Question 2345: Integral , Fractals , Pi

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Abstract

An integral for pi

Resumen

Una integral para pi

1. Introduction

$$f(z) = z\ln z - z - 1 \quad , z \in \mathbb{C} \tag{1}$$

$$\alpha > 0 \wedge f(\alpha) = 0 \Rightarrow \alpha = 3.5911214766686221... \tag{2}$$

$$\alpha = e^{1+e^{-1-e^{-1-e^{-1-...}}}} = \exp(1+\exp(-1-\exp(-1-\exp(-1-...)))) \tag{3}$$

$$\alpha = e^{1+W\left(e^{-1}\right)} \quad , W(x) \; Lambert \;\; function \tag{4}$$

$$W(x)e^{W(x)} = x \tag{5}$$

$$\alpha = \exp\left(1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^{n-1} e^{-n}}{n!}\right) \tag{6}$$

$$\alpha = \frac{1}{2\pi}\int_0^{2\pi} g\left(3+e^{ix}\right)e^{ix}dx \quad , g(z) = \frac{z\ln z}{z\ln z - z - 1} \tag{7}$$

$$\alpha = \frac{1}{2\pi}\int_0^{2\pi} g\left(3+e^{ix}\right)e^{ix}dx \quad , g(z) = \frac{z^2 + e^{1+z^{-1}}}{z^2 - ze^{1+z^{-1}}} \tag{8}$$

$$\alpha = \exp\left(2\pi\left(\int_0^{2\pi} g\left(3+e^{ix}\right)e^{ix}dx\right)^{-1}\right) \quad , g(z) = \frac{1}{z\ln z - z - 1} \tag{9}$$

$$x_1 = 4 \quad , \quad x_{n+1} = e^{1+x_n^{-1}} \Rightarrow x_n \to \alpha \tag{10}$$

$$x_1 = 4 \quad , \quad x_{n+1} = \frac{1+x_n}{\ln x_n} \Rightarrow x_n \to \alpha \tag{11}$$

$$x_1 = 4 \quad , \quad x_{n+1} = \frac{4+9x_n-4x_n \ln x_n}{5} \Rightarrow x_n \to \alpha \tag{12}$$

$$x_1 = 4 \quad , \quad x_{n+1} = \frac{x_n + 4e^{1+x_n^{-1}}}{5} \Rightarrow x_n \to \alpha \tag{13}$$

$$x_1 = 4 \quad , \quad x_{n+1} = \frac{x_n\left(1+x_n\right)}{1+x_n^2 e^{-1-x_n^{-1}}} \Rightarrow x_n \to \alpha \tag{14}$$

2. The Integral for Pi

$$\pi = \int_0^\infty \ln x \ln\left(1+\frac{\alpha^2}{x^2}\right)dx \tag{15}$$

Related integrals and formulas:

$$\pi = \int_{-\infty}^{\infty} xe^x \ln\left(1+\alpha^2 e^{-2x}\right)dx \tag{16}$$

$$\pi = -\int_{-\infty}^{\infty} xe^{-x} \ln\left(1+\alpha^2 e^{2x}\right)dx \tag{17}$$

$$\pi = -\int_0^\infty \frac{\ln x \ln\left(1+\alpha^2 x^2\right)}{x^2}dx \tag{18}$$

$$\pi = \alpha\int_0^\infty \frac{\ln\left(1+x^2\right)}{x^2}\ln\left(\frac{\alpha}{x}\right)dx \tag{19}$$

$$\pi = \int_0^1 \ln x\left(\ln\left(1+\frac{\alpha^2}{x^2}\right)-\frac{\ln\left(1+\alpha^2x^2\right)}{x^2}\right)dx \tag{20}$$

$$\frac{\pi}{2}-\ln 2+2\alpha G = \int_\alpha^\infty \ln x \ln\left(1+\frac{\alpha^2}{x^2}\right)dx \tag{21}$$

$$\frac{\pi}{2}+\ln 2-2\alpha G = \int_0^\alpha \ln x \ln\left(1+\frac{\alpha^2}{x^2}\right)dx \tag{22}$$

$$G=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\ \text{, Catalan constant} \tag{23}$$

