

# Electron Stability Approach to Finite Quantum Electrodynamics

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This paper analyses electron stability and applies the resulting stability principle to resolve divergence issues in quantum electrodynamics (QED) without renormalization. Stability is enforced by requiring that the positive electromagnetic field energy be balanced by a negative interaction energy between the observed electron charge and a local vacuum potential. Then in addition to the observed core mechanical mass  $m$ , an electron system consists of two electromagnetic mass components of equal magnitude  $M$  but opposite sign; consequently, the net electromagnetic mass is zero. Two virtual, electromagnetically dressed mass levels  $m \pm M$ , constructed to form a complete set of mass levels and isolate the electron-vacuum interaction, provide essential S-matrix corrections for radiative processes involving infinite field actions. Total scattering amplitudes for radiative corrections are shown to be convergent in the limit  $M \rightarrow \infty$  and equal to renormalized amplitudes when Feynman diagrams for all mass levels are included. In each case the infinity in the core mass amplitude is canceled by the average amplitude for electromagnetically dressed mass levels, which become separated in intermediate states and account for the stabilizing interaction energy between an electron and its surrounding polarized vacuum. In this manner, S-matrix corrections are shown to be finite for any order diagram in perturbation theory; all the while, maintaining the mass and charge at their physically observed values.

## I. INTRODUCTION

A long-standing enigma in particle physics is how an elementary charged particle such as an electron can be stable in the presence of its own electromagnetic field (see [1, 2] and cited references). Critical accounting for electron stability is essential since radiative corrections in quantum field theory involve self-interactions that can change the mass and charge of an electron. This analysis seeks to identify, understand, and account for the hidden interaction that energetically stabilizes an electron such that its mass and charge assume their physically observed values.

The agreement between renormalized QED theory and experiment confirms the effect of vacuum fluctuations on the dynamics of elementary particles to astounding accuracy. For example, electron anomalous magnetic moment calculations currently agree with experiment to about 1 part in a trillion [3, 4]. This achievement is the result of more than six decades of effort since the relativistically invariant form of the theory took shape in the works of Feynman, Schwinger, and Tomonaga (see Dyson's unified account [5]). The agreement leaves little doubt that QED predictions are correct; however, the renormalization technique [6, 7] used to overcome divergence issues in radiative corrections offers little insight into the underlying physics behind electron stability in the high-energy regime. Recall that divergent integrals occur in scattering amplitudes for self-energy processes and arise in sums over intermediate states of arbitrarily high-energy virtual particles. This stymied progress until theoretical improvements were melded with renormalization to isolate the physically significant parts of radiative corrections by absorbing the infinities into the electron mass and charge. Although the renormalization method used to eliminate ultraviolet divergences results in numerical predictions in remarkable agreement with experiments, redefinition of fundamental physical constants remains an undesirable feature of the current theory.

Our main purpose is to develop an alternative to mass and charge renormalization in QED. We begin by revisiting the classical self-energy problem where we define an energetically stable charge. From the resulting stability principle, we construct two virtual, electromagnetically dressed mass levels to isolate the interaction between the observed charge and the polarized vacuum. The total scattering amplitude for a radiative correction with a primitive divergence includes contributions from observed core mechanical mass and electromagnetically dressed core (EDC) mass levels; therefore, three Feynman diagrams are defined, one for each mass level. In particular, S-matrix corrections for the EDC are simply constructed using core amplitudes from the literature and account for the action of the vacuum back on the electron via an

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opposing vacuum current. After defining divergent integrals for EDC amplitudes, we verify that net S-matrix corrections for vacuum polarization, electron self-energy, and vertex processes are finite to all orders in perturbation theory.

## II. FORMULATION

Regarding an electron as a point particle [8], the classical electrostatic self-energy  $e^2/2a \equiv \alpha\Lambda_o$  diverges linearly as the shell radius  $a \rightarrow 0$ , or energy cutoff  $\Lambda_o \rightarrow \infty$ , where  $-e$  is the charge and  $\alpha = e^2/4\pi\hbar c$  is the fine-structure constant. However, Weisskopf [9, 10] showed using Dirac's theory [11] that the charge is effectively dispersed over a region the size of the Compton wavelength due to pair creation in the vacuum near an electron, and the self-energy only diverges logarithmically. Feynman's calculation [12] in covariant QED yields an electromagnetic mass-energy

$$m_{em} = \frac{3\alpha m}{2\pi} \left( \ln \frac{\Lambda_o}{mc^2} + \frac{1}{4} \right), \quad (1)$$

where  $m$  is the electron mass. In the absence of a compensating negative energy, (1) signals an energetically unstable electron. It is the key ultraviolet divergence problem in QED, whose general resolution will result in finite amplitudes for all radiative corrections. In this section we derive a stability condition and apply it to develop corrections to scattering amplitudes for otherwise divergent processes.

