

*Panta rei
conditio sine
qua non est*



Panta Rei: The Evolution Equation

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Abstract

It seems possible to suggest the [evolution equation in cosmology](#), which permits unlimited *creatio ex nihilo* from the quantum vacuum, without leading to catastrophic events. The *physicalized* presentation of the Universe is interpreted as ‘Brain of the Universe’, which gives access to ‘[God’s thoughts](#)’ (Albert Einstein) with spacetime engineering performed by the [human brain](#) embedded in the Brain of the Universe.

1. Introduction

The idea (*noêma*) of ‘nothing’ means ‘something that has no inherent properties’, such as an [empty set](#) (if any). You can’t get something from nothing. In Latin, *ex nihilo nihil fit*, or ‘out of nothing, nothing becomes’.

Well, it depends on what we mean by ‘nothing’ (if any). For example, if we look at a flat line, we can say that, obviously, there are no waves in it, although we know that waves can cancel each other completely due to [destructive interference](#), leading to a flat line. Taking this example further, imagine that back in 19th century, long before [Max Planck](#) was born, some philosopher tried to relate the concept of ‘nothingness’ with the example of a flat line that contains no waves whatsoever: his argument will be logically correct, as even today people strive to explain (not define) the concept of ‘nothingness’ as ‘*something that is not there*’, like an [empty set](#) (if any). He may even try to speculate that the ancient ideas of ‘atom’ and ‘point’ (“that which has no part”, [Euclid](#)) may be related to this kind of ‘nothingness’ or ‘[vacuum](#)’. I believe nobody from the established scientific community in 19th century would have paid attention to such metaphysical exercise, yet it might have helped in our understanding of the quantum vacuum² and its [zero-point energy](#).

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I would like to offer a similar metaphysical exercise (Path II), based on a new *relativistic vacuum* (Fig. 3), and will try to explain a new *evolution equation*. I will have to avoid the generic case of ‘zero’ as The Noumenon¹, which is **not explicitly** present in Fig. 1, because it cannot be a *set* in principle, not even an “empty” one. The equation (Sec. 3) presumes specific coupling of matter (*res extensa*) to its potential states (*res potentia*)¹, and offers conceptual solutions to many problems in our understanding of cosmology, gravity, and the alleged “dark energy”⁶. How was the Universe created? And why is it **larger than a football**?

Take a closer look at *res potentia*¹ known as ‘God’s thoughts’ (Albert Einstein), facilitated by the quantum vacuum² (Slide 13¹). To quote Sir Arthur Eddington³,

A star is drawing on some vast reservoir of energy by means unknown to us. This reservoir can scarcely be other than the subatomic energy which, it is known exists abundantly in all matter; we sometimes dream that man will one day learn how to release it and use it for his service. The store is well-nigh inexhaustible, if only it could be tapped. (...) If, indeed, the sub-atomic energy in the stars is being freely used to maintain their great furnaces, it seems to bring a little nearer to fulfillment our dream of controlling this latent power for the well-being of the human race – or for its **suicide**.

I will argue that the inexhaustible “reservoir of energy” is related to gravity⁸ as well, because the genuine gravitational energy is not *directly* observable, much like the genuine ‘quantum state’, as stressed by Erwin Schrödinger in 1935¹. In a nutshell, the conservation of energy, including the input from gravity, is perpetually violated⁸ in the physical world, yet it is *always* conserved in the Platonic world of *res potentia*¹: have our cake and eat it. How could this be possible? With a new evolution equation (Sec. 3)¹. The initial idea comes from Plato, with some minor modifications (Fig. 4), such as ‘chained Eskimos’ (Slide 14¹).

Now let me briefly mention two approaches to cosmology, dubbed Path I and Path II. Consider the topological dimensions of 4D spacetime: if we look at a **clock**, we can always pinpoint an instant of the **cosmic time**, and if we look along any direction in 3D space, we can see as far as we like⁴. Yet if we apply our current mathematical models¹⁵ to The Beginning of spacetime (Path I), we will hit an insurmountable problem: “Long time ago, there was a brief period of time during which there was still no time at all” (Yakov Zeldovich, private communication, 1986; translation mine). With Path I, we inevitably hit some “very special state”⁵ of the universe, which was perfectly smooth and gravity was still (Sic!) absent, and prior to such “very special” proto-state, there was “no time at all.” One would need some Biblical “miracle” to reproduce the world from “no time at all.”

We believe that Path I, despite being based on mathematical models, is not acceptable. Thus, we will pursue Path II by suggesting a phenomenological theory of spacetime, which is free from any problems and inadmissible errors, Biblical “miracles” included. Our goal is to suggest *conceptual* solutions to *conceptual* problems, such as “the worst theoretical prediction in the history of physics!”⁶. On the flip side, Path II still lacks mathematical description, firstly because the so-called hyperimaginary numbers¹ are not yet unraveled.

2. Path II: Vacuum Energy

There is something truly peculiar about the vacuum²: we can observe only its energy **differences**⁷ (Fig. 5). If we could gain access to the *complex* phase of quantum waves and tweak their **destructive interference** leading to “vacuum”, perhaps we could evoke real

physical stuff⁸ to emerge at macroscopic level as ‘free lunch’, like *creatio ex nihilo*. But of course, we need [quantum gravity](#) in the first place, to eventually fulfill “our dream of controlling this latent power for the well-being of the human race – or for its [suicide](#)”³.

The point here is that we can *never* observe the vacuum itself, so the expression ‘vacuum energy’ is **false**. To explain the puzzle, I suggested in September 2000 the parable of John’s jackets.

Suppose you chase somebody on the street (let’s call him John), and any time you catch him, he leaves his **jacket** in your hands. You can’t catch John himself. Only his jacket. You believe that John has a set (or is it strictly a set?) of physical jackets with different probabilities for catching, and you deeply believe that this set can be normalized, i.e., the sum of probabilities for catching his jackets is unity. Yet John does not wear *any* jacket by default – neither before nor after you catch his current jacket (Schrödinger, Slide 6¹). John is simply the [Platonic Idea](#) and ‘the true monad without windows’ (Leibniz, Slide 13¹).

The parable of John’s jackets applies to gravity⁸ as well – we certainly observe various gravitational ‘jackets’ in the right-hand side of Einstein’s field equations, despite the fact that there is no gravitational “spring or sink for matter energy-momentum anywhere in spacetime”⁹: if we try to present John *himself* with a **tensor**, as we do it for matter and fields in [classical physics](#), we have to admit that there is no gravitational stress-energy **tensor**¹⁰ to describe [John-the-Gravity](#). We can only observe his *physicalized* ‘jackets’, say, from “positive energy density of about 6×10^{-10} joules per cubic meter”⁷ to 8.8×10^{47} joules (app. 4.9 times the sun’s mass turned to energy), in the case of [GRB 080916C](#).

To cut the long story short, in our theory of quantum gravity we offer a common ‘John’ (*res potentia*) for all quantum-gravitational ‘jackets’ (*res extensa*), stressing that ‘John’ cannot be **physically** observed due to the “speed” of light (**A2** in Slide 19¹). If people insist on modeling ‘John’ as some *physical* stuff, they will immediately hit “the worst theoretical prediction in the history of physics!”⁶. To explain why, let me offer a simple explanation, starting with the opposite case in which ‘John’ did not exist, only his ‘jackets’.

Suppose that you have €1000 in your bank account, and decide to withdraw €80 from it. You go to some cash machine on the street, insert your debit card, dial your password, and get your €80: the total amount of your €1000 remains conserved; you just have €80 less in your bank account, matching the same €80 in your wallet. All your money and those in the bank are *physical* stuff. Also, you can’t withdraw more than €1000 with your debit card, and the total amount of money in the bank is, say, €1.000.000.000. Simple and clear.