$$\pi=-4-2\ln\alpha+\int_0^1\ln x\ln\left(1+\frac{x^2}{\alpha^2}\right)dx+\int_1^{\infty}\ln x\ln\left(1+\frac{\alpha^2}{x^2}\right)dx \tag{24}$$

$$\pi=-4-2\ln\alpha-\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\alpha^{-2n}}{n(2n+1)^2}+\int_1^{\infty}\ln x\ln\left(1+\frac{\alpha^2}{x^2}\right)dx \tag{25}$$

$$\int_0^{\infty}\ln\left(1+\frac{e^2}{x^2}\right)dx=\pi e \tag{26}$$

$$\int_0^{\infty}\ln x\ln\left(1+\frac{e^2}{x^2}\right)dx=0 \tag{27}$$

- Ref.1. p.530 ,formula 4.222-3:

$$\int_0^{\infty}\ln x\ln\left(1+\frac{b^2}{x^2}\right)dx=\pi b(\ln b-1)\quad ,b>0 \tag{28}$$

$$\pi=2\int_{\alpha}^{\infty}\frac{\ln x}{\cosh(1+x-x\ln x)}dx \tag{29}$$

$$\pi=2\int_0^{\alpha}\frac{1+x}{x^2\cosh(1+x^{-1}-\ln x)}dx \tag{30}$$

$$\pi=-2\int_{1-\ln\alpha}^{\infty}\frac{x-1}{x\cosh(x-1-\ln x)}dx \tag{31}$$

$$\pi\ln 2=\frac{1}{4\alpha}\int_0^{\infty}\left(\ln\left(1+\frac{\alpha^2}{x^2}\right)\right)^2dx \tag{32}$$

$$\pi=\frac{\alpha-1}{4}\int_0^{\infty}\ln\left(1+(\alpha-1)^2x^2\right)\ln\left(1+\frac{1}{x^2}\right)dx \tag{33}$$

- Remark: $\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.1415...$; $e=\sum_{n=0}^{\infty}\frac{1}{n!}=2.7182...$