To ensure that the total electron mass is its observed value, renormalization theory posits that a negatively infinite 'bare' mass must exist to counterbalance  $m_{em}$ . For lack of physical evidence, negative matter is naturally met with some skepticism (see Dirac's discussion [13] of the classical problem, for example). Nevertheless, energies that hold an electron together are expected to be negative, and we can understand their origin by first considering the source for the electrical energy required to assemble a classical charge in the rest frame. Recall that the work done in assembling a charge from infinitesimal parts is equal to the electromagnetic field energy. Since the agents that do the work must draw an equivalent amount of energy from an external energy source (well), the well's energy is depleted and the total energy

$$\mathcal{E} = mc^2 + \mathcal{E}_{em}^+ + \mathcal{E}_w \quad (2)$$

of the system including matter, electromagnetic field  $\mathcal{E}_{em}^+$ , and energy well  $\mathcal{E}_w$  is constant. For an elementary particle, could the depleted energy well be the surrounding vacuum?

Specifically, consider an electron and its neighboring vacuum treated as two distinct systems that can act on one another. Suppose the electron acts on the vacuum to polarize it creating a potential well, then there must be opposing reaction of vacuum back on the electron. The resulting vacuum potential  $\Phi_{vac}$  confines the observed core charge akin to a spherical capacitor as shown in Fig. 1, and the interaction energy

$$\mathcal{E}_w \rightarrow \mathcal{E}_{em}^- \equiv -e\Phi_{vac} \quad (3)$$

is assumed to just balance  $\mathcal{E}_{em}^+$  resulting in a stability condition

$$\begin{aligned} \mathcal{E}_{em}^+ + \mathcal{E}_{em}^- &= 0 \\ m_{em}^+ + m_{em}^- &= 0, \end{aligned} \quad (4)$$

where the mass-energy equivalence  $\mathcal{E}_{em}^\pm = m_{em}^\pm c^2$  has been used to obtain an equivalent expression in terms of electromagnetic masses. Therefore, the net mass-energy of a free electron is attributed entirely to the observed core mechanical mass  $m$ . In contrast to Poincaré's theory [14] wherein internal non-electromagnetic stresses hold an electron together, external vacuum electrical forces are assumed to provide charge stabilization and energy balance via a steady state polarization field surrounding the electron. Corresponding to a divergent self-action process, we require a mechanism whereby the core charge interacts locally with the polarized vacuum according to (3).

The energy of the core charge in the potential well of Fig. 1 is

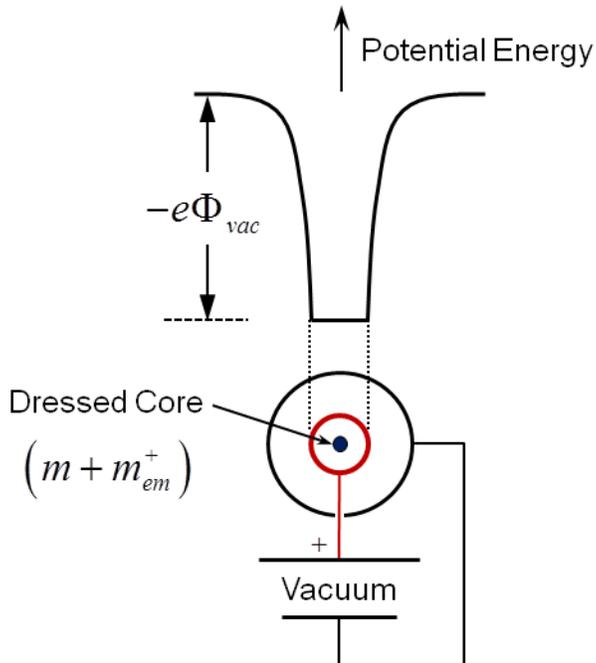


FIG. 1: Effective vacuum potential confines core electron charge similarly to spherical capacitor. Since the stability principle requires  $\mathcal{E}_{em}^+ - e\Phi_{vac} = 0$ , the total energy of core electron in the well and dressed in its electromagnetic field is just its observed mass-energy.

