# A new approach to prime numbers

Pedro Caceres
Longmeadow, MA- March 20th, 2017
Email: pcaceres@comcast.net
Phone: +1 (763) 412-8915

**Abstract:** A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic, which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors. Primes can thus be considered the “basic building blocks”, the atoms, of the natural numbers.

There are infinitely many primes, as demonstrated by Euclid around 300 BC. There is no known simple formula that separates prime numbers from composite numbers. However, the distribution of primes, that is to say, the statistical behavior of primes in the large, can be modelled. The first result in that direction is the prime number theorem, proven at the end of the 19th century, which says that the probability that a given, randomly chosen number n is prime is inversely proportional to its number of digits, or to the logarithm of n.

The way to build the sequence of prime numbers uses sieves, an algorithm yielding all primes up to a given limit, using only trial division method which consists of dividing n by each integer m that is greater than 1 and less than or equal to the square root of n. If the result of any of these divisions is an integer, then n is not a prime, otherwise it is a prime.

This paper introduces a new way to approach prime numbers, called the DNA-prime structure because of its intertwined nature, and a new process to create the sequence of primes without direct division or multiplication, which will allow us to introduce a new primality test, and a new factorization algorithm.

As a consequence of the DNA-prime structure, we will be able to provide a potential proof of Golbach’s conjecture.

## A. INTRODUCTION

### 1. Prime sequence

A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself. The sequence of prime numbers is infinite, is composed only by odd numbers and there is no known formula to generate it.

The first 25 prime numbers are given by:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, …

The infinite sequence has been studied in many different ways:

## 2. Differences between consecutive primes

The differences between two of these consecutive primes is calculated to be:

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 | 13 | 19 | 23 | 29 | 31 | 37 | 41 | 47 | 53 | 59 | 61 | 67 | 71 | 73 | 79 | 83 | 89 | 97 |
| | 1 | 2 | 2 | 4 | 2 | 6 | 4 | 6 | 2 | 6 | 4 | 6 | 6 | 6 | 2 | 6 | 4 | 2 | 6 | 4 | 6 | 8 |

With:

$$g_n = p_n - p_{n-1}$$

Verifying that:

$$\lim_{n\to\infty} g_n = \infty$$

And:

$$\lim_{n\to\infty} \frac{g_n}{p_n} = 0$$

The differences between primes are increasing and the prime number theorem proved that these gaps grows with the logarithm of n. The function is neither multiplicative nor additive.

As of March 2017 the largest known prime gap with identified probable prime gap ends has length 5103138, with 216849-digit probable primes found by Robert W. Smith.[3] This gap has merit M=10.2203. The largest known prime gap with identified proven primes as gap ends has length 1113106, with 18662-digit primes found by P. Cami, M. Jansen and J. K. Andersen [4][5]

## 3. Ratios between consecutive primes

The ratios between two consecutive primes is given by:

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 7 | 11 | 13 | 19 | 23 | 29 | 31 | 37 | 41 | 47 | 53 | 59 |
| | 1.50 | 1.67 | 1.40 | 1.57 | 1.18 | 1.46 | 1.21 | 1.26 | 1.07 | 1.19 | 1.11 | 1.15 | 1.13 | 1.11 |

These ratios are decreasing with:

$$\lim_{n\to\infty} \frac{p_n}{p_{n-1}} = 1$$

The gaps are not consistently decreasing and important research has been done on the limits of those gaps. This research is related to the counting of the number of primes less than a given number.

## 4. Known algebraic expressions for primes

The following expression by Euler connects all prime numbers with the Riemann Zeta function over all natural numbers:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p\ prime}^{\infty} \frac{1}{1-\frac{1}{p^s}}$$

One of the most important advance in the study of Prime numbers was the paper by Bernhard Riemann in November 1859 called “Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse” (On the number of primes less than a given quantity) [2].

In this paper, Riemann gave a formula for the number of primes less than x in terms the integral of 1/log(x) and the roots (zeros) of the zeta function in the complex plane, defined by:

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$$

In this paper, Riemann formulated his famous hypothesis that all zeros of ζ(z) have Re(z)=1/2.

In a paper not published yet [12] “An Engineer’s approach to the Riemann Hypothesis and why it is true” (Feb 2017) I formulated that the following expressions valid for the non-trivial zeros of Riemann ζ(z):

1. Any zero of ζ(z) with z=α+iß meet these two conditions:

   (condition 1) α=1/2

   (condition 2) Calculating ß:

   If $S=\frac{1}{[ß^2+1/4]}$ then for n=1/S →

   $$\left(\sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\frac{1}{2}} * j^{-\frac{1}{2}} * \cos\left(ß\left(\ln\left(\frac{k}{j}\right)\right)\right)\right) = 0$$

   This is a finite sum, with n∈ $[1, \frac{1}{S}]$

2. All zeros of ζ(z) are related through the following algebraic expression where $z_1=\alpha_1+iß_1$ and $z_2=\alpha_2+iß_2$ are non-trivial zeros of Riemann ζ(z):

   $$\frac{n}{[ß_2^2+(1-\alpha)^2]} - \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * \cos(ß_2 (\ln\left(\frac{k}{j}\right)) = \frac{n}{[ß_1^2+(1-\alpha)^2]} - \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * \cos(ß_1 (\ln\left(\frac{k}{j}\right))$$

   when $n \to \infty$.