3. Related Fractals : $f(z) = z\ln z - z - 1$

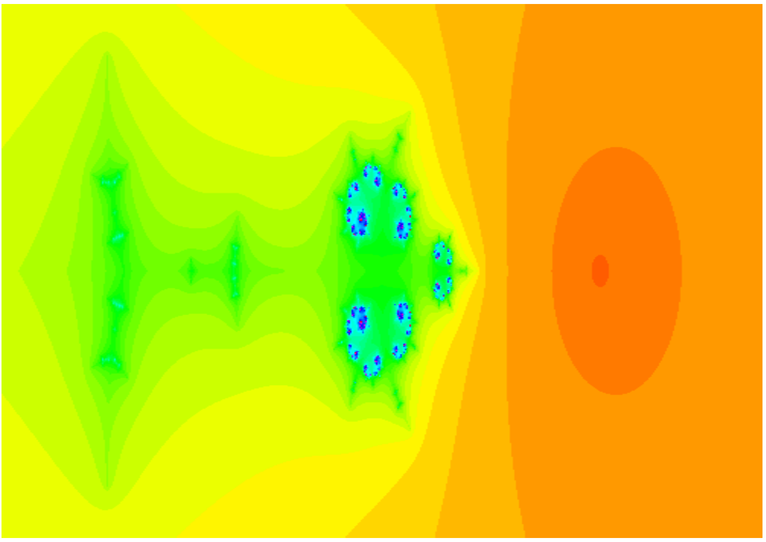

Fig1. $(bl, ur) = (-9-3i, 7+3i)$

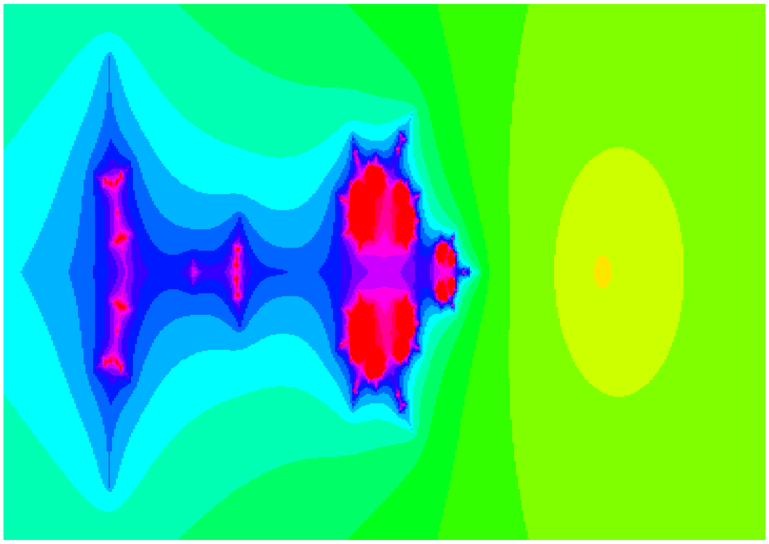

Fig2. $(bl, ur) = (-9-3i, 7+3i)$

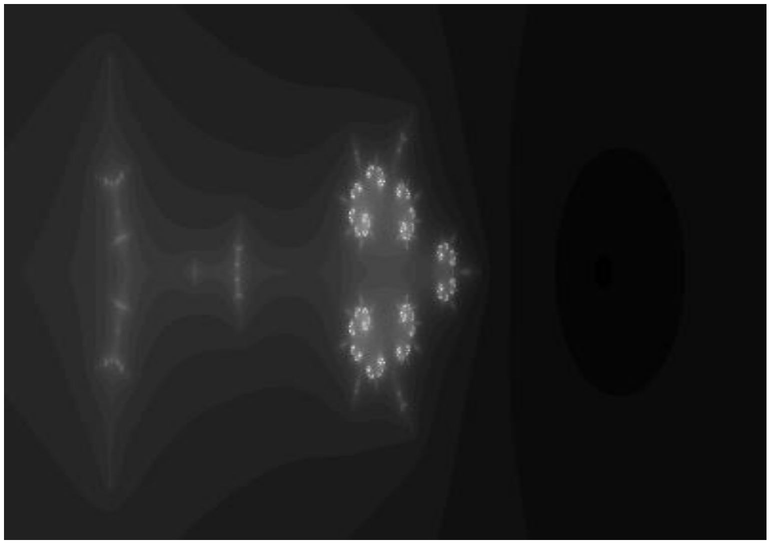

Fig3. $(bl, ur) = (-9-3i, 7+3i)$

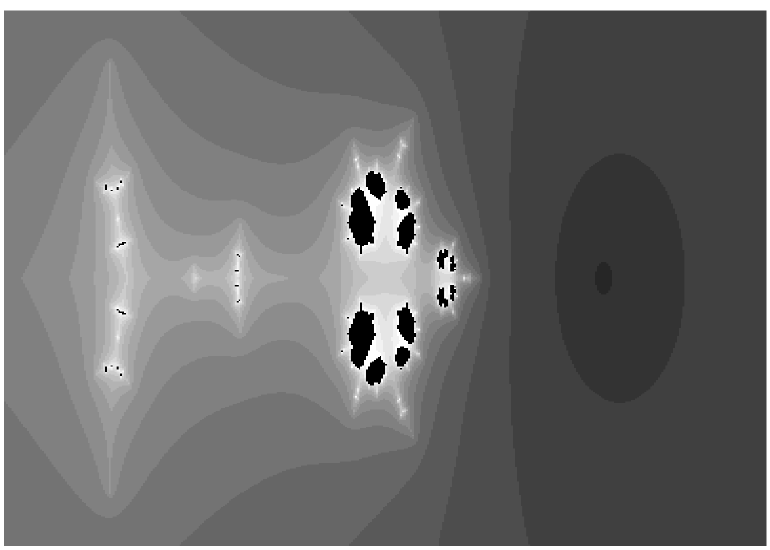