Now, suppose your money in the bank (not in your wallet) *and* bank’s money are ‘John’s jackets’ (**Res potentia**, Slide 13¹), and the requirements for withdrawing *physical* money (physical ‘jackets’) from your bank are that (i) you must possess the initial physical ‘quantum of money’ (similar to ‘one drop of petrol’⁶) in your wallet, which is one cent, and (ii) you can withdraw only ‘money differences’ ([Fig. 5](#)), akin to energy differences⁷. This case is totally different from the one above, because now you can withdraw **indefinite** amount of *physicalized* money, provided that the latter has *finite* value, neither “zero” nor “infinite”. It doesn’t matter if you withdraw €80 or crack the lottery jackpot of €80M.

Notice that there can be no conservation of *physical* money, because your money in the bank (not in your wallet) *and* bank’s money are **indefinable**, just like the “total amount” of “[vacuum energy](#)”. Thus, you may withdraw a colossal amount of *physicalized* money, say, €1B (similar to 8.8×10^{47} joules from GRBs in the example [above](#)), provided that you

have the initial *infinitesimal* ‘quantum of money’ in your wallet. Even more: you may create a *physicalized* universe of ‘money’ with what some (otherwise smart) people call “inflation” (Slide 12¹). There can be no “violation” of the “initial amount” of money, simply because one cannot violate something that does **not** exist. Simple and clear.

The big puzzle, however, is the initial physical ‘quantum of energy’ in *cosmology*, which should coincide with The Beginning¹³ (Fig. 12). We associate the ‘quantum of energy’ with the *elementary transition* dt (Fig. 1) of the **self-acting physicalized universe**¹³ along the Arrow of Space (p. 10 in Hyperimaginary Numbers¹), from any given instant/frame to the **next** one (Fig. 4). The **elementary transition dt** equates to **work**, and we expect that the ‘quantum of energy’ of the ‘atom of geometry’ has an *infinitesimal* value, many orders of magnitude smaller than “positive energy density of about 6×10^{-10} joules per cubic meter”⁷.

But what is ‘negative energy density’? It is John’s jackets with respect to **Res extensa** (Slide 13¹) viz. the “nose” (Slide 14¹) made of *positive* energy density, which brings us to the *evolution equation* and the huge bundle of unsolved challenges related to the three types of mass – positive, negative, and imaginary (see p. 7 in Hyperimaginary Numbers¹).

3. The Evolution Equation

The evolution equation, proposed previously¹, reads

$$|\mathbf{w}|^2 = |\mathbf{m}|^2 + |\mathbf{m}_i|^2 \quad (\text{Eq. 1}).$$

It is a symbolic equation (see Path II [above](#)) about two *atemporal* offer and confirmation waves, producing the **elementary transition dt** , $AB = dt$, depicted in Fig. 1 below.

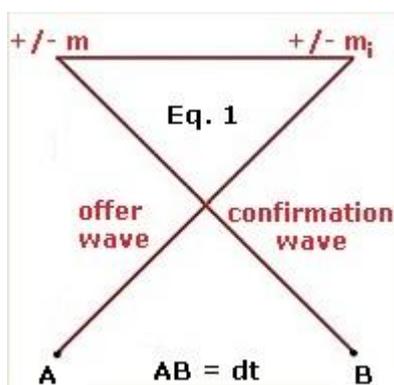


Fig. 1

See Fig. 2, p. 5 in Hyperimaginary Numbers¹ and John C. Polkinghorne [below](#).

There is no **natural measure** along null intervals¹⁹ in Fig. 1, and the **proper time** of the offer and confirmation waves with *hypercomplex* phases and amplitudes ($+/- \mathbf{m}$ and $+/- \mathbf{m}_i$) will be “frozen” or “stand still”¹¹ to all inanimate clocks (*not* to the **human brain**).

The term $|\mathbf{m}|^2$ presents the real (positive and negative) mass produced at B “after” the confirmation wave, whereas $|\mathbf{m}_i|^2$ shows the **imaginary mass**. The **prototype** of Eq. 1 is

$$0 = (+1) + (-1) \quad (\text{Eq. 2}).$$

Say, $0 = 3/3 - 5/5$ or $0 = 9/9 - 25/25 = 1 - 1$. Notice that $(+/-3)^2$ or $|3|^2 = 9$ and $(+/-5)^2$ or $|5|^2 = 25$. We postulate that the real and imaginary terms in the right-hand side of Eq. 1 belong to two entirely different worlds¹¹, and that the **ratio** of their amplitudes (Fig. 2) is always equal to unity, e.g., $9/9 (+/- m) = 25/25 (+/- m_i)$.

Suppose that at t_1 we have $0 = 9/9 - 9/9$ (Eq. 2), and later at t_2 the imaginary term has increased, for whatever reason, to $25/25$. Now there is **more** negative mass from **squared** imaginary mass $|m_i|^2$ to feed (Sic!) the negative mass in $|m|^2$ (Eq. 1): $|w|^2 = |5|^2 + |5_i|^2$, and we will have **more physicalized** or “positive” mass – $|5|^2 > |3|^2$.

It’s all in the **phase** (Fig. 2). We can also produce the so-called “inflation” (Slide 12¹) and no “violation” of mass-energy “conservation” can occur, ever.

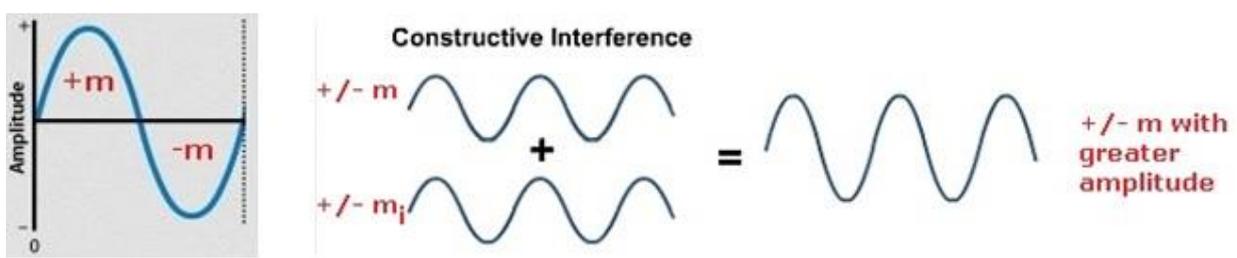


Fig. 2

The evolution equation works in the opposite way (destructive interference) as well: if at t_1 we have $0 = 9/9 - 9/9$, and later at t_2 the imaginary term has decreased to $4/4$, there will be **less** negative mass from **squared** imaginary mass $|m_i|^2$ to feed (Sic!) the negative mass in $|m|^2$, and the **physicalized** or “positive” mass-energy will decrease – $0 = 4/4 - 4/4$ (Eq. 2) or $|w|^2 = |2|^2 + |2_i|^2$ (Eq. 1). Again, it’s all in the **phase**, and no “violation” of mass-energy “conservation” can occur. Hence we can think about gravitational radiation⁸ and perhaps try to reproduce it with **spacetime engineering** (Fig. 8). **Mark my words.**

As of today, however, Eq. 1 and Fig. 7 are not clear, in addition to the condition $|w|^2 = 0$, where **w** involves the so-called hyperimaginary unit¹. We claim that, relative to the platform, time on the train **completely stops** and is “stand still”¹¹, which means that the train has entered the **atemporal** realm of **Res potentia** (Slide 13¹) along $+/- w$. This is a new **relativistic vacuum**, which is hidden by the “speed” of light (A2 in Slide 19¹). You cannot look twice at the same river (Heraclitus). *Panta rei conditio sine qua non est.*