$$\mathcal{E}_{core}^- = mc^2 + \mathcal{E}_{em}^- \equiv m_b c^2, \quad (5)$$

where  $m_b$  may be identified with the bare mass, and

$$m_b + m_{em}^+ = m \quad (6)$$

captures the mass renormalization condition which is equivalent to (2) with (3) and (4). However, notice that the bare mass corresponds to a core electron dressed in negative electromagnetic energy; hence, its characterization as a 'bare mechanical mass' is a misnomer (see [15] for example). Only the core mass is observable, and only it is expected to appear in the Lagrangian if one takes (4) seriously. In renormalization theory, however, one starts with a bare electron, self-interaction dresses it with positive electromagnetic energy, and (6) is subsequently applied to redefine the mass. On the other hand, suppose we start with the observed electron charge; then taking into account (2), (3), and (4),  $m_{em}^+$  and  $m_{em}^-$  are always present, and the total mass reduces to the observed core mechanical mass. Starting with this premise, we can formulate a finite theory of radiative corrections that accounts for all possible electromagnetically dressed intermediate states, and no asymmetry necessitating a redefinition of mass and charge is introduced. For the ensuing development, relativistic notation defined in [16] is employed, and natural units are assumed; that is,  $\hbar = c = 1$ .

Equations (2) and (4) suggest that a stable electron consists of three rest mass components: a core mass  $m$  and two electromagnetic masses  $m_{em}^\pm$  that are assumed large in magnitude but finite until the final step of the development. We can think of  $m_{em}^\pm$  as components of an electromagnetic vacuum (zero net energy) which are tightly bound to the core mass and inseparable from the core and each other, at least for finite field actions. Considering all non-vanishing masses constructed from the set  $\{m, m_{em}^+, m_{em}^-\}$ , we are led to define a complete set of mass levels  $m + \lambda M$ , where  $\lambda = \{0, \pm 1\}$  and  $M \equiv |m_{em}^\pm|$ . Associated 4-momenta are  $p + \lambda P_M$ , where  $\{p, P_M\}$  correspond to  $\{m, M\}$ , respectively. In the following, an electromagnetically dressed core (EDC) refers to a composite particle with mass level  $m \pm M$ .

To implement the EDC model in quantum theory, first consider a free particle state  $|p, m\rangle$  with momentum satisfying  $p^2 \equiv p_\mu p^\mu = m^2$ , where  $p^\mu = (p^0, \vec{p})$  and  $p_\mu = g_{\mu\nu} p^\nu$  are contravariant and covariant momentum 4-vectors, respectively. Metric tensor  $g_{\mu\nu}$  has non-zero components

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1.$$

Spin is omitted in  $|p, m\rangle$  since it is inessential to the subsequent development, and the rest mass is included because it is the fundamental particle characteristic which varies in EDC corrections to the S-matrix [see Eq. (16)]. We employ the relativistic normalization

$$\langle p', m | p, m \rangle = 2E(\vec{p}, m) (2\pi)^3 \delta(\vec{p} - \vec{p}'),$$

where  $E(\vec{p}, m) = \sqrt{\vec{p}^2 + m^2}$ .

Now construct the superposition

$$|\chi\rangle = \frac{1}{\sqrt{2}} \sum_{\lambda=\pm 1} |\Upsilon_\lambda^{edc}(p)\rangle \quad (7)$$

of EDC states

$$|\Upsilon_\lambda^{edc}(p)\rangle = |p + \lambda P_M, m + \lambda M\rangle, \quad (8)$$

where the core 4-momentum is dispersed per an uncertainty  $\Delta p \equiv \lambda P_M$ . Electromagnetically dressed states are normalized according to

$$\begin{aligned} \langle \Upsilon_{\lambda'}^{edc}(p') | \Upsilon_\lambda^{edc}(p) \rangle &= 2E(\vec{p} + \lambda \vec{P}_M, m + \lambda M) (2\pi)^3 \delta(\vec{p} - \vec{p}' + (\lambda - \lambda') \vec{P}_M), \\ &\simeq 2E(\vec{P}_M, M) (2\pi)^3 \delta(\vec{p} - \vec{p}') \delta_{\lambda\lambda'} \end{aligned}$$