3. The harmonic function $H_n$ can be expressed in infinite ways as a function of each non-trivial zeros of Riemann ζ(z):

$$H_n = \frac{n}{[ß^2+(1-\alpha)^2]} - \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * \cos(ß\,(\ln\left(\frac{k}{j}\right)) \qquad \text{when } n \to \infty$$

4. The Imaginary part of the non-trivial zeros of the Riemann ζ(z) is equal to the zeros of the following function in the Real plane:

$$P(n)= \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\frac{1}{2}} * j^{-\frac{1}{2}} * \cos\left(ß\,(\ln\left(\frac{k}{j}\right)\right)$$

where 1/2+iß would be a non-trivial zeros of Riemann ζ(z).

In the same paper, the author also provided a potential proof of the Riemann Hypothesis.

## 5. Number of primes less than a given number

Let's call $\pi(n)$ the number of primes less than n. The prime number theorem says that:

$$\lim_{n\to\infty} \pi(n)/(n/\ln n) = 1$$

Which can be written as:

$$\lim_{n\to\infty} \pi(n)/li(n) = 1$$

Where:

$$li(n) = \int_0^n \frac{dt}{t}$$

The following table shows the results of this approximation [6 modified]:

| *x* | *π*(*x*) | *π*(*x*) − *x* / ln *x* | li(*x*) − *π*(*x*) | *π*(*x*) / li(x) |
|---|---|---|---|---|
| 1E+01 | 4 | -0.34 | 2.200 | 0.645161290 |
| 1E+02 | 25 | 3.29 | 5.100 | 0.830564784 |
| 1E+03 | 168 | 23.24 | 10.000 | 0.943820225 |
| 1E+04 | 1,229 | 143.26 | 17.000 | 0.986356340 |
| 1E+05 | 9,592 | 906.11 | 38.000 | 0.996053998 |
| 1E+06 | 78,498 | 6115.59 | 130.000 | 0.998346645 |
| 1E+07 | 664,579 | 44158.31 | 339.000 | 0.999490163 |
| 1E+08 | 5,761,455 | 332773.98 | 754.000 | 0.999869147 |
| 1E+09 | 50,847,534 | 2592591.57 | 1701.000 | 0.999966548 |
| 1E+10 | 455,052,511 | 20758029.10 | 3104.000 | 0.999993179 |
| 1E+11 | 4,118,054,813 | 169923159.33 | 11588.000 | 0.999997186 |

Table 1

The effort in this direction is to find more accurate approximations to $\pi(n)$. All these expressions involve complex algebraic expressions of ln(n), or the Riemann Zeta function, and li(x).

As an example, the Riemann hypothesis is equivalent to a much tighter bound on the error in the estimate for $\pi(n)$. and hence to a more regular distribution of prime numbers, Specifically, [9]

$$|\pi(n) - li(x)| < \frac{1}{8\pi}\sqrt{x}\ln x \quad \text{for all x>2657}$$

## 6. Prime factorization

In number theory, the prime factors of a positive integer are the prime numbers that divide that integer with no remainder.

If n is divided by p, there is a $k, r \in Z$ such that:

n = k*p + r

p is a prime factor of n, if and only if r = 0, which can also be expressed using the mod(ulo) function by:

$$n \; mod \; p = 0$$

Where the function mod(ulo) is defined as follows:

$$r = p - n * trunc(\frac{p}{n})$$

The prime factorization of a positive integer is a list of the integer's prime factors, together with their multiplicities; the process of determining these factors is called integer factorization. The fundamental theorem of arithmetic says that every positive integer has a single unique prime factorization.[7]

To find prime factors requires to know if a number is a prime. This type of test is called primality test. Among other fields of mathematics, it is used extensively in cryptography.

The RSA codes in cryptography consists of very large composite numbers that have exactly two prime factors. These numbers are called semiprimes. Finding those two factors require very complex algorithms as the numbers are composed by two prime numbers of more than one hundred digits. As an example:

RSA-220 =
2601385262034057849416540486101975135080389157197767183211977681094456418179666766085931213065825772506315628866769704480700018111497118630021124879281994874820660701310665866460833279828035603792053919801399464969552 61

FACTOR 1 of RSA-220 =
68636564122675662743823714992884378001308422399791648446212449933215410614414642667938213644208420192054999687

FACTOR 2 of RSA-220 =
32929074394863498120493015492129352919164551965362339524626860511692903493094652463337824866390738191765712603

The simple factorization method is the trial division method which consists in dividing sequentially by all known primes until we find a factor. Then we reduce the number by the factor and start again. This method is unpractical for large primes.

The fastest-known fully proven deterministic algorithm is the Pollard-Strassen method (Pomerance 1982; Hardy et al. 1990). [8]

Wolfram Math World mentions the following list of factorization methods: [9]

- Brent's Factorization Method,
- Class Group Factorization Method,
- Continued Fraction Factorization Algorithm,
- Direct Search Factorization,
- Dixon's Factorization Method,
- Elliptic Curve Factorization Method,
- Euler's Factorization Method,
- Excludent Factorization Method,
- Fermat's Factorization Method,
- Legendre's Factorization Method,
- Number Field Sieve,
- Pollard p-1 Factorization Method,
- Pollard rho Factorization Algorithm,
- Quadratic Sieve,
- Trial Division,
- Veryprime,
- Williams p+1 Factorization Method

## B. A new approach to Prime Numbers. Defining the DNA-Prime Sequences P+ and P-

So far, the sequence of primes is a mystery. We have been able to find some patterns, predict the number of primes, factor very large integers but we are still in the same position as Eratosthenes was more than 2000 years ago, when he introduced his sieve to identify prime numbers.