Fig4. $(bl, ur) = (-9-3i, 7+3i)$

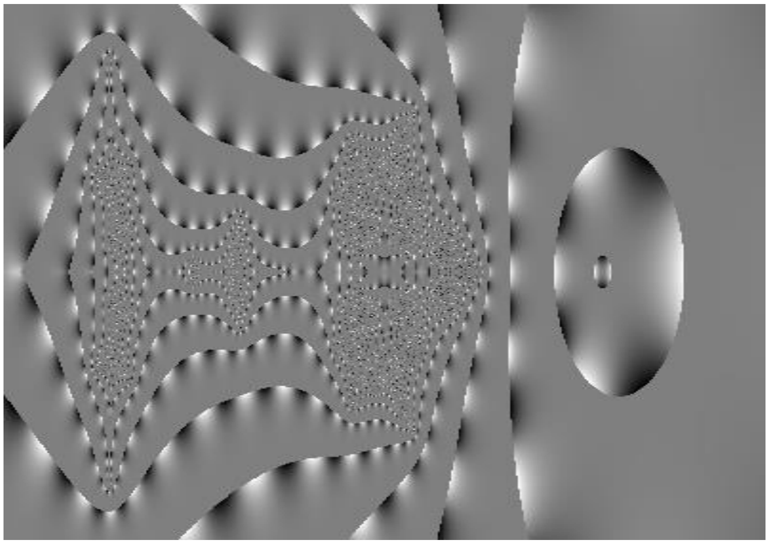

Fig5. $(bl,ur)=(-9-3i,7+3i)$

4. Related Fractals: $f(z)=z^z-e^{1+z}$

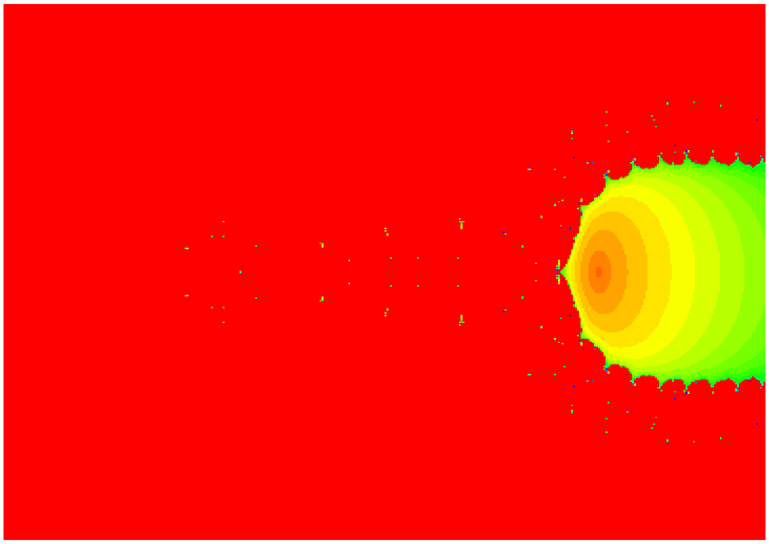

Fig6. $(bl,ur)=(-5-2i,6+2i)$

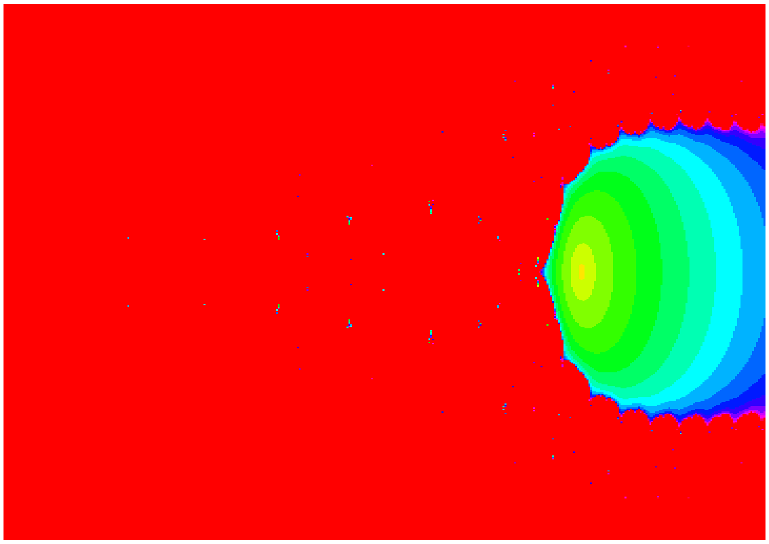

Fig7. $\left(bl,ur\right)=\left(-4-1.5i,6+1.5i\right)$