where the latter form follows upon assuming  $M \gg m$  and requiring the vector components satisfy

$$|P_M^i| \gg |p^i - p'^i|, \quad i = 1, 2, 3$$

thereby excluding a zero in the delta-function argument at infinity for  $\lambda' \neq \lambda$ . The expected momentum and mass are given by

$$\frac{\langle \chi | \{p_{op}, m_{op}\} | \chi \rangle}{\langle \chi | \chi \rangle} = \{p, m\},$$

where  $\{p_{op}, m_{op}\}$  are corresponding operators. Therefore, the composite state (7) is energetically equivalent to the core mass state  $|p, m\rangle$  as required by (2) and (4). A core electron dressed with positive or negative energy as in (8) is a transient state that is sharply localized within a spacial interaction region  $r \simeq \hbar/Mc$  in accordance with Heisenberg's uncertainty principle [17]  $\Delta p^\mu \Delta x^\mu \geq \hbar/2$  (no implied sum over  $\mu$ ). Scattering amplitudes for low-energy processes are assumed unaffected because the energies are insufficient to induce a separation of tightly bundled states (8) in (7). For infinite field actions, however, EDC states may become separated in intermediate states with infinitesimally small lifetimes; in this case, we shall need to account for both core and EDC scattering amplitudes. To account for all possible intermediate states in QED and satisfy (4), both mass levels  $m \pm M$  are required; this generalizes the classical model depicted in Fig. 1 which assumed that only a positive energy electron interacts with the vacuum potential well.

Since the interaction region reduces to a point as  $M \rightarrow \infty$  for EDC states, self-interaction effects vanish, and a electromagnetically dressed electron interacts only with the polarized vacuum. The vacuum potential is generated by a net positive current in close proximity to the core electron charge since  $\Phi_{vac} > 0$ . Therefore, suppose a dressed electron is located at space-time position  $x_1$  such that it is constrained to interact only with an opposing vacuum current as indicated in Fig. 2. The current density at a neighboring point  $x_2 \neq x_1$  is distinct from that of the dressed core and reversed in sign; that is,

$$\text{sgn}[j_\mu(x_2)] = -\text{sgn}[j_\mu(x_1)]. \quad (9)$$

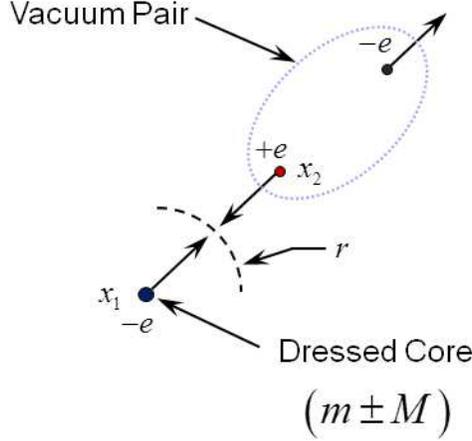


FIG. 2: Dressed core electron interacts with opposing vacuum current resulting in an exchange of the core and vacuum electrons and a sign reversal of the EDC scattering amplitude relative to the core.

With core current defined by the normal product [18, 19]

$$j_\mu(x_1) = -\frac{e}{2} [\bar{\psi}\gamma_\mu\psi - \bar{\psi}_c\gamma_\mu\psi_c]_{x_1} = -eN [\bar{\psi}\gamma_\mu\psi]_{x_1} ,$$

where  $\gamma^\mu$  are Dirac matrices, the vacuum current operator at  $x_2$  may be generated by interchanging the field operator  $\psi$  with its charge conjugate  $\psi_c$  to satisfy (9) and model an exchange of core and vacuum electrons via the  $e^+e^-$  annihilation process suggested in Fig. 2 , then

$$j_\mu(x_2) = eN [\bar{\psi}\gamma_\mu\psi]_{x_2} .$$

Similarly to (9), the Hamiltonian density at nearby points must satisfy

$$\text{sgn} [\mathcal{H}_{int}(x_2)] = -\text{sgn} [\mathcal{H}_{int}(x_1)] , \quad (10)$$

where  $\mathcal{H}_{int}(x) = j_\mu(x) A^\mu(x)$  in the interaction representation [20], and  $A^\mu(x)$  is the radiation field. From (10) we anticipate a sign reversal in the EDC scattering amplitude relative to that for the core mass since second-order S-matrix [21] corrections involve a product  $\mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)$ .

