The Physical Electron-Positron Model in Geometric Algebra

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Abstract

In an earlier paper on a Geometrical Algebraic solution to the Klein Gordon expression,[1] that was a generic outline to mass particle showing a physical model and its electromagnetic interactions.

This paper will focus on illustrating a Geometric Algebra model of an electron illustrating the physical properties including the source of the charge. It will be shown that a particle such as an electron can be modeled as a pair of interaction photons bound in circular motion by their own interaction vectors, and having a Lorentz Scalar rest mass. This model is not QM, and does not have a probability interpretation, but rather it is defined in terms of an electromagnetic gauge. The notation used here is generally Feynman Slash [2] and the conventions are those of Doran, Lasenby [3].

Technically the interacting speed of light constituents within a particle should not be referred to as photons, but that does seems to be the most useful mnemonic designation.

Systemfunction

The Systemfunction $\bar{\Theta}$ representing a particle is defined as a function that properly models both real and vector properties of a particle. The scalar factor is real and the vector is defined in terms of GA rotors [3].

Preliminary Constructs

The instantaneous null action vector for a photon can be defined as:

$$\not{S} = m\left(\gamma^k c x_k + \gamma^0 cct \right) \tag{1}$$

For convenience and clarity of concepts a mass m assigned to a photon and defined as $m = \hbar\omega / c^2$.

A Systemfunction for a photon is defined as:

$$\bar{\Theta}_F = e^{\left(\not{S} + I_3 \right) \not{S}} \tag{2}$$

Where I_3 is the grade three pseudoscalar $\gamma^1\gamma^2\gamma^3$

This function has is composed of both the standard action of QM and a scalar square of the action that functions as a Gaussian kernel defining the location of the particle moving along the light cone. It satisfies the physical as well as the vector properties of a photon.

Noting that $I_3 \not{S}$ is the QM action mapped into GA and that the magnitude of I_3 times the action is a solution of the Schrodinger equation.

Explicitly writing the function in GA gives:

$$\bar{\Theta}_F = e^{m_2^2 \left(\gamma^k c x_k + \gamma^0 cct \right)^2 + I_3 m \left(\gamma^k c x_k + \gamma^0 cct \right)} \tag{3}$$

This function has all the features of a photon. The first term is a null Gaussian having a half-width of 1.4 λ, and moving at c. the second term defines a rotor in the σ^k plane perpendicular to the direction of travel with the frequency defined by $\hbar\omega = mc^2$, and $m = \hbar\omega / c^2$

In the proposed model of the Electron and Positron, it will be shown that when the particles co-located at the same point the Systemfunction can be factored into a pair of opposite going photons i.e.:

$$\bar{\Theta} = \bar{\Theta}_E \bar{\Theta}_P = \bar{\Theta}_{F1} \bar{\Theta}_{F1} \tag{4}$$

Preliminaries

As in earlier papers, the functional model for the particle is a physical model, and has been designated as the Systemfunction. The use of the Systemfunction designation is to avoid confusion with QM wavefunctions.

Photon Momentum

The four momentum of a particle is the instantaneous value of the derivative of the action of a particle, and thus a starting point for physical particle properties
It has been shown by the author that opposite going photons locked together exhibit dynamical propertied s of massive particles. [4] thus a starting point for dynamical particles.

For two photons going in opposite directions in Euclidian space the momentum is:

$$\not{P}_1 = m_1 \left(\gamma^k c_{1k} + \gamma^0 c \right) \tag{5}$$

$$\not{P}_2 = m_2 \left(-\gamma^k c_{2k} + \gamma^0 c \right) \tag{6}$$

The sum is:

$$\not{P} = \not{P}_1 + \not{P}_2 = (m_1 + m_2)\gamma^0 c + (m_1 - m_2)\gamma^k c_{1k} \tag{7}$$

Squaring:

$$(\not{P}_1 + \not{P}_2)^2 = 4\not{P}_1 \not{P}_2 = 4m_1 m_2 c^2 = m_0^2 c^2 \tag{8}$$

The fact that this is the rest mass comes from the fact that the product of two Lorentz invariant vectors is a Lorentz scalar constant, and thus invariant under a Lorentz transformation. [5]. This is the definition of a rest mass for two opposite photons. Note that there is a defined rest mass even if the photons are not physically in the same location.