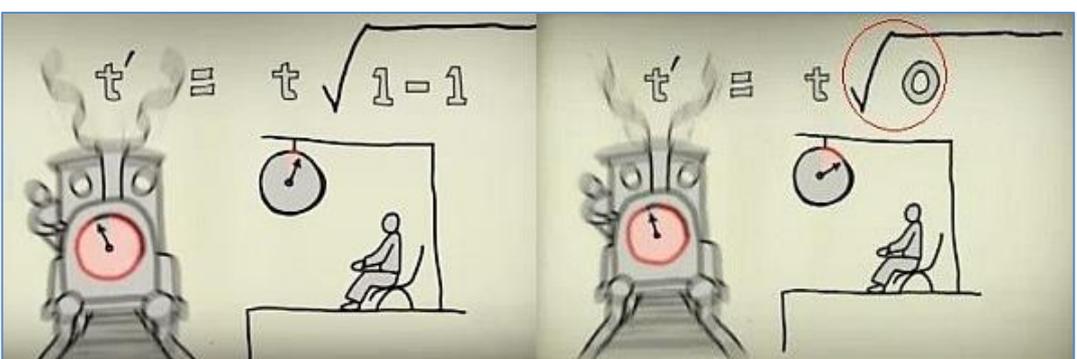


Fig. 3

We are like chained Eskimos (Slide 15¹) and the “speed” of light (A2 Slide 19¹) does not allow us to ‘turn around’ and see the Platonic world (Fig. 4) “inside” dt (Fig. 1).

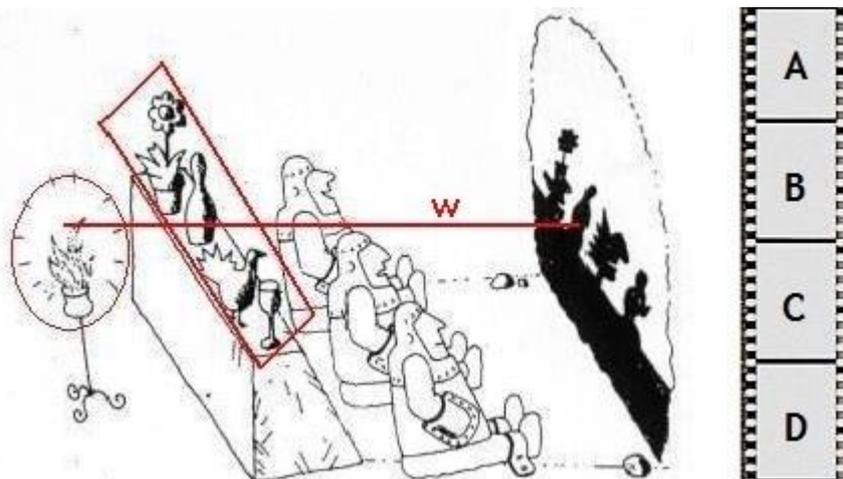


Fig. 4

To make the *real line*/film reel *perfectly* smooth (see Fig. 7 in Hyperimaginary Numbers¹) or “infinitely differentiable”, and speculate that every point/frame from it corresponds to a ‘number’, the current math textbooks offer two and only two alternatives: the dark strip separating any neighboring points/frames (Fig. 4) is either (i) “zero” or (ii) non-zero. Case (i) leads to only one point/frame, and no change in time is possible. Bad idea. Case (ii) will insert a finite, non-zero gap]between[all points/frames. Bad idea, too.

The only possible solution to the fundamental *flow of events* $A \neq B \neq C \neq D, \dots$ (Fig. 4) is combination of (i) and (ii), meaning that every 4D event ‘here and now’, pertaining to the physical world (*Res extensa*, Slide 13¹), must pass through a **gap** “during” which there is **no spacetime at all**¹³ – the **dark gap** is **not** an event – so at the *next* ‘tick of time’ dt , the *next* 4D ‘here and now’ can and will be **different**: the *flow of events* requires *change*. Thus, we suggest to place the horizontal **dark gaps** in Fig. 4 along the hyperimaginary axis W^1 erected on null spacetime distances¹⁹, and to treat W as **non-event** – The Beginning¹³ (Fig. 12) is eternally residing “inside” us (John 1:1; Luke 17:21).

Let me reiterate that we introduce (Slide 13¹) fundamental *flow of events* (“you cannot look twice at the same river”, *Heraclitus*), as a result of which the atom of geometry (“that which has no part”, Euclid) is endowed with *internal structure* (Fig. 1): check out Sec. 2 and Fig. 7 in Hyperimaginary Numbers¹, and A2 in Slide 19¹. Many “intuitively clear” axioms used to construct the *topological manifold* and then the differentiable or “smooth” manifold need painstaking revision, starting with the “intuitively clear” axiom of *mapping numbers to points*: the hyperimaginary numbers¹ cannot be mapped to ‘points’ from a line; only their *physicalized* “jackets” can cast their *physicalized* 4D footprints on the points from the *number line*, as they belong to the *irreversible* past (Slide 13¹). Recall Plato’s ‘*allegory of the cave*’: the world is not just what we can see (Fig. 4).

We are ‘Eskimos’ (Slide 14¹) and need new Mathematics to present three ontologically different ‘elements of reality’ with the so-called hyperimaginary numbers (hi-numbers, in addition to *q-numbers* and *c-numbers*): (i) points mapped to *numbers*, as in *classical*

physics, (ii) points mapped to ‘John’s jackets’ with hyperimaginary numbers, and (iii) John (not his ‘jackets’) emerging from the **Noumenon** viewed as non-reality. I hope that the new hyperimaginary numbers will be revealed by **Christmas 2018**, at the latest.

4. Questions and Answers

Q1: What do you mean by “increased” and “decreased” stuff?

A1: Right, there is no metric in the Platonic realm of hyperimaginary waves (Fig. 2). Think about the *idea* of a tree and the *idea* of a mountain: there is no metric in the human memory, yet the *idea* of a tree corresponds to lighter physical object, compared to a mountain. Likewise with $|m|^2$ and $|m_i|^2$: you operate with Platonic objects as well, and should be able, for example, to reduce the weight of your body (switch from ‘mountain’ to ‘tree’) and even cancel it for a few minutes, in order to fly in the air (REIM). Many people can fly, but most of them unfortunately prefer to present it as some “magic”, for profit.

Q2: I don’t understand your “waves”. What are they?

A2: Two hyperimaginary waves, corresponding to two *potential* (**Res potentia** in Slide 13¹) mirror worlds¹¹. At every 4D instant ‘here and now’ in the physical universe (A2 in Slide 19¹) made exclusively by positive mass-energy, the offer and confirmation waves (Fig. 1) have *already* “squared” their amplitudes, yielding positive mass-energy, $|m|^2$ in Eq. 1.

Q3: What do you mean by ‘quantum of energy’? Is it related to Planck constant?

A3: I can only try to answer your first question. By ‘quantum of energy’ I mean the minimal “push” by the **self-acting physicalized** universe¹³: see ref. [9] in Hyperimaginary Numbers¹. As Banesh Hoffmann suggested in 1964, “If the universe is such that negative-mass particles can, on balance, “escape to infinity” (Sic! - D.C.) there will be an effect of continual creation of positive energy in the observed region” (pp. 95-96). Even in 1920, Sir Arthur S. Eddington spoke about ‘etheral energy’ and explained that “though ether waves are not usually classed as material, they have the chief mechanical properties of matter – viz., mass and momentum” (p. 345). Thus, the “creation field” in Eq. 1 is *always* producing gravitational radiation⁸ ($|m|^2$ in Eq. 1), but because Sir Arthur could not trace it to some physical process known in 1920, he opted for ‘ether waves’ and ‘etheral energy’. Nowadays we can interpret Eq. 1 as quantum-gravitational “creation field” emerging from some kind of hyperimaginary plasma composed of positive and negative *propensities* (cf. A1 and A2 above), which supposedly fluctuate² about their mean values of **zero** (Eq. 2).