In this paper, we want to introduce a new way to look at primes and develop a new primality test and factorization method from it.

One characteristic that all primes greater than 3 have is that they can be expressed with one of the two following expressions:

$$p = 6k_n + 1 \qquad k_n \in N \qquad (1)$$

$$p = 6k_m - 1 \qquad k_m \in N \qquad (2)$$

In theory, if we knew the sequences $k_n$ and $k_m$ we would know all primes.

In this paper, we are going to provide a unique formulation for the two "generator" series $k_n$ and $k_m$.

As we can see in the following table:

| P+ | P- | | |
|---|---|---|---|
| 6kn+1 | 6km-1 | kn | km |
| | | | |
| | 5 | 1 | |
| 7 | | | 1 |
| | 11 | 2 | |
| 13 | | | 2 |
| | 17 | 3 | |
| 19 | | | 3 |
| | 23 | 4 | |
| | 29 | | 5 |
| 31 | | 5 | |
| 37 | | | 6 |
| | 41 | 7 | |
| 43 | | | 7 |
| | 47 | 8 | |
| | 53 | | 9 |
| | 59 | 10 | |
| 61 | | | 10 |

Table 2

The prime numbers belong to either $P^+$ or $P^-$ series.

It is shocking that all primes can be generated from the numbers 1,2 ,3, as:

$P^+$ series $p = 1+2*3*k_n$

$P^-$ series $p= -1+2*3*k_m$

I will call these two sequences the DNA-Prime Sequences as it resembles the intertwined DNA helix.

## C. Characteristics of the DNA-Prime Sequences

a. The difference between two primes in either sequence $P^+$ and $P^-$ is a multiple of 6. In the following table we show the two DNA-prime series, the difference between two consecutive elements of the series, the difference divided by 6 and the cumulative difference divided by 6.

We must observe that the cumulative difference from any element of the series and the first element, that we will call $R_n$ for the $P^+$ series, and $R_m$ for the $P^-$ series, is equal to the K generator minus 1. This key fact will help us formulate a way to generate the prime sequence.

| P+ | | | | | P- | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Pn+** | **Kn** | **P(n)-P(n-1)** | **(P(n)-P(n-1))/6** | **Rn** | **Pm-** | **Km** | **P(m)-P(m-1)** | **(P(m)-P(m-1))/6** | **Rm** |
| **7** | **1** | | | | **5** | **1** | | | |
| **13** | **2** | 6 | 1 | 1 | **11** | **2** | 6 | 1 | **1** |
| **19** | **3** | 6 | 1 | 2 | **17** | **3** | 6 | 1 | **2** |
| **31** | **5** | 12 | 2 | 4 | **23** | **4** | 6 | 1 | **3** |
| **37** | **6** | 6 | 1 | 5 | **29** | **5** | 6 | 1 | **4** |
| **43** | **7** | 6 | 1 | 6 | **41** | **7** | 12 | 2 | **6** |
| **61** | **10** | 18 | 3 | 9 | **47** | **8** | 6 | 1 | **7** |
| **67** | **11** | 6 | 1 | 10 | **53** | **9** | 6 | 1 | **8** |
| **73** | **12** | 6 | 1 | 11 | **59** | **10** | 6 | 1 | **9** |
| **79** | **13** | 6 | 1 | 12 | **71** | **12** | 12 | 2 | **11** |
| **97** | **16** | 18 | 3 | 15 | **83** | **14** | 12 | 2 | **13** |
| **103** | **17** | 6 | 1 | 16 | **89** | **15** | 6 | 1 | **14** |
| **109** | **18** | 6 | 1 | 17 | **101** | **17** | 12 | 2 | **16** |
| **127** | **21** | 18 | 3 | 20 | **107** | **18** | 6 | 1 | **17** |
| **139** | **23** | 12 | 2 | 22 | **113** | **19** | 6 | 1 | **18** |
| **151** | **25** | 12 | 2 | 24 | **131** | **22** | 18 | 3 | **21** |
| **157** | **26** | 6 | 1 | 25 | **137** | **23** | 6 | 1 | **22** |
| **163** | **27** | 6 | 1 | 26 | **149** | **25** | 12 | 2 | **24** |
| **181** | **30** | 18 | 3 | 29 | **167** | **28** | 18 | 3 | **27** |
| **193** | **32** | 12 | 2 | 31 | **173** | **29** | 6 | 1 | **28** |

Table 3

We can see in the chart that:

$P^+$ series $\quad P(n)-P(n-1) \bmod 6 = 0$
$P^-$ series $\quad P(m)-P(m-1) \bmod 6 = 0$

b. The difference between any two primes in either series is given by:

$P^+$ series $\quad$ if $p_1=6*k_1+1$ $\quad$ and $\quad$ $p_2=6*k_2+1$
$P^+$ series $\quad$ if $p_1=6*k_1-1$ $\quad$ and $\quad$ $p_2=6*k_2-1$

Then $\quad p_2-p_1 = 6*(k_2-k_1)$

c. The difference between any prime in either sequence $P^+$ and P and the first one in the series is a multiple of the generators:

$P^+$ series $\quad P_n = 7 + 6*R_n$
$P^-$ series $\quad P_m = 5 + 6*R_m$

Where $R_n = k_n-1$ $\quad$ and $\quad$ $R_m=k_m-1$

If we can find a formula for the generators, then we will have a formula for the primes. Let's take a look at the $R_n$ and $R_m$ sequences eliminating (in color) all those that don't generate a prime using previous formulas:

**$R_n$**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 |
| 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 |
| 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 |