Eq.(7), can also be written as:

$$\not{P} = (m_1 + m_2)\left(\gamma^0 c + \frac{(m_1 - m_2)}{(m_1 + m_2)}\gamma^k c_{1k}\right) \tag{9}$$

For two such particles the velocity of the center of gravity v_C can be determined from:

$$(m_1 + m_2) v_C = m_1 c - m_2 c \tag{10}$$

or:

$$\frac{v}{c} = \frac{(m_1 - m_2)}{(m_1 + m_2)} \tag{11}$$

Thus squaring the square of the four momentum for the two photons becomes the standard invariant mass form:

$$\not{P}^2 = m^2 c^2 \left(1 - \frac{v^2}{c^2}\right) = m_0^2 c^2 . \tag{12}$$

This is the best indicator that two bound sped of light particles can be considered to have rest mass.

Action Four-Vector

The instantaneous four-derivative of the action must necessarily be the four vector momentum defined above Eq(6), thus:

$$\not{\partial}\,\not{S} = \not{P} \tag{13}$$

or:

$$dS = m\left(\gamma^0 cdt + \gamma^k c_1 dx_k\right) \tag{14}$$

The differential particle action of the two photons Eq.(5), and Eq.(6), is then.

$$S_1 = m_1\left(\gamma^k cx_k + \gamma^0 cct\right) \tag{15}$$

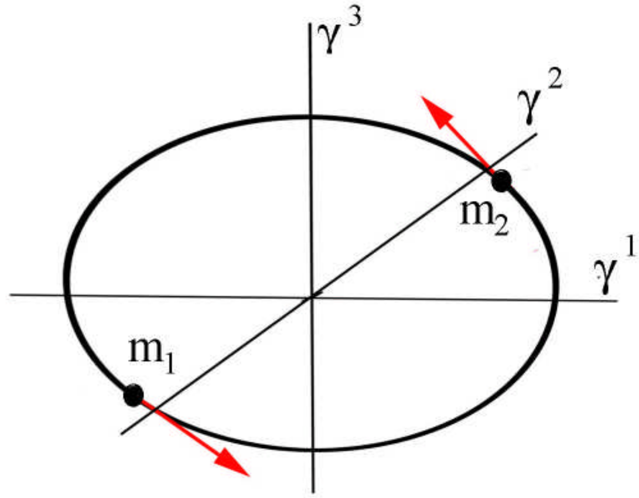
$$S_2 = m_2\left(-\gamma^k cx_k + \gamma^0 cct\right) \tag{16}$$

And the instantaneous particle action of the two photons Eq.(9), is then:

$$\not{S} = \not{S}_1 + \not{S}_2 = m\left(\frac{v}{c}\gamma^k c_1 x_k + \gamma^0 c^2 t\right) \tag{17}$$

Proposed Structure of Electron

It is proposed that a pair of photons moving in opposite directions $\not{S}_1$ and $\not{S}_2$, as discussed above are, not moving along linear trajectories , but are engaged in such a way that there is a separation between the trajectories, and are orbiting in the $\gamma^1\gamma^2$ plane.



The Electron Systemfunction

It is proposed that the electron Systemfunction is composed of the action of two opposite going photons Eq.(15), and Eq.(16), and is:

$$\bar{\Theta}_E = A e^{-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]^2 / 2} \tag{18}$$

Explicitly the exponent is:

$$-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]^2 / 2 = \frac{-\left[\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + \left(\not{S}_1\not{S}_2 + \not{S}_2\not{S}_1\right) + I_3^2 \; + \; I_3\left(\not{S}_1 + \not{S}_2\right) + \left(\not{S}_1 + \not{S}_2\right)I_3\right]}{2} \tag{19}$$

For convenience his can be separated into three functions the first a real scalar Gaussian, and two vector rotors.

The Gaussian:

$$\rho = \exp - \frac{\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + \left(\not{S}_1 \not{S}_2 + \not{S}_2 \not{S}_1\right) + 1}{2} \tag{20}$$

The first square terms are the null action vectors that define the Gaussian locations of the individual spin one photons.