For radiative corrections containing primitive divergences, evaluation of S-matrix corrections for EDC states entails a core mass replacement

$$m \rightarrow m + \lambda M \quad (11)$$

in fermion lines internal to loops as indicated in Fig. 3; that is, in each fermion propagator [22]

$$iS_F(p, m) = \frac{i}{\not{p} - m + i\varepsilon} ,$$

where  $\not{p} = \gamma_\mu p^\mu$ . Resulting loop-operator amplitudes are averaged over mass levels; that is,  $\lambda = \pm 1$ . For an external line entering a loop, the momentum is similarly modified

$$p \rightarrow p + \lambda P_M , \quad (12)$$

since the propagator is required to have a pole at  $m + \lambda M$ . For processes containing infrared divergences, in which a fictitious photon mass  $m_\gamma \neq 0$  is introduced [12] to regulate singularities for soft photon emissions, inspection of fermion self-energy and vertex functions in Sec. IV reveals that a replacement

$$m_\gamma \rightarrow m_\gamma + \lambda M_\gamma \quad (13)$$

is also needed in the modified photon propagator

$$iD_F^{\alpha\beta}(k) = \frac{-ig^{\alpha\beta}}{k^2 - m_\gamma^2 + i\varepsilon},$$

wherein the Feynman gauge is assumed. Since resulting amplitudes involve a ratio  $m/m_\gamma$ , we require  $M_\gamma = \eta m_\gamma$  with  $\eta = M/m$  to ensure reduction to known results.

In summary, the total second-order loop-operator associated with a self-energy or vertex part is defined by

$$\Omega = \Omega_{core} + \Omega_{edc}, \quad (14)$$

where  $\Omega_{core}$  accounts for self-interaction effects involving the core mass and  $\Omega_{edc}$  enforces stability via interaction of EDC states with the polarized vacuum.  $\Omega_{edc}$  is evaluated by substituting (11), (12), and (13) into known  $\Omega_{core}$ . In addition to mass  $m$ ,  $\Omega_{core}$  depends on external momenta  $\{p, q, k\}$  for Feynman diagrams in Fig. 3. For notational simplicity, any dependence on an external momentum parameter is suppressed during construction of  $\Omega_{edc}$  because  $\{p, q\}$  are implicitly dependent on the core mass. Since  $\Omega_{core}$  and  $\Omega_{edc}$  are both divergent for loop corrections, their improper integrals must be temporarily regulated using an energy cutoff  $\Lambda_o$  or by dimensional regularization. Assuming an energy cutoff, the net amplitude (14) is convergent and reduces to expected results if we define

$$\Omega = \lim_{\Lambda_o \rightarrow \infty} [\Omega_{core}(m, \Lambda_o) + \Omega_{edc}(m, \Lambda_o)], \quad (15)$$

where

$$\Omega_{edc}(m, \Lambda_o) = -\frac{1}{2} \lim_{\eta \rightarrow \infty} \sum_{\lambda=\pm 1} \Omega_{core}(m + \lambda M, \Lambda) |_{M=\eta m, \Lambda=\eta \Lambda_o}. \quad (16)$$

The overall minus sign in (16) ensures that the core charge associated with an EDC state interacts with an opposing vacuum current as required by (9). Scaling rules

$$M = \eta m \quad (17)$$

$$\Lambda = \eta \Lambda_o \quad (18)$$

are required for consistent definition of the integrals – they ensure that  $\Lambda \gg M$  for arbitrarily large  $M$ , synchronize cutoff to  $\Lambda_o$ , and yield a well defined limit as  $\eta \rightarrow \infty$  in (16). As verified in Sec. IV, the operator  $\Omega_{edc}$  is independent of  $\{P_M, M\}$  for  $M \gg m$ . In contrast to the regulator technique of Pauli and Villars [23], the above method employs physically meaningful EDC mass levels (albeit virtual only), and the same principle applies to all self-energy processes in QED without introduction of auxiliary constraints.