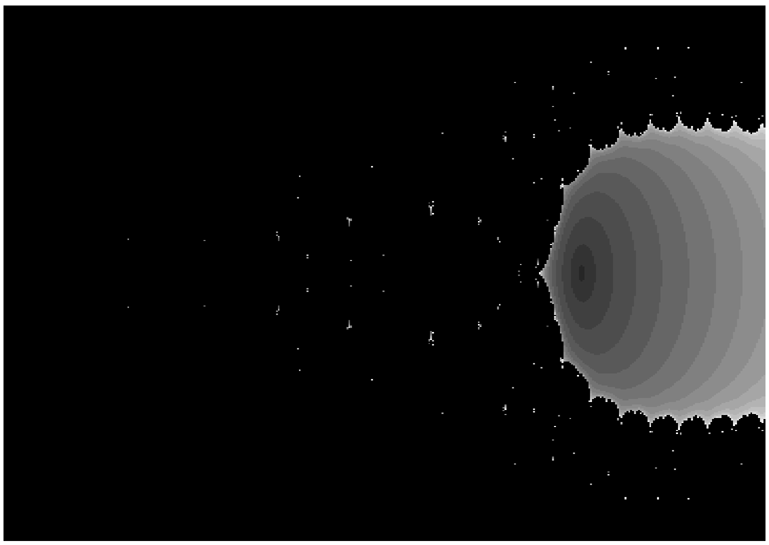

Fig8. $\left(bl,ur\right)=\left(-4-1.5i,6+1.5i\right)$

5. Related Fractals: $f(z)=\frac{z\ln z}{1+z}-1$

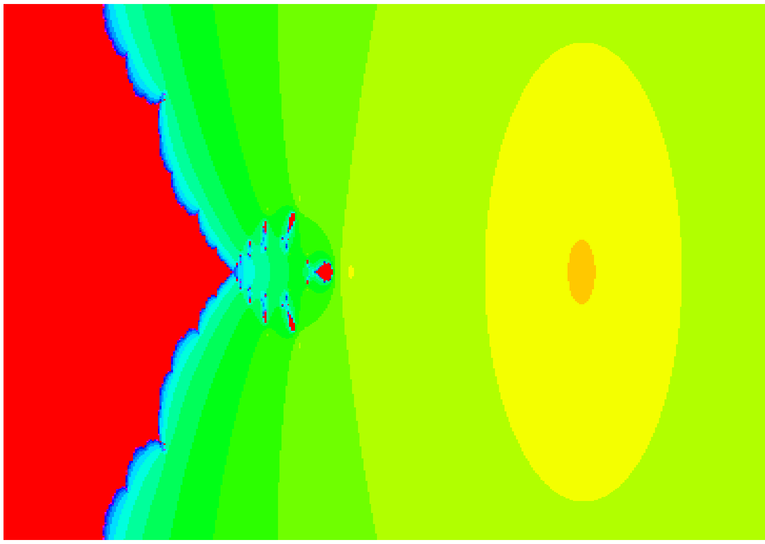

Fig9. $(bl,ur)=(-4-1.5i,6+1.5i)$

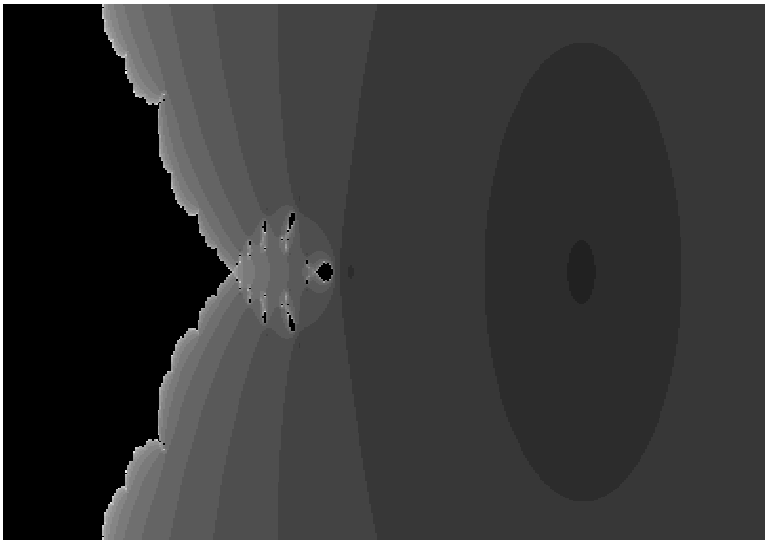

Fig10. $(bl,ur)=(-4-1.5i,6+1.5i)$