The second bracket is the commutator of the photon action. Since the product of two Lorentz null vectors is a Lorentz constant this is a constant or twice the dot product of the opposite going particles. Using Eq.(8), this is just non kinetic rest mass, and for spin one photons the value of this term is :

$$\frac{\left(S_1 S_2 + S_2 S_1\right)}{2} = \frac{2 S_2 \bullet S_1}{2} = m_1 m_2 c^2 \left[\lambda\!\!\!^-{}_1^{\,2} + \left(cT\right)^2 \right] = 2 m_1 m_2 c^2 \lambda\!\!\!^-{}_1 \lambda\!\!\!^-{}_2 \tag{21}$$

From Eq.(8), we can then find that the value of the scalar simplifies to:

$$\rho = \exp - \left(\frac{\not{S}_1^{\,2} + \not{S}_2^{\,2}}{2} + \frac{m_0^{\,2} c^2 \lambda\!\!\!^-{}_1^{\,2}}{2} + \frac{1}{2} \right) \tag{22}$$

The two defined rotors in Eq.(19), are:

$$R_{E1} = \exp\left[I_3 \left(\not{S}_1 + \not{S}_2 \right) / 2 \right] = \exp\left[I_3 \not{S} / 2 \right] \tag{23}$$

and:

$$R_{E2} = \exp\left[\left(\not{S}_1 + \not{S}_2 \right) I_3 / 2 \right] = \exp\left[\not{S} I_3 / 2 \right] \tag{24}$$

The product of these two rotors is not a null and thus the term is not Lorentz null invariant.

$$\bar{\Theta}_E = \rho R_{E1} R_{E2} \tag{25}$$

That is because R_{E1} and R_{E2}, are time reversals, and the product has only the three space vector momentum, not the time energy terms.

This is addressed by adding the initial condition of a π phase shift between $\not{S}_1$ and $\not{S}_2$. Eq.(25), then becomes:

$$\bar{\Theta}_E = \exp-\left[\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) I_3 / 2 \;+\; I_3\left(\not{S}_1 + \not{S}_2\right)/2 + \left(\not{S}_1 + \not{S}_2\right) I_3 / 2 + I\sigma^3 \frac{\pi}{2}\right] \qquad (26)$$

Which can be written [3].as

$$\bar{\Theta}_E = \left(\rho^{1/2} R_{E1}\right) I_3 \sigma^3 \left(\rho^{1/2} R_{E2}\right) \quad = \quad \left(\rho^{1/2} R_{E1}\right)\left(\rho^{1/2} \tilde{R}_{E2}\right) I_3 \sigma^3 \qquad (27)$$

This is Lorentz invariant. The physical meaning of the phase shift will become apparent in the physical description of the function

Electric Charge

For the electric charge we turn to the vector terms in the action and note that:

$$R_{E1} = e^{\left[I_3\left(\not{S}_1 + \not{S}_2\right)/2\right]}, \qquad (28)$$

or

$$R_{E1} = e^{\left[I_3 \gamma^3\left(\omega_1 t + \omega_1 t\right)\right]/2} \qquad (29)$$

Adding in the mechanical action of the particles revolving around the center of mass gives:

$$I_3 \gamma^3 \left(\omega_1 t + \omega_2 t + \left(\frac{m_1 c^2}{n\hbar}\right) t + \left(\frac{m_2 c^2}{n\hbar} t\right)\right) \qquad (30)$$

Where n is the number of wavelengths of the photons from the center of mass:

$$\rightarrow I_3\gamma^3\left(\omega_1 t+\left(\omega_2 t+\pi\right)+\left(\frac{m_1c^2}{n\hbar}\right)t+\left(\frac{m_2c^2t}{n\hbar}\right)\right), \tag{31}$$

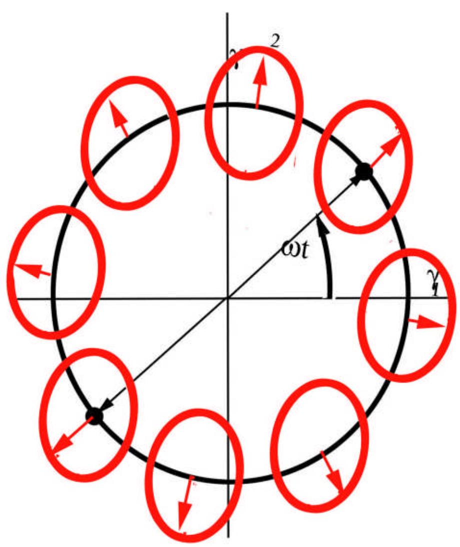
or:

$$\rightarrow I_3\gamma^3\left(\left(\omega_1 t-\omega_2 t\right)+\left(\frac{\left(m_1+m_2c\right)}{n\hbar}\right)ct\right) \tag{32}$$

or

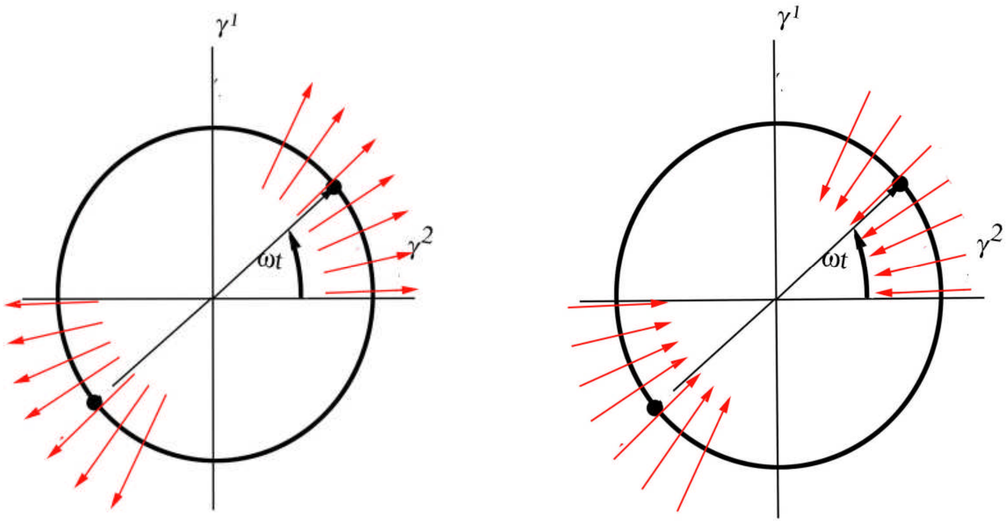
$$\rightarrow I_3\gamma^3\left(\omega_1 t-\omega_2 t+\omega_R t\right) \tag{33}$$

From the model Fig 1. the two photons are presumed to revolve about their center of mass, and noting from Eq.(15), and (16) that the photon internal rotation vectors are anti-aligned but in phase with the rotation phase, thus as the particles rotate around the center of mass the electric vector is always in a positive radial direction ie:



A first thought is that the vectors cancel and have are null, but since they are not co-located there is a Gaussian spatial relation. The vectors are only null at the center of mass.

Two photons orbiting in phase with the rotation can be either in phase or out with a phase difference of π in the rotational angle.



We can designate one of these as having the charge of an Electron and the other as a Positron.

These last configurations have electric vectors continuously positive or negative in the radial direction and a value of zero at the center of mass. The effect of one on the other is as two opposite charges and thus providing a radial binding force between the photons which is discussed in the next section.

Each of the photon rotations direction has the option of positive or negative.

Positron Systemfunction

The Positron Systemfunction is nearly identical to the electron accept for a space time reversal. From the definition of the Electron Systemfunction Eq.(18), a simple reflection of the spacetime of one of the factors gives the positron to be:

$$\bar{\Theta}_E = Ae^{-\left[\left(\not{S}_1+\not{S}_2\right)+I_3\right]\left[\left(\not{S}_1-\not{S}_2\right)-I_3\right]/2} \tag{34}$$

Note that the negative signs just reverse the time & space of $\not{S}_2$ and I_3 otherwise Eq. (18), and Eq.(34), are identical. The photon designations for the Positron of 1 and 2 are kept the same since they are equivalent.

Displaying the functions together illustrates the common terms.

$$\bar{\Theta}_E = \exp\frac{-\left[\left(\not{S}_1^{\,2}+\not{S}_2^{\,2}\right) + \left(\not{S}_1\not{S}_2+\not{S}_2\not{S}_1\right) + 1 + I_3\left(\not{S}_1+\not{S}_2\right) + \left(\not{S}_1+\not{S}_2\right)I_3 + I\sigma^3\frac{\pi}{2}\right]}{2}$$

$$\bar{\Theta}_P = \exp\frac{-\left[\left(\not{S}_1^{\,2}+\not{S}_2^{\,2}\right) - \left(\not{S}_1\not{S}_2+\not{S}_2\not{S}_1\right) - 1 + I_3\left(\not{S}_1-\not{S}_2\right) + \left(\not{S}_1-\not{S}_2\right)I_3 - I\sigma^3\frac{\pi}{2}\right]}{2}$$