The ‘quantum of energy’ must have some finite, albeit extremely small, positive energy, because it cannot be dead zero: *ex nihilo nihil fit*. As an analogy, recall that we widely speculate about some minimal **Planck length**, app. $1.6 \times 10^{-35}m$, which may be interpreted here as the *infinitesimal* ‘quantum of length’, although we cannot reproduce 1m with $1.6 \times 10^{-35} \times 1.6 \times 10^{35} = 1$. Ditto to the buildup of $|m|^2$ in Eq. 1.

Example: proton’s mass¹⁴ shown in Fig. 5 with **AB** = 938 Mev; the *sliding* cutoff **Z** stands for “zero” in Eq. 2.



Fig. 5

Now comes some advanced math: $(B - Z) - (A - Z) = AB \pm 0 = 938 \text{ Mev}$ “with precision of one part to 10^{45} ”¹⁴ (Slide 10¹) – “one of the greatest mysteries of Nature”¹⁴. We cannot speculate that the error margin here matches the *infinitesimal* ‘quantum of energy’, which in the case of proton’s mass is effectively “zero”² or 10^{-45} . As to your second question – sorry, I don’t know the origin of Quantum Inequalities (QIs)¹².

Let me go back to the genuine continuum endowed with **non**-events (Fig. 4).

5. The Continuum

In my opinion, the so-called “smooth”¹⁵ or “infinitely differentiable” manifold (C^∞) is a **joke**. Why? Because we *all* know, very well indeed, that neither option (i) nor option (ii) in Fig. 4 could work – people acknowledge the fundamental problem of the *continuum* even in textbooks²². To understand the genuine continuum made by ‘atoms of geometry’ dt (Fig. 1), consider the causal horizon of spacetime – the “boundary”²¹ for *causality* – denoted with **AB** in the drawing at [this http URL](#): how “large” is the dark strip dt in Fig. 4 viz. Fig. 1 ?

It cannot be dead zero, due to **Planck’s constant**. So if we instruct **A** to tend *asymptotically* (potential infinity) toward **Z** in Fig. 5, then $AZ = dt$ in Fig. 4. Likewise if we instruct **B** to tend *asymptotically* (potential infinity) toward **Z** in Fig. 6 below, $BZ = dt$ in Fig. 4 as well.

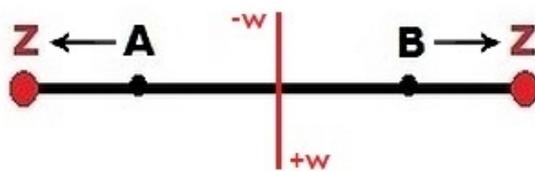


Fig. 6

We can picture the atom of geometry dt as $[dt (...) dt]$, as shown in the drawing at [this http URL](#). In Fig. 6 and Fig. 7 below, **Z** denotes The Beginning & The End (John 1:1; Luke 17:21) at the intersection **Z** of $-w$ and $+w$ (shown in Fig. 9.2). The two hypercomplex waves $-w$ and $+w$ pertain to the two mirroring worlds¹¹ in Eq. 1, and are erected along **W** on null spacetime distances¹⁹ at every physical point dt (Fig. 1) in Fig. 6.

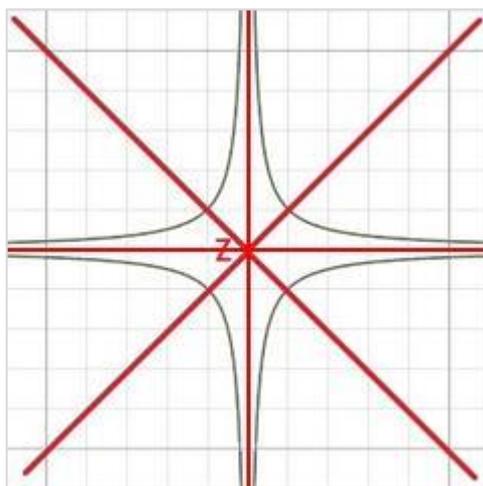


Fig. 7

Imagine the universal “drummer”¹⁸ as a water lily with four leaves having two modes: *open* (Fig. 7) and *closed* leading to two **squared** terms in the right-hand side of Eq. 1. This *atemporal* phenomenon is also known as the “breathing” of the Universe: Inhaling (open leaves, Fig. 7) and Exhaling (closed leaves, Fig. 1). The latter leads to **squared** parameters in the **invariant spacetime interval**, including **squared** “speed” of light c^2 , which makes it impossible *in principle* to detect any *physical aether* endowed with the fundamental **asymmetry** of the Heraclitean flow of events (Fig. 4) and Aristotle’s Unmoved Mover¹³ at absolute rest. Which is why the genuine **cosmic time** is *perfectly hidden*, including in GR.

For example, we face two *equally* important ‘**components**’ in the definition of causality, and we can only label one of them with “future pointing” and the other with “past pointing” to present the obvious “time-orientability” of **causality**. See also ref. [13] and p. 19¹: we have spatial inside-out symmetry (Fig. 8 and Fig. 9.2), exactly like the temporal symmetry in **causality** – the *physicalized 3D space* is **squared** (Eq. 1) as well.

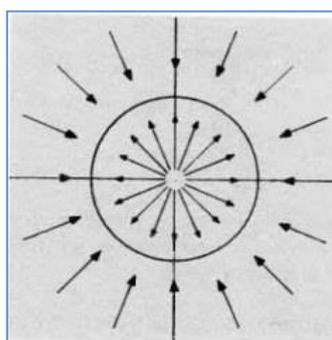


Fig. 8

The omnidirectional “force”, producing the *flow of events* (Fig. 4) in a “moving universe” (cf. Dennis Sciama and the note on p. 89 in [gravity.pdf](#)), has **topological** origin, resulting from “inflated” (outward-pointing) and “contracted” (inward-pointing) **3D balloon** turned “inside-out” (Fig. 9.2) with respect to the circle in Fig. 8 above, depicted with [AB] in Fig. 6 and with the “running guys” in Fig. 12.

It is hard to overestimate the fact that the **human brain** (Slide 11¹) can contact the **non-squared, atemporal global mode of spacetime**: if we place ourselves along the circle in Fig. 8 viz. along **-w/+w** in Fig. 6, we can “look” simultaneously along **-t/+t**, like the ancient god **Janus**, and will “see” all points in **3D space** as well, including the inner structure of solid objects, e.g., “all six sides of an opaque box **simultaneously**, and in fact, what is inside the box at the same time, just as we can see the interior of a square on a piece of paper” (Wikipedia). Physically, such *atemporal* approach to the **intact** quantum world (Slide 14¹), located “inside” **dt** (Fig. 1), is banned by **energy conditions** and QIs¹²: the **propellantless propulsion** might be achieved only with **REIM** and **BAVER**.

The hyperimaginary numbers, needed to define the brand new “**phase space**” (Fig. 7) of **Res potentia** (Slide 13¹), are expected to be derived from **4D sphere** \Leftrightarrow saddle transitions (Fig. 9.1) passing through God (Fig. 9.2) “during” every infinitesimal instant **dt** (Fig. 1).