6. Related Fractals: $f(z) = z^2 - ze^{1+1/z}$

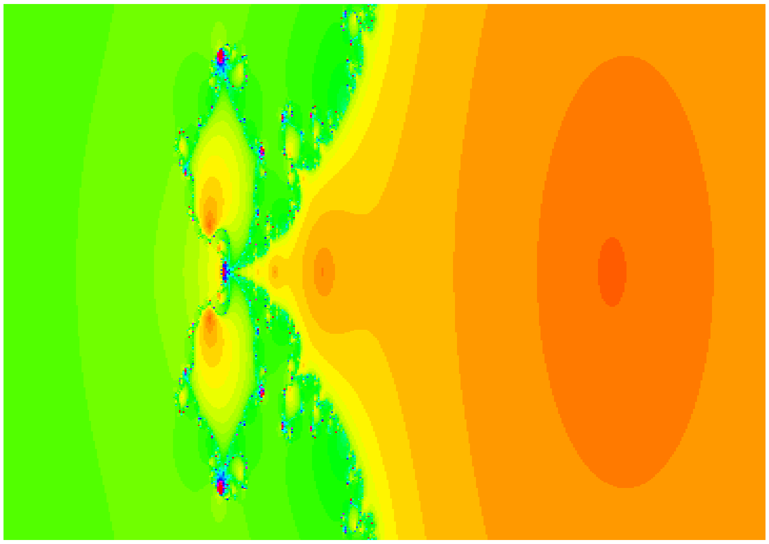

Fig11. $(bl, ur) = (-2-i, 5+i)$

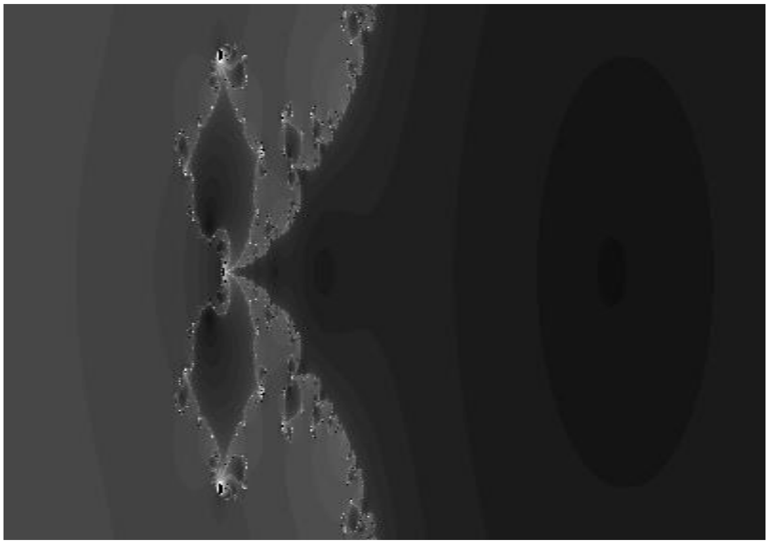

Fig12. $(bl, ur) = (-2-i, 5+i)$

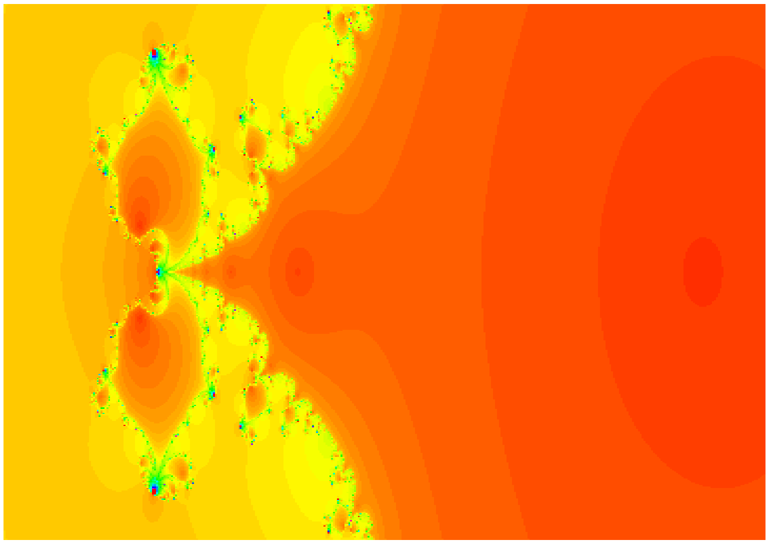

Fig13. $(bl, ur) = (-1-i, 4+i)$