(35)

For clarity: The first bracketed terms are the null light speed photons, the second is a rest mass associated with the binding, the 1 is a designation of the positive or negative mass, the third and fourth are the electric rotation vectors and the last are the phase relation between the photons

Noting that :

$$I_3\not{S}_2 - \not{S}_2 I_3 = 0 \tag{36}$$

The product an electron and a photon located at the same point is:

$$\bar{\Theta}_E\bar{\Theta}_P = \exp-\left(\not{S}_1^{\,2} + \not{S}_2^{\,2} + I_3\not{S}_1 + \not{S}_2 I_3\right) \tag{37}$$

Re-factoring shows the function to be the collocation 0f two opposite going spin-one photons;

$$\bar{\Theta}_{F1}\bar{\Theta}_{F2} = \exp\left(\not{S}_1^{\,2} + I_3\not{S}_1 + \not{S}_2^{\,2} + \not{S}_2 I_3\right) \tag{38}$$

This illustrates the photon electron annihilation process

Note that in Eq.(35), the sign of the sign of the commutator ant square of the I_3 term are the same and indicate positive or negative mass and the sign of the π phase term indicates the charge. Those terms cancel in annihilation.

$$\bar{\Theta}_E = \exp\frac{-\left[\left(\not{S}_1^{\,2}+\not{S}_2^{\,2}\right) + \left(\not{S}_1\not{S}_2+\not{S}_2\not{S}_1\right) + 1 + I_3\left(\not{S}_1+\not{S}_2\right) + \left(\not{S}_1+\not{S}_2\right)I_3 + I\sigma^3\frac{\pi}{2}\right]}{2}$$

(39)

Rest mass and binding energy

The invariant constant terms of the photon commutator can be recognized as a binding energy.

$$\left(\not{S}_1\not{S}_2+\not{S}_2\not{S}_1\right) \qquad (40)$$

If the photons are going in the same direction the dot product is zero.

In this development the interaction which is the providence of QFT is not included, but the value of this term must be inversely proportional to the distance to a positron, thus providing the binding energy between the particles. For Eq.(38), it is presumed that the particles are going in opposite directions, but co-located at the same point.

As the photons rotate the electric vectors are opposite and create an attractive binding force between the opposite photon. Since the force must be equivalent to the centrifugal force the magnitude of this force can be evaluated

$$F_c = \frac{m_1 v^2}{r} = \frac{m_1 c^2}{\bar{\lambda}_1} = \frac{m_1 c}{\bar{\lambda}_1 \hbar} c\hbar = \frac{c\hbar}{\bar{\lambda}_1^{\,2}} = \frac{1}{\alpha}\frac{Q^2}{r} \qquad (41)$$

This is equivalent to the force on a charge at the classical electron radius, or 137 times greater than a charge-charge binding force.

The external electric vector or charge vector is the differential of the photon electric vectors summed over all the possible action paths requires treated by QFT. The proposed path of the electron orbits used here is a classical path but should be the optimum path determined by QFT [6], and it should not be a surprise that the external electric vector is a factor of α less than the electric vector binding the photons.

Corresponding Dirac KG Differential Solutions

At the center of mass, the square of the action is zero or at least constant and thus ρ can be considered constant

$$\bar{\Theta}_{E} = \rho R_{E1} I\sigma^{3} R_{E2} \tag{42}$$

$$R_{E1} = \exp\left[I_{3} \not{S} / 2 \right] \tag{43}$$

and:

$$R_{E2} = \exp\left[\not{S} I_{3} / 2 \right] \tag{44}$$

Then the solution for the Dirac equivalent expression is [7]:

$$\not{\partial}\ \bar{\Theta} = \not{\partial}\ \rho R_{E1} I_{3} \sigma^{3} R_{E2} = \not{P} \bar{\Theta} \tag{45}$$

The magnitude of which is the rest mass.