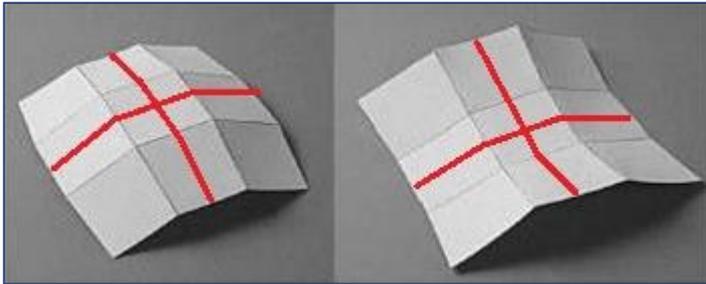


Fig. 9.1

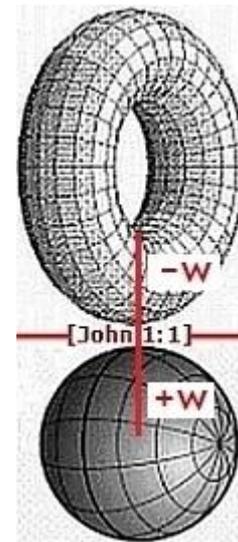


Fig. 9.2

The horizontal line in Fig. 9.2 (i) marks the sphere \Leftrightarrow saddle (Fig. 9.1) transition at the instant at which the [hypersphere and torus](#) are inflated exactly to *completed* infinity (“a totality of things which exists all at once”, [David Hilbert](#)), and (ii) produces 3D “slice” (Sic!) of the sphere \Leftrightarrow saddle transition, such that the 3D “slice” tends *asymptotically* toward the horizontal line in Fig. 9.2 along $\pm w$, from “south” ([hypersphere](#) in Fig. 9.2 and in [Fig. 13](#)) and from “north” ([hypertorus](#)). Hence the *physicalized*, asymptotically flat 4D spacetime is the arena at which the hyperimaginary sphere and torus “clash” into each other, like two waves ([Fig. 2](#)), leading to their **cancellation** in the physical world at dt ([Fig. 1](#)) and explication of only one *physicalized* ‘jacket’ from them – one **re-created** 4D ‘jacket’ at a time, as read with a **clock**. There is no need for “[tangent vectors](#)” and “[curvature of spacetime](#)” (Sic!) to model [gravity](#): the *physicalized* clocks and rulers are only ‘jackets’ (p. 14 in [spacetime.pdf](#), and p. 77 and p. 90 in [gravity.pdf](#)), and they can slow down or speed up viz. shrink or expand (ref. [63] on p. 41 in [rs_spacetime.pdf](#)).

To practice spacetime engineering ([A1](#) and Sec. 4 in [Hyperimaginary Numbers¹](#)), keep in mind that the only way to “predict” the future is to create it, for the future is not fixed but *flexible* (Slide [8¹](#)) and open to brand new events, including ‘the unknown unknown’. Then in addition to working as [Janus](#) ([Fig. 8](#)), you’ll have to overcome at least two challenges from the [Brain of the Universe](#).

Firstly, use *only* the Law of Reversed Effort: “To the mind that is still, the whole universe surrenders” ([Lao Tzu](#)). Allow your preferred state in the **future** to *unfold* toward you. If you choose to apply your free will and volition in the opposite “direction”, you will inevitably block the Law of Reversed Effort and will move toward the dead end of [parapsychology](#). And secondly, design and build your preferred state in the **future** *only* with the Law of Reversed Effort. Metaphorically, you have to swing – effortlessly – the flexible “carrot” ([Fig. 10](#)) toward your desired destination, and the “donkey” will carry you there (not effortlessly, but this is not your problem).



Fig. 10

The second non-action does not require physical work either: it's all in the **phase** (Fig. 2). The “carrot”, however, is a *very* tricky “steering wheel”, and you may need years of hard work to learn how to handle the “breathing” of the Universe (Fig. 7) and its infinitesimal **quantum of energy** to produce scalable ‘**brain-aided vacuum energy** release’ (BAVER).

How do you know whether you have learned to practice spacetime engineering with the Law of Reversed Effort? Your subjective passage of time will slow down for a few seconds, and you will be able to “**see**” yourself and your *potential* state with astonishing clarity, as if you watch a video clip in **slow motion**. As **Michael Flaherty** explained, “high levels of concentration and meditation can influence the subjective passage of time. Various athletes, for example, perceive time to pass slowly when they are “in the zone.” Yet people who are adept at meditation can produce comparable effects”.

Well, I was never able to learn any **meditation** or “**magic**”, only spacetime engineering (Sec. 4 in *Hyperimaginary Numbers*¹). In my opinion, **parapsychology** relates to spacetime engineering like astrology to astronomy.

Most importantly, spacetime engineering cannot be misused, like the suicidal (**Sir Arthur Eddington**) **nuclear energy** – not only because it is based on God as Love (1 John 4:8), but also because if you are *entangled* with people, you cannot hurt them without hurting you as well. So you are either *disentangled* from people and can do whatever you want (say, **flip your glasses**), or you are *entangled* with them and cannot even *think* of acting against them – not because you are some super ethical guy with super high moral standards, but because the “boomerang” from your intended action will hit you as well, at the same instant dt (Fig. 1) you decide to hurt them.

This is utterly important issue based on the fundamental phenomenon called *entanglement* (*Verschränkung*) by **Erwin Schrödinger** – “*the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.*” If we try to use concepts derived from the inanimate world at the length scale of tables and chairs, we cannot even imagine the *quantum* nature of entanglement, firstly because our imagination will require some **additional** stuff to “entangle” two or more objects, say, additional **bridges** connecting river’s banks or cables connecting **computers**. It just **doesn’t work**.

The quantum “**waves**” (Fig. 11) are not *physical* stuff (Slide 6¹), just as there are no “computers” and “cables” in your **brain** to **EPR-like correlate** your **glia** and 100+ trillion **synapses** (p. 2 in *HBP.pdf*), so that you can read these lines.

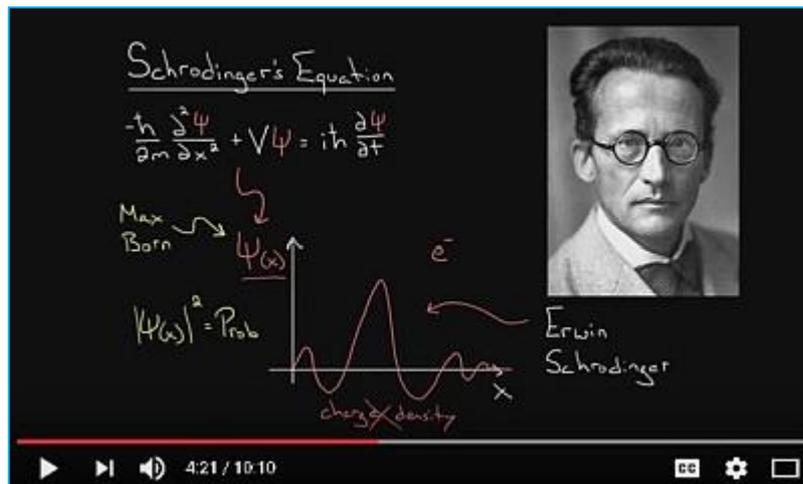


Fig. 11

I can offer very specific facts about the *entanglement* of ‘John’s jackets’, which boil down to the fact that the *cosmic time* itself is **not directly observable**: the fundamental *flow of events* $A \neq B \neq C \neq D, \dots$ (Fig. 4) is made *exclusively* by EPR-like correlated ‘jackets’. As John C. Polkinghorne explained²⁰:

It is as if a singer at 1 was singing a random series of notes and a singer at 2 was also singing a random series of notes and only if one were able to hear them **both together** would one realize that the two singers were in some kind of harmony with each other.

This *harmony* is Leibniz’ *harmonia praestabilita* (Slide 14¹), and the *atemporal* ‘common cause’ (Reichenbach’s *Common Cause Principle*) is John himself, located in the *global mode of spacetime*. Alternatively, if we abandon the Heraclitean *flow of events* (Fig. 4), the present instant ‘here and now’ (e.g., G.F.R. Ellis) would be some “*moving spotlight*” and we would have to *explain* the obvious time-orientability and *causality* by reducing it to some “irreversible” *physical process* that moves the “spotlight”, just like we *explain* ‘temperature’ by reducing it to *kinetic energy*, and trace the origin of time-orientability and *causality* back to the alleged “big bang”⁵. Forget it²¹.