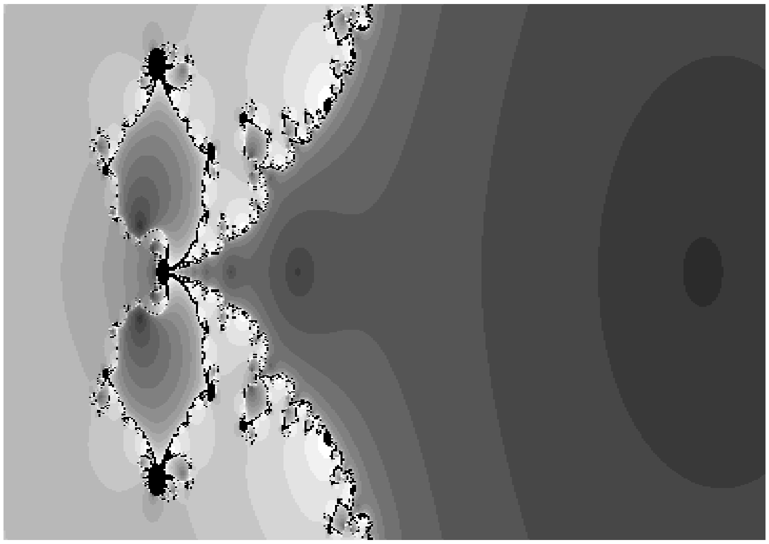

Fig14. $(bl, ur) = (-1-i, 4+i)$

7. Related Fractals: $f(z)=\sin\left(\frac{\pi}{2}z\ln\left(\frac{z}{e}\right)\right)-1$

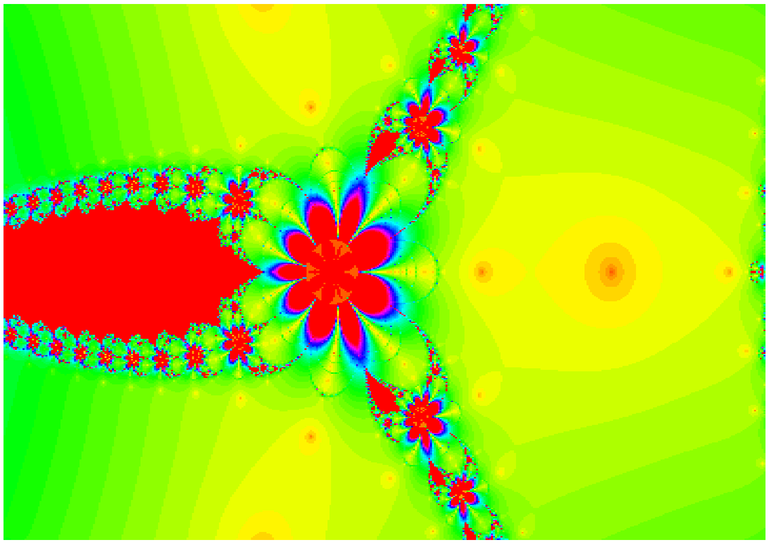

Fig15. $(bl,ur)=(-2-2i,5+2i)$

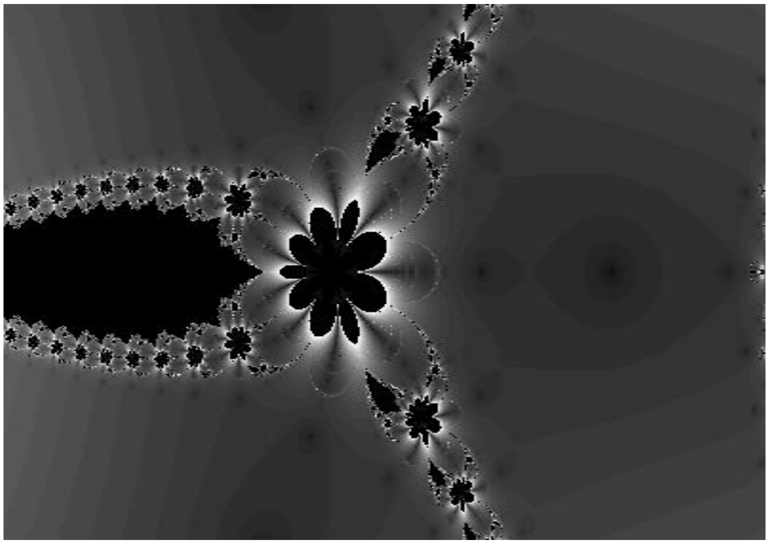

Fig16. $(bl,ur)=(-2-2i,5+2i)$

