Affirmative resolve of Legendre's conjecture if Riemann Hypothesis is true.

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Abstract

Near m, the destance of primes is lower order than $\log m$. This is the key to solve the Legendre's conjecture.

1

Theorem 1.1. *Legendre's conjecture*
There is at least 1 prime n^2 *and* $(n+1)^2$

Definition 1.1.

$$Li(x) := \int_2^\infty \frac{1}{\log x} dx$$

Remark:Asymptotic expansion
$Li(m) = \frac{m}{logm} + \frac{1!m}{\log m^2} + \cdots \frac{(n-1)!m}{\log m^n} + O(\frac{x}{\log x^{n+1}})$

Next result is Riemann Hypothesis.

Theorem 1.2. *The prime number less than* m *is*

$$\pi(m) = Li(m) + O(\sqrt{m} \log m)$$

More,

Theorem 1.3. *Littlewood*
The prime number less than m satiosfies

$$\pi(m) - Li(m) = \Omega_+(\sqrt{m}\frac{\log\log\log m}{\log m})$$

$f(m) = \Omega_+(g(m))$ means for large m,there exists c. c satisfies $f(m) > cg(m)$

Theorem 1.4. *Near m,destance of two prime number is order* $\log m$

$$\frac{m}{m/\log m} = \log m$$

We think constant K,for enough large m,"destance of primes"$< K \log m$.K is not depend on m.So, if $(n+1)^2 - n^2 >> K \log n^2$, then Legendre's conjecture is true for m.