Needless to say, I am by no means satisfied with the *evolution equation*. It might look a bit more “substantial” than the *symbolic Einstein’s equation*, but it is still a symbolic equation (Path II) and cannot be used for calculating proton’s mass¹⁴ (Slide 10¹) or the “dark” effects of quantum-gravitational vacuum⁶. At this moment, I can only argue that what was called here ‘*quantum of energy*’ is related to *work* (see *above*), referring to the **self-acting human brain** – check out the experiment on p. 2 in *Hyperimaginary Numbers*¹ and those in Slide 11¹. If the *physicalized universe* ($|m|^2$ in Eq. 1) is designed as the Brain of the Universe, it should possess **self-acting** faculty¹³ as well, and therefore could act on itself to produce the elementary ‘*tick of time*’ dt (Fig. 1) matching the *quantum of energy*.

One major corollary is that if the *human brain* is indeed part and parcel of the Brain of the Universe, we should be able to access (Fig. 8) the *atemporal quantum vacuum*² and practice *spacetime engineering* – effortlessly, because it’s all in the **phase** (Fig. 2). Check out the story about the yellow button on p. 15 in *Hyperimaginary Numbers*¹. The ‘yellow button’ is not made by “magic” but by exploring the quantum spacetime¹. But how can we explore the spacetime?

In addition to the *dynamics* of spacetime (Fig. 4), we need to know its *structure*, which brings us to the suggestion above that the spacetime is “squared”, as shown in Eq. 1.

6. The internal structure of spacetime points

The trivial answer to the question of why the spacetime (not the Finsler space) looks “squared” is that the parameters in the invariant spacetime interval show up squared after the Pythagorean Theorem. Here we offer an entirely different hypothesis about the *atemporal*, non-squared origin of the *physicalized* spacetime (Fig. 14) and the *internal structure* of spacetime points or ‘atoms of geometry’ (Fig. 7): Janus (Fig. 8) is depicted with the *atemporal* red axis along $-w/+w$ in Fig. 6 and Fig. 9.2. Let me try to explain.

Imagine AB in Fig. 6 as the diameter of 3D balloon shown with the circle in Fig. 8: Janus will occupy the circle and will *simultaneously* see the shrinking 3D balloon along all inward-pointing directions, and the inflating (inside-out, Fig. 9.2) 3D balloon along all outward-pointing directions in Fig. 8, like the running guys in Fig. 12.

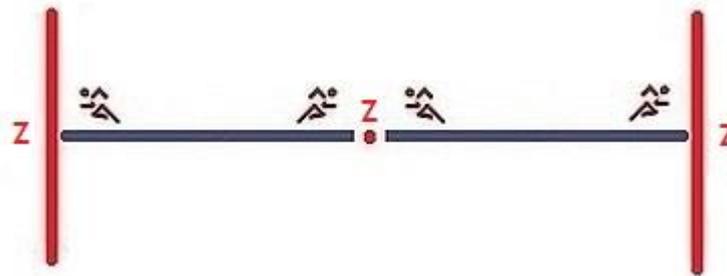


Fig. 12

The Universe is like an unbroken ring with no circumference, for the circumference is nowhere and the center is everywhere.

From the perspective of the running guys (Fig. 12) located in the local mode of spacetime, their “final” endpoint Z will look like The Beginning and The End of their spacetime, because they are confined “in the train” (ref. [54] on p. 34 in spacetime.pdf) and cannot escape from it by switching to their massless luxonic state (Fig. 3) along null intervals¹⁹.

The endpoint Z in Fig. 12 belongs to the *atemporal*, non-squared *physicalized* spacetime: Z does not belong²¹ to the *physicalized* spacetime. It is both “inside” it and “outside” it.

How this could be possible? We have to resort to the 1933 ‘balloon analogy’ by Sir Arthur Eddington. Since we cannot imagine 3D balloon, we can try to imagine 2D balloon with infinitesimal “thickness” matching the “distance” between the running guys and their endpoint Z in Fig. 12.

The case of 3-sphere (Fig. 9.2) is depicted in Fig. 13 below, and the circle in Fig. 8 corresponds to the case in which the “balloon” below has been collapsed along +/- Y.

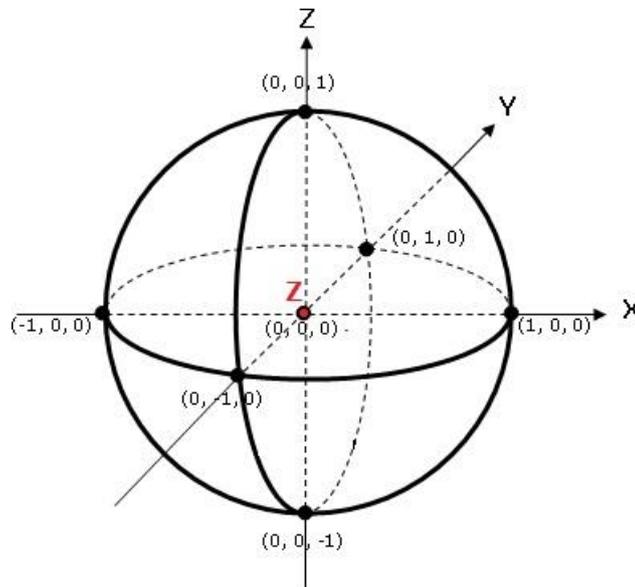


Fig. 13

Notice the **red remnant** “inside”²¹ the center of 3-sphere and check out Fig. 9.1 and p. 19 in Hyperimaginary Numbers¹.

If we collapse the closed *surface* in Fig. 13 along +/- Y, we will obtain a X/Z *circle* shown previously in Fig. 8, and if we collapse it further along both +/- Z and +/- X, we will entrap **Z** “inside” $AB = dt$ shown in Fig. 1. Yet The Beginning & The End at **Z** cannot disappear, because **John** is residing as **non**-event in Fig. 4. This object is dubbed ‘**dimensionless point**’, but it has internal structure and non-trivial topology, as its “thickness” along **W** in Fig. 9.2 matches those of the 2D images on a movie screen (Fig. 4), or the extent to which the asymptotically flat spacetime (*local mode*) is both “closed” ($\Omega > 1$) and “open” ($\Omega < 1$) in the infinitesimal neighborhood of the flat line in Fig. 9.2. Let me try to elaborate.

The X/Z section in Fig. 13 is shown in Fig. 14 with the middle circle with radius R_s below. Fig. 14 also shows the cross section of a **torus** with minor radius R_t , and the two **red dots** correspond to two instances of Fig. 9.1. In reality, there are infinitely many **red dots**.

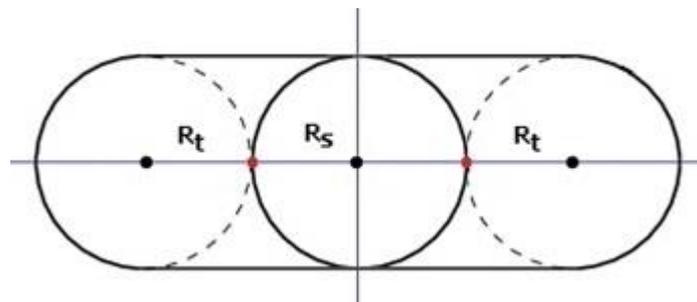


Fig. 14

If we inflate R_s *exactly* to infinity, all points will **break apart** and fuse into the **non**-event in Fig. 4.