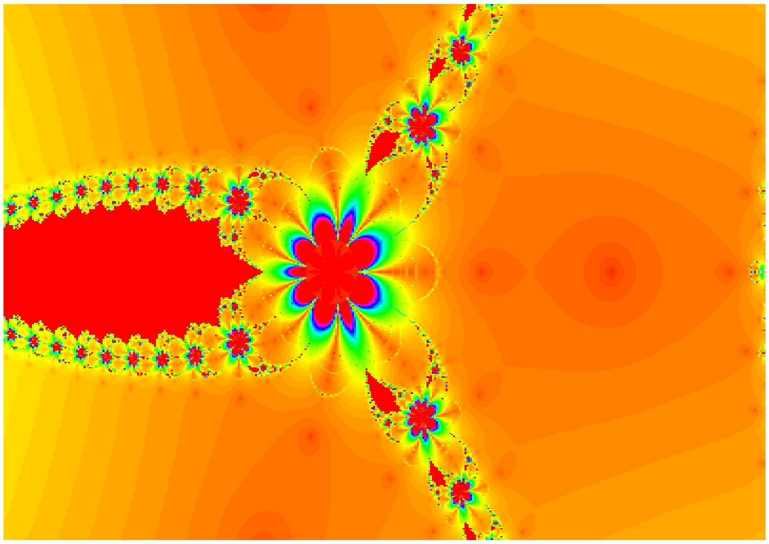

Fig17. $(bl,ur)=(-2-2i,5+2i)$

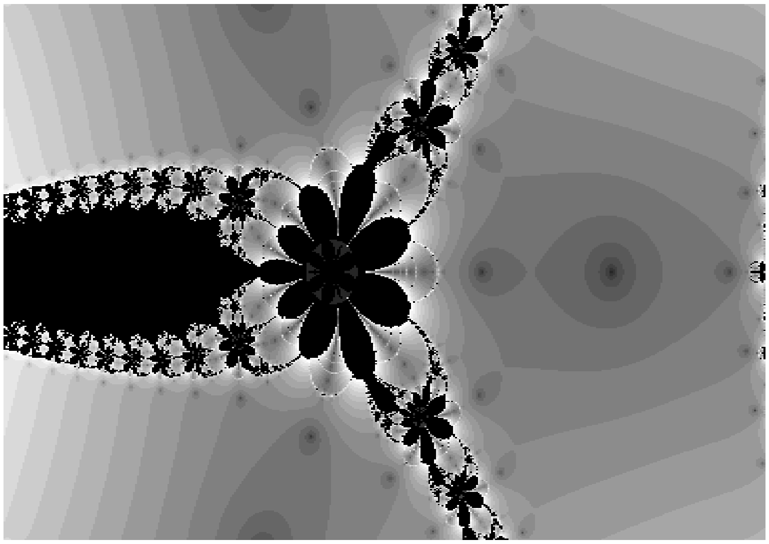

Fig18. $(bl,ur)=(-2-2i,5+2i)$

References

1. Gradshteyn,I.S., and Ryzhik, I.M., Table of Integrals,Series, and Products.Seventh edition, Edited by Alan Jeffrey and Daniel Zwillinger.Academic Press,2007.