Table 4

**$R_m$**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 |
| 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 |
| 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 |
| 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 |
| 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 |

Table 5

## D. The prime series can be formulated algebraically

From the tables above and testing many potential combinations, we have concluded that the sequence of DNA-primes and their generators $R_n$ and $R_m$ can be formulated algebraically as follows:

| | | |
|---|---|---|
| **$P^+$ series** | **$P_n = 7 + 6*R_n$** | |
| **$P^+$ series** | $R_n \neq x + (6x + 7) * y$ | $x > 0, y > 1 \in N$ |
| | $R_n \neq -x + (6x - 7) * y$ | $x > 1, y > 1 \in N$ |
| **$P^-$ series** | **$P_m = 5 + 6*R_m$** | |
| **$P^+$ series** | $R_m \neq x + (6x + 5) * y$ | $x > 0, y > 1 \in N$ |
| | $R_m \neq -(x + 1) + (6x + 1) * y$ | $x > 0, y > 1 \in N$ |

These expressions are equivalent to:

| | | |
|---|---|---|
| $P^+$ series | $P_n = 1 + 6*K_n$ | with $K_n=R_n+1$ |
| $P^-$ series | $P_m = -1 + 6*K_m$ | with $Km=R_m+1$ |

The first numbers in the generator series are $R_n$ and $R_m$:

**$R_n$=** 1, 2, 4, 5, 6, 9, 10, 11, 12, 15, 16, 17, 20, 22, 24, 25, 26, 29, 31, 32, 34, 36, 37, 39, 44, 45, 46, 50, 51, 54, 55, 57, 60, 61, 62, 65,…

**$R_m$=** 1, 2, 3, 4, 6, 7, 8, 9, 11, 13, 14, 16, 17, 18, 21, 22, 24, 27, 28, 29, 31, 32, 37, 38, 39, 41, 42, 43, 44, 46, 48, 51, 52, 57, 58, 59, 63,…

And the primes generated by these generators:

**Primes generated=** [2, 3, 5, 7], 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149,…

We prepared a simple code to run the prime generation and we this is a test for the prime numbers less than n=1,000,000

```
PEDRO CACERES - pcaceres@comcast.net - +1 (763) 412-8915
Version 03/14/2017
----------------------------------------------------
GENERATING PRIME SERIES Start @  2017-03-21 11:44:11
----------------------------------------------------
Number of Primes      = 78498
Number of Prime Pairs = 8168 %Pi(x)= 10.405360646131111
----------------------------------------------------
My series generated    @  2017-03-21 11:49:27
---> GENERATED Prime list is correct
----------------------------------------------------
Last prime position:  78498 -> 999983
----------------------------------------------------
GENERATING PRIME SERIES Ends   @  2017-03-21 11:49:27
----------------------------------------------------
```

## E. The gaps between elements of the DNA-prime series

The gaps between elements of Rn and Rm can be described using the Prime Number Theorem as the DNA-prime series and the prime series have a bijective relationship.

We can affirm that:

$$R_n - R_{n-1} \sim \ln(6*R_n+7) < R_{n-1} < R_n$$

$$R_m - R_{m-1} \sim \ln(6*R_m+5) < R_{m-1} < R_m$$

## F. DNA-Prime twins

The definition of the two DNA-prime series implies that any $S_{2n}$ such that $S_{2n}=R_n=R_m$ will create a pair of twin primes.

The list of the first $S_{2n}$ is:

$S_{2n}$ = [1,2,4,6,9,11,16,17,22,24,29,31,32,37,39,44,46,…]

And the pairs generated:

| | | |
|---|---|---|
| Twin Primes from S2n = | 1 | =[ 11 , 13 ] |
| Twin Primes from S2n = | 2 | =[ 17 , 19 ] |
| Twin Primes from S2n = | 4 | =[ 29 , 31 ] |
| Twin Primes from S2n = | 6 | =[ 41 , 43 ] |
| Twin Primes from S2n = | 9 | =[ 59 , 61 ] |
| Twin Primes from S2n = | 11 | =[ 71 , 73 ] |
| Twin Primes from S2n = | 16 | =[ 101 , 103 ] |
| Twin Primes from S2n = | 17 | =[ 107 , 109 ] |

| | | |
|---|---|---|
| Twin Primes from S2n = | 22 | =[ 137 , 139 ] |
| Twin Primes from S2n = | 24 | =[ 149 , 151 ] |
| Twin Primes from S2n = | 29 | =[ 179 , 181 ] |
| Twin Primes from S2n = | 31 | =[ 191 , 193 ] |
| Twin Primes from S2n = | 32 | =[ 197 , 199 ] |
| Twin Primes from S2n = | 37 | =[ 227 , 229 ] |
| Twin Primes from S2n = | 39 | =[ 239 , 241 ] |
| Twin Primes from S2n = | 44 | =[ 269 , 271 ] |
| Twin Primes from S2n = | 46 | =[ 281 , 283 ] |

We must add [3,5] and [5,7] to complete the series.

The series $S_{2n}$ has infinite terms as $R_n$ and $R_m$ are also infinite and the condition for $R_n=R_m$ does not have a maximum limit. The generation of $R_n$ and $R_m$ is done over all x, y $\in N$. The proof will only require to define a x,y that makes all 4 conditions of the DNA-series not equal. The length of the gap among twin prime sets can be calculated from the difference between elements in $S_{2n}$.