We set $R_s = 1/R_t$, and *inflate* the 3-sphere in Fig. 13 and Fig. 14 up to the **breaking** point at actual/completed infinity, shown with the **red** horizontal line in Fig. 9.2 and Fig. 15

below. In Fig. 14 and in its section in Fig. 15 (rotated 90° clockwise), $R_s = R_t$, which corresponds to four points with coordinates $(|1|, |1|)$ in Fig. 7.

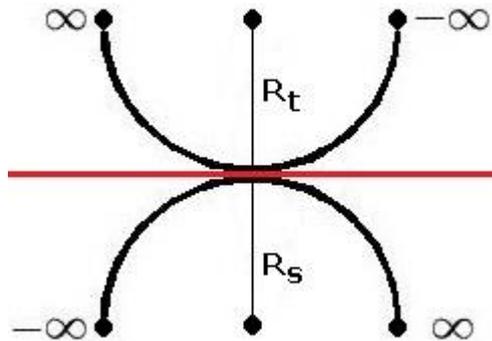


Fig. 15, adapted from Eric Schechter.

It goes without saying that there are many issues (summary in Sec. 7), which have to be thoroughly studied and explained. I hope we will unravel the hyperimaginary numbers¹ by Christmas 2018, to solve many outstanding problems of current Mathematics, and then proceed to quantum gravity and BAVER, well before 2022 (p. 94 in gravity.pdf).

Notice also that, in the local mode of spacetime (cf. the Archimedean topology, ref. [31] on p. 18 in Hyperimaginary Numbers¹), the guys in Fig. 12 run with potential infinity and can only approach asymptotically (Fig. 7) their “final” endpoint Z , but if they use actual or completed infinity, they can calculate the sliding cutoff Z in Fig. 5 as the infinitesimal quantum of energy at ‘time zero’, as well as the largest volume of spacetime “bounded” by the causal horizon. Yet The Beginning & The End is always residing at Z , in line with the dual age cosmology (p. 4 in Hyperimaginary Numbers¹ and p. 67 in gravity.pdf): once created (John 1:1; 1 John 4:8), the Universe is already (Sic!) eternal and can never reach its Beginning & End residing inside us (Luke 17:21), as non-event inside dt (Fig. 1) and inside the center Z of 3-sphere in Fig. 13. Hence we can see “as far as we like”⁴, as if we were living in Euclidean space, and pinpoint the sliding ‘jacket’ of the cosmic time.

“In a certain sense, everything is everywhere at all times” (Alfred North Whitehead). This is the motto of dual age cosmology. We only need new Mathematics¹, as stated above.

I tried many times to contact Kevin Brown¹⁹, but he did not reply. My model of causality, dubbed ‘biocausality’ (January 1990), requires two modes of spacetime: local (time-like) mode and global atemporal mode along null intervals. Example: the school of fish in ref. [11] in Hyperimaginary Numbers¹ and pp. 89-90 in gravity.pdf.

Dead matter makes quantum jumps; the living-and-quantum matter is smarter (Slide 8¹). Check out also p. 34 and Table 1, p. 14 in spacetime.pdf.

To reach the so-called global mode of spacetime, you don’t have to dive into some “meditation”, but to picture the 3-sphere ensuing from its 2D analog (Fig. 13 and Fig. 9.1) and move to Fig. 8. You can never “see” the atemporal hypersphere \Leftrightarrow torus transitions in Fig. 14, yet your brain will (hopefully) produce UNSpeakable yet distinctive images from them, similar to the UNSpeakable cases of ‘meanings’ we keep in our memory: check out the experiment with your brain on p. 2 in Hyperimaginary Numbers¹. Then perhaps you will

be able to fly (A1) by tweaking the ‘matrix’ (p. 3 in Hyperimaginary Numbers¹) fixing the [weight](#) of your body.

Here people will probably say, ‘naah, you can’t bypass [Newton’s third law](#) and [fly in the air](#)’, as if they knew the *interaction* (Sic!) by which ‘[mass there governs spacetime geometry here](#)’ (p. 89 in [gravity.pdf](#)). Yes we can fly, even though nobody has explained the *origin* of inertia. It is still a ‘[yellow button](#)’, just like the human brain (Fig. 11).

If you keep quiet because you believe spacetime engineering is difficult, recall Henry Ford: “Whether you believe you can do a thing or believe you can’t, you are right.”

7. Summary

We endorse the metaphysical ideas of Plato, Heraclitus, and Aristotle, and introduce the Aristotelian [potentia](#) (acknowledged in physics after quantum mechanics, Slide 6¹) in the theory of spacetime, as ‘potential reality’ (Slide 6¹) or *Res potentia* (Slide 13¹). It resides in the **potential** future of the [fundamental flow of events](#) (Fig. 4) and determines causality together (Sic!) with the **past** states (*Res extensa*), which requires brand new *quantization* of spacetime continuum (Fig. 1) to **avoid** faster-than-light causality. We also suggest the *evolution equation* of the Universe (Eq. 1) and the structure and topology of the ‘atom of geometry’ (Fig. 1 and Sec. 5), and finally elaborate on the **squared** and **non-squared** components of spacetime (Sec. 6). However, the theory is still formulated at conceptual level (Path II). To complete the theory and quantify its predictions related to theoretical physics and cosmology, first we need new [Mathematics](#). First things first.

Acknowledgements

I am deeply grateful to Eugene Higgins Professor Emeritus of Physics and Natural Philosophy at Yale University [Henry Margenau](#) for his moral support and encouragement in June 1990, and to my beloved parents Gocho G. Chakalov and Dany Chakalova for their longstanding moral and financial support. They went back home and are now with Jesus.

References and Notes

1. D. Chakalov, Hyperimaginary Numbers, 31 December 2016. Available at [this http URL](#). *Idem*, Quantum Spacetime, 14 March 2017. Slides in .pdf format, available at [this http URL](#).
2. Peter W. Milonni, *The Quantum Vacuum*, Academic Press, 1993, [Ch. 2.6](#).
3. Sir Arthur Stanley Eddington, The Internal Constitution of the Stars. Presidential Address to Section A of the British Association at Cardiff, 24 August 1920; *The Scientific Monthly*, Vol. 11, No. 4 (October 1920), [pp. 297-303](#).
4. Lee Smolin, *Three Roads to Quantum Gravity*, Phoenix, London, 2000, [p. 205](#).

One of the biggest mysteries is that we live in a world in which it is possible to look around, and see as far as we like.

5. Robert M. Wald, The Arrow of Time and the Initial Conditions of the Universe. 21 July 2005, [arXiv:gr-qc/0507094v1](#), p. 5.

Since a mere minus sign distinguishes space from time, the remaining case $(n,m) = (1, 3)$ is mathematically equivalent to the case where $(n,m) = (3, 1)$ and all particles are tachyons [14] with imaginary rest mass.

Footnote 4: The only remaining possibility is the rather contrived case where data is specified on a null hypersurface. To measure such data, an observer would need to “live on the light cone”, i.e., travel with the speed of light, which means that it would subjectively not perceive any time at all (its proper time would **stand still**). (Emphasis mine; see A2 in Slide 19¹ - D.C.)

12. Thomas A. Roman, Some Thoughts on Energy Conditions and Wormholes, 23 September 2004, [arXiv:gr-qc/0409090v1](https://arxiv.org/abs/gr-qc/0409090v1).

If such fields are truly physical, then why does Nature bother to enforce QIs at all? The fascinating mysteries and subtleties of negative energy should keep us all busy for a while yet.

13. According to Aristotle [*Poetics* VII 1450b27-29], The Beginning is that which does not have anything necessarily before it, but does have something necessarily following from it. The Beginning is believed to possess **self-acting** faculty, since it is also the **Unmoved Mover** (that which moves without being moved). Thus, it (not “He”) can only be presented as *purely* mathematical object residing “between” (cf. the dark strips in Fig. 4) any two primary events connected by cause-and-effect relations, but without being an intermediate event – The Beginning is **not** an event. It is *both* “no time at all” (Yakov Zeldovich) and the causal horizon of spacetime, a “boundary” for causal influence and processes²¹. It is also **The Noumenon** and **John 1:1**: check out ‘John’s jackets’ **above**, endowed with *infinitesimal* ‘quantum of energy’.

