert space $\mathcal{L}^2$ simply, because it is conveniently self-dual. Looking at the mathematical steps from a pragmatic point of view, all one needs is a dual pair (X, X') of functional spaces, on which $\sum_\mu \gamma_\mu \partial_\mu$ and its adjoint are well-defined. The more we restrict X, the larger X' may become, and vice versa. So, if we replace $X = \mathcal{L}^2$ with the space of continuous functions on $\{\sum_\mu \gamma_\mu x_\mu | x_\mu \in \mathbb{R}\}$, which vanish as $|x| \to \infty$, then X' extends to Borel measures with compact support, and $j' = q_e(1, \vec{v'})\delta(\sum_\mu \gamma_\mu x'_\mu)$ would be a valid choice, resulting in the ordinary equations $\Box A = j$. (Of course, in there j wouldn't need to be square integrable, per se.)

5. Putting it all together

There are several non-trivial consequences from the above:

First, because the complex conjugate of j' is the time inverse of j', the time inverse of the interaction $< Sj', Sj >$ is its canonical conjugate $< Sj, Sj' >= \overline{< Sj', Sj >}$, which generally won't be equal. In other words, time reversal will not be a symmetry any more. But, as the complex value and its conjugate have the same absolute values, we still have a $U(1)$ symmetry in time (see [6]). The irreversibility is of course due to the light electrons within j and j': if at some point $(\tau, \vec{x})$ of interaction of $j(\tau, \vec{x})$ with $j'(\tau - \left|\vec{x'} - \vec{x}\right| /c, \vec{x'})$ the fluxes $\vec{j'}(\tau - \left|\vec{x'} - \vec{x}\right| /c, \vec{x'})$ and $\vec{j}(\tau, \vec{x})$ are unequal, then one of it receives more energy than it sends to the other, and this will continue until a state of equilibrium sets in - by which we then already ordered time - so, what's the direction of time? What appears to be obvious, is at least non-trivial to prove, so I state it as

Conjecture 5.1. *Given a closed system as above, let time be ordered such that interaction is from source to target in positive time direction, only. (This defines a partial ordering of the time axis.) Then the entropy of the system increases in that positive time direction.*

Second, take a look at what is called "electromagnetic field tensor" $F_{\mu\nu}$ (see e.g: [1]), which has 0 on its diagonal, and otherwise coincides with the Euclidean derivative $DA = \Box^{-1} Dj$ of the electromagnetic 4-vector A, where the diagonal elements of DA are the $\Box$-inverses of $(\partial_0 j_0, \ldots, \partial_3 j_3)$. These we expect to be generally unequal zero. Obviously, these diagonal elements aren't needed in electrodynamics, so in there are considered to be zero. But why? The whole theory is governed by the three inversions $\mathcal{P}$, $\mathcal{C}$, and $\mathcal{T}$. And evidently, the anti-symmetry of the electromagnetic field tensor just states that the electromagnetic force is to be anti-symmetric w.r.t. $\mathcal{P}$ and $\mathcal{T}$, wheras the diagonal elements are symmetric w.r.t $\mathcal{P}$ and $\mathcal{T}$. And that is exactly what we would expect for neutral matter. So, these components do describe neutral mass, and then the field tensor for these diagonal components must define the gravitational force between masses. And it is attractive: $< Sj', Sj >$ is the (product of) energies of the $\mathcal{T}$- inverse of j' and j. Given that $\mathcal{T}$ is an invariant of closed, purely neutral systems - as is in conformance with all physical experience today - the equation $< j'_0, A_0 j_0 > - \cdots - < j'_3, A_3 j_3 >= - < Sj', Sj >$ predicts mutual attraction in this case. (This is different from a charged system, where neglecting its neutral content - $\mathcal{T} = \mathcal{CP}$ inverts the net charge of j', resulting into $- < Sj', Sj > = < \mathcal{P}Sj', Sj >$.) It is the brokenness of $\mathcal{C}$-inversion that is causing this phenomenon.

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