The Brun's theorem [11] proved that the sum of the reciprocals of these twin primes converges to a finite value known as Brun's constant, usually denoted by $B_2$. In 2002 Pascal Sebah and Patrick Demichel used all twin primes up to $10^{16}$ to give the estimate:

$$B_2 = \sum_{p,p+2\ pairs}\left(\frac{1}{p}+\frac{1}{p+2}\right) = 1.902160583104 \quad [6]$$

We can express this equation with the DNA-prime expression as:

$$B_2 = \sum_{0,R=Rn=Rm}\left(\frac{1}{(6*R+7)}+\frac{1}{(6*R+5)}\right) = 1.902160583104 \quad [6]$$

Adding (1/3+1/5).

Using again the definition of the two DNA-prime series we can calculate other partial consecutive sequences of primes $S_{kn}$.

For instances, series of 4 consecutive primes will require that $S_{4n}$ and $S_{4n+1}$ to be both in $R_n$, and $R_m$.

The list of the first $S_{4n}$ is:

S4n= [1, 16, 31, 136, 246, 311, 346, 541, 576, 941, 1571,…]

With:

| | | | |
|---|---|---|---|
| S4n = | 1 | = | [ 11 , 13 , 17 , 19 ] |
| S4n= | 16 | = | [ 101 , 103 , 107 , 109 ] |
| S4n= | 31 | = | [ 191 , 193 , 197 , 199 ] |
| S4n= | 136 | = | [ 821 , 823 , 827 , 829 ] |
| S4n= | 246 | = | [ 1481 , 1483 , 1487 , 1489 ] |
| S4n= | 311 | = | [ 1871 , 1873 , 1877 , 1879 ] |
| S4n= | 346 | = | [ 2081 , 2083 , 2087 , 2089 ] |
| S4n= | 541 | = | [ 3251 , 3253 , 3257 , 3259 ] |
| S4n= | 576 | = | [ 3461 , 3463 , 3467 , 3469 ] |
| S4n= | 941 | = | [ 5651 , 5653 , 5657 , 5659 ] |
| S4n= | 1571 | = | [ 9431 , 9433 , 9437 , 9439 ] |

We must add [ 5 , 7 , 11 , 13 ] to complete the series,

There is also a Brun's constant for these prime quadruplets with $B_4$ = 0.87058 838… Wolf derived an estimate for the Brun-type sums $B_n$ of 4/n.

I have not found any elements for sets for $S_{6n}$ and above yet.

## G. DNA-Prime primality test

For a number N to be prime, the following conditions must be met:

a) If (N-1) mod 6≠0 and (N+1) mod 6≠0 the number is not prime

b) If (N-1) mod 6=0 then N=6*$k_n$+1 and Rn=$K_n$-1

CONDITION 1 $(R_n - 7s)\ mod\ (6s + 1) \neq 0$ for $s \in N$<$k_n$

CONDITION 2 $(R_n + 7s)\ mod\ (6s - 1) \neq 0$ for $s \in N$<$k_n$

If s=1 or s=$k_n$ then N is Prime.

c) If (N+1) mod 6=0 then N=6*$k_m$+1 and $R_m$=$K_m$-1

CONDITION 3 $(R_m - (s - 1))\ mod\ (6s - 1) \neq 0$ for $s \in N$<$k_m$

CONDITION 4 $(R_m - 5s)\ mod\ (6s + 1) \neq 0$ for $s \in N$<$k_m$

If s=1 or s=$k_m$ then N is Prime.

Examples:

| | Kn | Km | Rn | Rm | s | Condition | |
|---|---|---|---|---|---|---|---|
| 4,489 | **748.00** | 748.33 | 747.00 | | 11 | 1 | No prime |
| 6,839 | 1139.67 | **1140.00** | | 1139.00 | 163 | 3 | No prime |
| 9,973 | **1662.00** | 1662.33 | 1661.00 | | 1662 | | Prime |

The maximum values of (s) for each condition can be formulated as a function of N to optimize the algorithm.

## H. DNA-Prime factorization process

For any given number N, the factors can be calculated with the following method:

1. Check if the number is divisible by 2,3,5,7
    a. If yes, divide N by factors and continue with N*=N/factors ($2^a$,$3^b$,$5^c$,$7^d$)
2. If N*>7, Determine if N* belongs to any DNA-Prime Sequence $P^+$ or $P^-$
    a. If $N \in P^+$ then calculate $k_n$=(N-1)/6 and $R_n$=$k_n$-1 and check Condition 1
        i. If Condition 1 is not met at s<$k_n$ then (6*s-1) is a prime factor of N*. Divide N* by the factor. N**=N*/factor
        ii. If condition 1 is met the check Condition 2
            1. If Condition 2 is not met at s<$k_n$ then (6*s+1) is a prime factor of N*. Divide N* by the factor. N**=N*/factor
            2. If condition 2 is met, N* is prime
    b. If $N \in P^-$ then then calculate $k_m$=(N+1)/6 and $R_m$=$k_m$-1 and check Condition 3
        i. If Condition 3 is not met at any s<$k_m$ then (6*s-1) is a prime factor of N*. Divide N* by the factor. N**=N*/factor
        ii. If condition 3 is met then Check Condition 4
            1. If Condition 4 is not met at s<$k_m$ then (6*s+1) is a prime factor of N*. Divide N* by the factor. N**=N*/factor
            2. If condition 4 is met, N* is prime
    c. If N* is not prime run process for new N**=N*/factor

Some examples of factorization with computer time:

**1. Factoring N = 1234567890 1234567890 showing all the steps:**

```
PEDRO CACERES - pcaceres@comcast.net - +1 (763) 412-8915
FACTORIZA Version 03/14/2017
-----------------------------------------------------------------
FACTORIZA Start @  2017-03-20 19:03:22
-----------------------------------------------------------------
N= 12345678901234567890
----------------------------------------------------------
```

```
PARTIAL N= 6172839450617283945 ->FACTORS+ [2]
----------------------------------------------------------
PARTIAL N= 2057613150205761315 ->FACTORS+ [2, 3]
----------------------------------------------------------
PARTIAL N= 411522630041152263 ->FACTORS+ [2, 3, 5]
----------------------------------------------------------
N= 411522630041152263 is NOT P+ or P-
----------------------------------------------------------
----------------------------------------------------------
PARTIAL N= 137174210013717421 ->FACTORS+ [2, 3, 5, 3]
----------------------------------------------------------
N= 137174210013717421 is in Series: P+
----------------------------------------------------------
N= 137174210013717421
k= 22862368335619570
r= 22862368335619569
----------------------------------------------------------
N: 137174210013717421  -> Failed Condition 1+  @ s= 590
----------------------------------------------------------
PARTIAL N= 38738833666681 ->FACTORS+ [2, 3, 5, 3, 3541]
----------------------------------------------------------
N= 38738833666681 is in Series: P+
----------------------------------------------------------
N= 38738833666681
k= 6456472277780
r= 6456472277779
----------------------------------------------------------
N: 38738833666681  -> Failed Condition 1+  @ s= 601
----------------------------------------------------------
PARTIAL N= 10739903983 ->FACTORS+ [2, 3, 5, 3, 3541, 3607]
----------------------------------------------------------
N= 10739903983 is in Series: P+
----------------------------------------------------------
N= 10739903983
k= 1789983997
r= 1789983996
----------------------------------------------------------
N: 10739903983  -> Failed Condition 1+  @ s= 4660
----------------------------------------------------------
PARTIAL N= 384103 ->FACTORS+ [2, 3, 5, 3, 3541, 3607, 27961]
----------------------------------------------------------
N= 384103 is in Series: P+
----------------------------------------------------------
N= 384103
k= 64017
r= 64016
----------------------------------------------------------
N: 384103  -> Failed Condition 2+  @ s= 17
----------------------------------------------------------
PARTIAL N= 3803 ->FACTORS+ [2, 3, 5, 3, 3541, 3607, 27961, 101]
----------------------------------------------------------
N= 3803 is in Series: P-
----------------------------------------------------------
N= 3803
k= 634
r= 633
----------------------------------------------------------
PARTIAL N= 3803 ->FACTORS+ [2, 3, 5, 3, 3541, 3607, 27961, 101]
----------------------------------------------------------
N= 12345678901234567890 --> Factors= 2*3^2*5*101*3541*3607
-----------------------------------------------------------------
FACTORIZA Ends   @  2017-03-20 19:03:33
```

### 2. Factorization of 20 consecutive numbers without factorization steps:

```
PEDRO CACERES - pcaceres@comcast.net - +1 (763) 412-8915
FACTORIZA Version 03/14/2017

-----------------------------------------------------------------
FACTORIZA Start @  2017-03-20 22:37:51
-----------------------------------------------------------------
N= 12345678901234567890 --> Factors= 2  3^2  5  101  3541  3607  3803  27961
N= 12345678901234567891 --> PRIME
N= 12345678901234567892 --> Factors= 2^2  3086419725308641973
N= 12345678901234567893 --> Factors= 3  14210467  289591207693
N= 12345678901234567894 --> Factors= 2  17  31  43  1189  229099455043
N= 12345678901234567895 --> Factors= 5  9577219  257813440441
N= 12345678901234567896 --> Factors= 2^3  3  5  19  389  757  7177  1830053
N= 12345678901234567897 --> Factors= 6373  1937184826805989
N= 12345678901234567898 --> Factors= 2  6172839450617283949
N= 12345678901234567899 --> Factors= 3^2  11261  73039  1667787409
N= 12345678901234567900 --> Factors= 2^2  5^2  11  12517  22147  40486211
N= 12345678901234567901 --> Factors= 314312807  39278319643
N= 12345678901234567902 --> Factors= 2  3  13^2  12175225740862493
N= 12345678901234567903 --> Factors= 5  59  2917  10247749396943
N= 12345678901234567904 --> Factors= 2^5  10133  503779  75576521
N= 12345678901234567905 --> Factors= 3  5  4314587  190758758621
N= 12345678901234567906 --> Factors= 2  23  515629579  520498309
N= 12345678901234567907 --> Factors= 1151  1223  8770274702459
N= 12345678901234567908 --> Factors= 2^2  3^4  22727  1676593797071
N= 12345678901234567909 --> Factors= 488899381  25251983089
-----------------------------------------------------------------
FACTORIZA Ends   @  2017-03-20 23:40:26
```

## I. Use of DNA-Prime to prove Golbach's conjecture

The Golbach's conjecture says that every even integer greater than 2 can be expressed as the sum of two primes.[10]

From the definition of the two DNA-Prime sequences we know that any prime can be expressed as:

$$p = 6k_n + 1 \qquad k_n \in N$$

$$p = 6k_m - 1 \qquad k_m \in N$$

The addition of two odd prime numbers will always be even.