14. Alexander Dolgov, Cosmic antigravity, 17 June 2012, [arXiv:1206.3725v1](https://arxiv.org/abs/1206.3725v1) [[astro-ph.CO](https://arxiv.org/archive/astro)]; read an excerpt from pp. 13-14 at [this http URL](#).

15. *Mathematical Cosmology and Extragalactic Astronomy*, ed. by Irving Ezra Segal, Academic Press, 1976; read an excerpt from pp. 8-9 at [this http URL](#) and notice my note at the end. The alleged “smooth” or “infinitely differentiable” manifold is a joke, for reason explained with the film reel **above**. It shows *different* points/frames from the **real number line**: time requires *change*, $A \neq B \neq C \neq D, \dots$ (Fig. 4), as read with a clock.

We need new Mathematics to unravel the so-called hyperimaginary numbers¹ with which we can address, and hopefully solve, various problems in the existence of **limit**, **interval**, **infinity**, the **Thomson lamp paradox**, **point-set topology**, **set theory**, and **number theory**. Detailed information is available upon request.

If the reader of these lines is interested in **quantum gravity**, I would suggest to compare the interpretation of the “time-dependent” Schrödinger equation¹⁶ by **Britain’s greatest quantum gravity expert** to the interpretation in Slide 7¹. Then all pieces of the jigsaw puzzle should snap to their unique places, effortlessly.

Just keep in mind that no *physical* clock¹⁷ (**GR included**) can read the time in the *flow of events* (Fig. 4 and **John C. Polkinghorne**) composed by *identical* timelike displacements¹⁸ $AB = dt$ (Fig. 1): the universal “drummer”¹⁸ is not physical phenomenon (Fig. 7); see the example with ‘international second’ on p. 3 and the discussion on p. 10 in Hyperimaginary

Numbers¹. There is no *physical* reference frame to “look” at [the entire Universe en bloc](#) – the so-called [conformal infinity](#) is ‘[not even wrong](#)’.

16. C.J. Isham, Prima Facie Questions in Quantum Gravity, 22 October 1993, [arXiv:gr-qc/9310031v1](#), p. 14.

The background [Newtonian time](#) appears explicitly in the time-dependent Schrödinger equation (3), but it is pertinent to note that such a time is truly an abstraction in the sense that no *physical* clock can provide a precise measure of it [[UW89](#)]: there is always a small probability that a real clock will sometimes run backwards ($D \neq C \neq B \neq A$, cf. [Fig. 4](#) - D.C.) with respect to Newtonian time.

17. John Baez, The Time-Energy Uncertainty Relation. Online article, April 10, 2010, available at [this http URL](#).

The problem is, for physically realistic Hamiltonians H one can prove there is no operator T with

$$[H, T] = i \hbar$$

In other words, [there is no time observable!](#)

18. T.A. Jacobson, A Spacetime Primer. Online paper, September 2, 2014, available at [this http URL](#).

The existence of an intrinsic time interval associated to any timelike displacement is another deep mystery. The fact is that, in Nature, there are systems that can serve as [clocks](#). It seems to be the case that fundamental systems all march to the beat of the same [drummer](#).

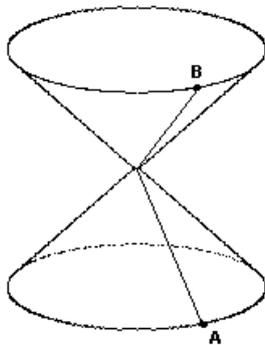
19. [Kevin Brown](#), [Reflections on Relativity](#), MathPages, 2017. Ch. 9.10 Spacetime Mediation of Quantum Interactions, pp. 678-698, available at [this http URL](#).

Most natural philosophers from Aristotle to Descartes held that material entities can influence each other only by coming into direct contact, i.e., “an object cannot act where it is not”. However, Newton’s theory of gravity undermined confidence in the doctrine of “direct contact”, because in Newton’s theory gravity is represented as an instantaneous universal force of attraction between every pair of objects, regardless of the distance between them, and regardless of whether the space between them contains any material substance.

According to this picture, a completely free massless particle - if such a thing existed - might just be represented by an entire null-cone, but a real photon is necessarily emitted and absorbed as a [quantum of action](#), so it corresponds to a bounded null interval in spacetime ([Fig. 1](#) - D.C.). (The quantum phase of a photon does not advance while in transit between its emission and absorption, unlike massive particles; the oscillatory nature of electromagnetic waves arises from the advancing phase of the source, rather than from any phase activity of a photon “in flight”.) Thus the field excitation corresponding to a massless particle propagates at the speed of light and **has no rest frame** ([Fig. 3](#) - D.C.). In contrast, a massive particle has a rest frame, following a time-like path through spacetime.

The “surface area” of this locus (the intersection of the two cones) is necessarily zero (Fig. 1 - D.C.), corresponding to the fact that these interactions represent the transits of massless particles.

In addition to the usual 3+1 dimensions, one could argue that spacetime operationally entails two more “curled up” dimensions of angular orientation to represent the possible directions in space. The motivation for treating these as dimensions in their own right arises from the non-transitive topology of the pseudo-Riemannian manifold. Each point $[t,x,y,z]$ actually consists of a two-dimensional orientation space, which can be parameterized (for any fixed frame) in terms of ordinary angular coordinates q and f . Then each point in the six-dimensional space with coordinates $[x,y,z,t,q,f]$ is a terminus for a **unique** pair of spacetime rays, one forward and one backward in time. We might imagine a tiny computer at each of these points, reading its input from the two rays and sending (matched conservative) outputs on the two rays, as illustrated below in the xyt space:



The point at the origin of these two views is on the **mediating surface** of events A and B. Each point in this space acts purely locally on the basis of purely local information. Specifying a preferred polarity for the two null rays terminating at each point in the 6D space, we automatically preclude causal loops and restrict information flow to the future null cone, while still preserving the symmetry of wave propagation.

Both components of a wave-pair could be regarded as “advanced”, in the sense that they originate on a spherical surface, one emanating forward and one backward in time, but both **converge inward** on the particles involved in the interaction.

According to this view, the “unoccupied points” of spacetime are elements of the 6D space, whereas an event or particle is an element of the 4D space (t,x,y,z) . In effect an event is the union of all the pairs of rays terminating at each point (x,y,z) .

One possible objection to the idea that quantum interactions occur locally between null-separated points is based on the observation that, although every point on the mediating surface is null-separated from each of the interacting events, they are **spacelike-separated** from each other, and hence unable to communicate or coordinate the generation of two equal and opposite outgoing quantum waves (one forward in time and one backward in time). However, communication between those events may not be required, because the “coordination” might arise naturally from the **context** (e.g., the **holomovement of fish** - D.C.).

21. José M.M. Senovilla, Singularity Theorems in General Relativity: Achievements and Open Questions, 30 April 2006, [arXiv:physics/0605007v1](https://arxiv.org/abs/physics/0605007v1), p. 6.

Singularities in the above sense clearly reach, or come from (notice the ambiguity – D.C.), **the edge of space-time**. This is some kind of boundary, or margin, which is not part of the space-time but that, **somehow**, it is accessible from within it. Thus the necessity of a rigorous definition of **the boundary of a space-time**.

22. Karel Hrbacek and Thomas J. Jech, *Introduction to Set Theory*, Basel, 1999, p. 269; excerpt at [this http URL](#).

Tuesday, 2 May 2017