Also note that:

$$\not{\partial} I_{3} R_{E1} + \not{\partial} R_{E2} I_{3} = \not{P}\left(R_{E1} + R_{E2} \right) \tag{46}$$

The second derivative is:

$$\not{\partial}\not{\partial}\ \bar{\Theta} = \not{\partial}\not{\partial}\ R_{E1} I\sigma^{3} R_{E2} = \not{P} \bullet \not{P}\ \bar{\Theta} = -m_{0}^{2}\ \bar{\Theta} \tag{47}$$

The function thus satisfies the GA equivalent of the Dirac and the Klein Gordon expression.

Spin

The spin of the particles is just the time derivative of the rotor, thus:

$$\frac{d}{dt}R_{E1} = \frac{d}{dt}\exp\left[I_3 \not{S} / 2\right] = \frac{1}{2}\hbar R_{E1}\sigma^3 \tag{48}$$

Identifiable as the proper spin for the electron moving along the γ^3 axis.

Neutrino

To illustrate the utility of the GA functional model of the electron, a plausible model for a neutrino can be constructed. Starting with the same function defining the electron, Eq.(18):

$$\bar{\Theta}_N = Ae^{-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]^2}, \tag{49}$$

but altering the action of the second photon $\not{S}_2$, such that it is traveling in the same direction as $\not{S}_1$ but with an opposite phase, and having an energy half the energy of $\not{S}_1$.

Expanding then:

$$\bar{\Theta}_N = \exp\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]^2 / 2 = \exp - \begin{bmatrix} \left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + 1 + I_3\sigma^3\dfrac{\pi}{2} \\ + I_3\left(\not{S}_1 + \not{S}_2\right) + \left(\not{S}_1 + \not{S}_2\right)I_3 \end{bmatrix} \tag{50}$$

With an opposite phase and $m_2 = m_1 / 2$ the Euler terms including a $\pi / 2$ phase shift then are:

$$\begin{aligned} & I_3\left[m_1\left(\gamma^k c_1 x_k + \gamma^0 c^2 t\right) - \frac{1}{2}m_1\left(\gamma^k c_1 x_k + \gamma^0 c^2 t\right)\right] \\ & + \left[m_1\left(\gamma^k c_1 x_k + \gamma^0 c^2 t\right) - \frac{1}{2}m_1\left(\gamma^k c_1 x_k \gamma^0 c^2 t\right)\right] I_3 + I_3\sigma^3\pi / 2 \end{aligned}, \tag{51}$$

Integrated the actions together gives:

$$I_3\left[\left(m_1-\frac{1}{2}m_1\right)\oint\left(\gamma^k c_1 dx_k+\gamma^0 c^2 dt\right)\right] \tag{52}$$

$$I_3\left[\frac{S_1}{2}\right]+\left[\frac{S_1}{2}\right]I_3+I_3\sigma^3 \tag{53}$$

The Systemfunction for the neutrino would then be:

$$\bar{\Theta}_N=\exp-\left[\left(\not{S}_1^{\,2}+\not{S}_2^{\,2}\right)\quad+\quad I_3\frac{\not{S}}{2}+\frac{\not{S}}{2}I_3\quad+\;1+\;I_3\sigma^3\frac{\pi}{2}\quad\right] \tag{54}$$

Thus this two light speed photons traveling together, producing an action that is half the spin of a single photon.

As the two particles travel together the function can be evaluated at the time and location of particle 1.that is $t\,,x_1=0$ so if:

$$\left(\frac{x_2}{\lambda_2}\right)^2-1\;=\;0 \tag{55}$$

Thus if particle 2 is at a distance of λ_2 from particle 1, the Gaussian has a single node and a value of one at the location of particle 1. Similarly if the evaluation is at particle 2, it is found that if particle 1 is at a distance of λ_1, there is a single node at particle 2 and the particle.

This gives two separate stable modes that bind the pair of photons into a single particle with a unit value Gaussian moving at c the same as a single photon. It could be suggested that the two modes are different flavors.

From the rotors, the interval between the positions of the two spin one photons, correlates to a $1/2\hbar$ total spin for the angular momentum.

As with the Positron shown above, the square of the pseudoscalar could be a reverse and thus produce a negative neutrino with a -1 with negative spin.

Conclusion

This paper presents a plausible complete GA model of photons, electrons, positrons, and the electron-positron pair annihilation process. Also presented is a plausible neutrino model, as well as a structure for other particles. The purpose is to provide to more mechanical view of QM, which with the incorporation of QFT could lead to a solution of the mass ratio problem.

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