If N=2q is any even number, for it to be the addition of two primes, the following needs to be true:

$$N = 2q = p_1 + p_2$$

The possible combinations of odd numbers that added together amount to N are:

| a | b | a+b=N |
|---|---|---|
| 1 | N-1 | N |
| 3 | N-3 | N |
| 5 | N-5 | N |
| … | … | … |
| N-1 | 1 | N |

To illustrate the problem, let's build a simple table for N=18

| N | p1 | p2 |
|---|---|---|
| 18 | 1 | 17 |
| | 3 | 15 |
| | 5 | 13 |
| | 7 | 11 |
| | 9 | 9 |
| | 11 | 7 |
| | 13 | 5 |
| | 15 | 3 |
| | 17 | 1 |

We can see that there are N/2 combinations of two odd numbers that add up to N.

We can also see that there are 2 combinations involving 1 and 1 is not a prime.

The option N/2+N/2=N does not involve addition of primes and we can disregard it.

Of the remaining combinations, they repeat themselves due to the commutative property of the addition in N.

So, the net number of potential valid combinations of two odd numbers with one of them at least being prime is:

$$\frac{\frac{N}{2} - 3}{2}$$

If p is prime, based on the Prime number theorem, we can see that for N>76 the number of combinations is larger than the number of primes <N as:

$$\frac{\frac{N}{2} - 3}{2} > N/\ln N \qquad \text{for N>76}$$

So, the number of primes that meet Golbach's conjecture for any even number N are proportionally less than the number of combinations of odd numbers as N grows.

We know that if p1 and p2 are primes, we can use the DNA-Prime series to say:

$$p_1 = 6 * k_1 \pm 1$$
$$p_2 = 6 * k_2 \pm 1$$

And there are three possibilities:

$$N = p_1 + p2 = 6 * (k_1 + k_2)\text{-}2$$
$$N = p_1 + p2 = 6 * (k_1 + k_2)$$
$$\text{N} = p_1 + p2 = 6 * (k_1 + k_2)\text{+}2$$

Based on this, and assuming that p1 and p2 exist, we can affirm that:

| | | |
|---|---|---|
| If | N mod 6=0 | N is the addition of a $p_1 \in P^+$and $p_2 \in P^-$ |
| If | (N-2) mod 6=0 | N is the addition of a $p_1 \in P^+$and $p_2 \in P^+$ |
| If | (N+2) mod 6=0 | N is the addition of a $p_1 \in P^-$and $p_2 \in P^-$ |

given that for any even number, there is a $q \in N$ such that N=2q and the previous expressions are equivalent to:

q mod 3 =0

or (q-1) mod 3 =0

or (q+1) mod 3 =0

Which is obviously true as for any 3 consecutive numbers (q-1), q, (q+1), one of them must necessarily be divisible by 3.

The three possible combinations of primes mentioned earlier can be also reformulated as follows:

| | | |
|---|---|---|
| If q mod 3 =0 | $\frac{q}{3} = k_1 + k_2$ | $p_1 = 6k_1 + 1$ and $p_2 = 6k_2 - 1$ |
| If (q-1) mod 3 =0 | $\frac{q-1}{3} = k_1 + k_2$ | $p_1 = 6k_1 + 1$ and $p_2 = 6k_2 + 1$ |
| If (q+1) mod 3 =0 | $\frac{q+1}{3} = k_1 + k_2$ | $p_1 = 6k_1 - 1$ and $p_2 = 6k_2 - 1$ |

As examples of these expressions:

| | | | | Potential $k_1$ | $k_2$ | Primes | |
|---|---|---|---|---|---|---|---|
| N=54 | q=27 | q=3*9 | q/3=9 | 2 $P^+$ | 7$P^-$ | 13 | 41 |
| N=64 | q=32 | q=3*11-1 | (q+1)/3=11 | 3$P^-$ | 8$P^-$ | 17 | 47 |
| N=68 | q=34 | q=3*11+1 | (q-1)/3=11 | 5$P^+$ | 6$P^+$ | 31 | 37 |

To prove Golbach's conjecture we must prove that for any $n \in N$ we can find combinations of $R_n$ and $R_m$ in the DNA-Prime generator series such that:

$$q = k_n + k_m = R_n + R_m - 2$$

$$\text{or } q = k_{n1} + k_{n2} = R_n^1 + R_n^2 - 2$$

$$\text{or } q = k_{m1} + k_{m2} = R_m^1 + R_m^2 - 2$$

In other words, that for any $q \in N$, we can find two elements of $R_n$, or two elements of $R_m$, or one element of $R_n$ and one element of $R_m$, that add up to q, as all even numbers are of the form: N=2q, with $q \in N$

To prove it, we are going to use an induction proof.

We will define a condition that is observable and met for a certain k=k*, we will assume that the condition is met at k=n-1 and then we will prove that this means the condition is also true at k=n for any element n of the generator series.

Let's observe that in the following chart of ($R_m$+$R_m$), the square 2 $R_m$ *2 $R_m$ contains at least all naturals up to $R_m$.

| Rm Series | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| card(Rm) | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| | | **1** | **2** | **3** | **4** | **5** | **6** | **8** | **9** | **10** | **11** | **12** | **15** | **16** | **17** | **18** |
| 1 | **1** | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 11 | 12 | 13 | 16 | 17 | 18 | 19 |
| 2 | **2** | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 11 | 12 | 13 | 14 | 17 | 18 | 19 | 20 |
| 3 | **3** | 4 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 13 | 14 | 15 | 18 | 19 | 20 | 21 |
| 4 | **4** | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 13 | 14 | 15 | 16 | 19 | 20 | 21 | 22 |
| 5 | **5** | 6 | 7 | 8 | 9 | 10 | 11 | 13 | 14 | 15 | 16 | 17 | 20 | 21 | 22 | 23 |
| 6 | **6** | 7 | 8 | 9 | 10 | 11 | 12 | 14 | 15 | 16 | 17 | 18 | 21 | 22 | 23 | 24 |
| 7 | **8** | 9 | 10 | 11 | 12 | 13 | 14 | 16 | 17 | 18 | 19 | 20 | 23 | 24 | 25 | 26 |
| 8 | **9** | 10 | 11 | 12 | 13 | 14 | 15 | 17 | 18 | 19 | 20 | 21 | 24 | 25 | 26 | 27 |
| 9 | **10** | 11 | 12 | 13 | 14 | 15 | 16 | 18 | 19 | 20 | 21 | 22 | 25 | 26 | 27 | 28 |
| 10 | **11** | 12 | 13 | 14 | 15 | 16 | 17 | 19 | 20 | 21 | 22 | 23 | 26 | 27 | 28 | 29 |
| 11 | **12** | 13 | 14 | 15 | 16 | 17 | 18 | 20 | 21 | 22 | 23 | 24 | 27 | 28 | 29 | 30 |
| 12 | **15** | 16 | 17 | 18 | 19 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 30 | 31 | 32 | 33 |
| 13 | **16** | 17 | 18 | 19 | 20 | 21 | 22 | 24 | 25 | 26 | 27 | 28 | 31 | 32 | 33 | 34 |
| 14 | **17** | 18 | 19 | 20 | 21 | 22 | 23 | 25 | 26 | 27 | 28 | 29 | 32 | 33 | 34 | 35 |
| 15 | **18** | 19 | 20 | 21 | 22 | 23 | 24 | 26 | 27 | 28 | 29 | 30 | 33 | 34 | 35 | 36 |

Table 6

For example, the square for ($R_{12}$ x $R_{12}$)=(15x15) contains up to the natural number up to 15, i.e. contains [1],2,3,4,5,6,7,8,9,10,11,12,13,14,15

We could use any other cardinal to observe that this true.

Let's assume now that the condition is true for $R_{m-1}$, which means that the square $R_{m-1}$x$R_{m-1}$ contains all naturals up to $R_{m-1}$ generated by the addition of two given $R_m^1$ and $R_m^2$ both < $R_{m-1}$ and let's prove that the condition is met for the square $R_m$x $R_m$, which means that the square $R_m$x $R_m$ must contain all naturals up to $R_m$.

The set of natural numbers between $R_{m-1}$ and $R_m$ are, by definition of the matrix

$R_m$x $R_m$:

$$D_n = \{ R_m - R_{m-1}\} = \{R_{m-1}+1, R_{m-1}+2, R_{m-1}+3 \ldots, R_{m-1}+(R_m - R_{m-1})\}$$

We know that all naturals up to $R_{m-1}$ exists and for each n< $R_{m-1}$ there are two:

$$R_m^j = \{R_m^1 \ldots R_m^j\}$$

$$R_m^k = \{R_m^1 \ldots R_m^k\}$$

Such that n=$R_m^j$+$R_m^k$

The assumption over $R_{m-1}$ implies that the set of :$\{R_m^j + R_{m\}}^k$ generates all naturals up to $R_{m-1}$

$$\{R_m^j + R_m^k\} = \{2,3,4,5, \dots R_{m-1}\}$$

$$\text{As } R_m^j = \{1,2,3,4,5,6,8,9, \dots R_{m-1}\}$$

If we add $R_{m-1}$to any of the two $R_m^j + R_m^k$ we can say that:

$$\{R_{m-1}+R_m^j\}=\{R_{m-1} + 1, R_{m-1} + 2 \dots , 2 * R_{m-1}\}$$

$2 * R_{m-1}$is the last diagonal term of the defined and known matrix $R_{m-1}xR_{m-1}$

And we know from [D] that $R_m$- $R_{m-1}$<$2 * R_{m-1}$ so, therefore, $D_n$ is contained in the matrix $R_m x R_m$.

Same proof works for ($R_n$+$R_n$) and ($R_n$+$R_m$), which proves the Golbach's conjecture.

As an example, to find the primes that add up to 180:

N=180 N mod 6=0 so one prime belongs to $P^+$ and the other to $P^-$

q=N/6=30 and $R_n$+$R_m$=30-2=28

All the combinations of $R_n$+$R_m$ =28 are:

| Rn | Rm | P+ | P | P++P- |
|---|---|---|---|---|
| 1 | 27 | 13 | 167 | = 180 |
| 4 | 24 | 31 | 149 | = 180 |
| 6 | 22 | 43 | 137 | = 180 |
| 10 | 18 | 67 | 113 | = 180 |
| 11 | 17 | 73 | 107 | = 180 |
| 12 | 16 | 79 | 101 | = 180 |
| 15 | 13 | 97 | 83 | = 180 |
| 17 | 11 | 109 | 71 | = 180 |
| 20 | 8 | 127 | 53 | = 180 |
| 22 | 6 | 139 | 41 | = 180 |
| 24 | 4 | 151 | 29 | = 180 |
| 25 | 3 | 157 | 23 | = 180 |
| 26 | 2 | 163 | 17 | = 180 |

Table 7

## J. <u>Open challenges</u>

- Can a formula for π(x) counting all primes up to x be formulated exactly based on the 4 conditions of the DNA-prime series? To calculate this, we should be able to calculate the amount of repetitions among the four conditions.
- Can a formula for gaps between primes and twin primes be formulated based on the 4 conditions of the DNA-prime series?

Thanks

Pedro Caceres
pcaceres@comcast.net
+1 (763) 412-8915

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