### III. DIVERGENT INTEGRALS

Here we develop integration formulae required for evaluation of EDC corrections using cutoff and dimensional regularization. In the p-representation, loop diagrams involve four-dimensional integrals over momentum space, and the real parts of scattering amplitudes contain integrals of the form [24]

$$D(\Delta) = \frac{1}{i\pi^2} \int \frac{d^4 p}{(p^2 - \Delta)^n} = \frac{(-1)^n}{\pi^2} \int \frac{d^4 p_\varepsilon}{(p_\varepsilon^2 + \Delta)^n}, \quad (19)$$

where  $\Delta$  depends on the core mass, momentum parameters external to the loop, and integration variables. On the right side of (19), a Wick rotation has been performed via a change of variables  $p = (ip_\varepsilon^0, \vec{p}_\varepsilon)$ , so that

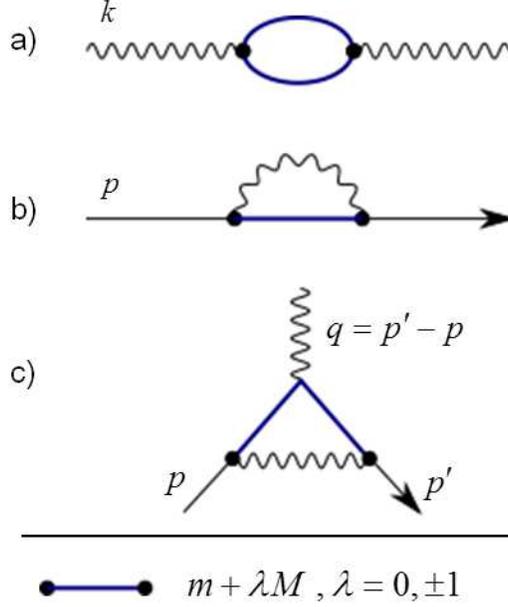


FIG. 3: Baseline radiative corrections a) Vacuum polarization, b) Fermion self-energy, and c) Vertex involve the core mass only in internal fermion lines. Two additional diagrams, obtained by replacing the core mass with electromagnetically dressed mass levels  $m \pm M$ , are required for each radiative process to account for interaction with an opposing vacuum current and ensure stability.

the integration can be performed in euclidean space where  $p_\varepsilon^2 = p_\varepsilon^\circ p_\varepsilon^\circ + \vec{p}_\varepsilon \cdot \vec{p}_\varepsilon$ . Integrals for the divergent case ( $n = 2$ ) must be regulated such that they are consistently defined for core and dressed core masses. For the core mass,  $D$  is regularized using a cutoff  $\Lambda_\circ$  on  $s = |p_\varepsilon|$ . In four-dimensional polar coordinates, we have

$$D(\Delta, \Lambda_\circ) = \frac{1}{\pi^2} \int d\Omega \int_0^{\Lambda_\circ} ds \frac{s^3}{[s^2 + \Delta]^2}. \quad (20)$$

For EDC states,  $\Delta$  depends on  $|m \pm M| \simeq \eta m$  with  $\eta \gg 1$ , and the domain of integration in (20) must be scaled according to (18); consequently, we need to evaluate

$$D_{edc} = D[\Delta(\eta m), \eta \Lambda_\circ].$$

With a change of variables  $s = \eta t$  and taking the limit  $\eta \rightarrow \infty$ , we obtain

$$D_{edc} = D(\Delta_\circ, \Lambda_\circ), \quad (21)$$

where

$$\Delta_\circ = \lim_{\eta \rightarrow \infty} \eta^{-2} \Delta(\eta m). \quad (22)$$

For example, considering the standard divergent integral [24]

$$D_\circ \equiv D(\Delta = m^2, \Lambda_\circ) = \ln \frac{\Lambda_\circ^2}{m^2} - 1 + O\left(\frac{m^2}{\Lambda_\circ^2}\right), \quad (23)$$

we see that  $D_\circ$  is invariant under scaling rules (17) and (18); that is,

$$D_\circ = D(M^2, \Lambda). \quad (24)$$

In contrast to the cutoff method, dimensional regularization evaluates a Feynman diagram as an analytic function of space-time dimension  $d$ . For  $n = 2$  and  $d^4 p \rightarrow d^d p$  in (19),  $D$  may be evaluated using [16, 25]

$$\begin{aligned} D(\Delta, \sigma) &= \pi^{-\sigma} \Gamma(\sigma) \Delta^{-\sigma} \\ &= \frac{1}{\sigma} - \gamma - \ln \Delta + O(\sigma) , \end{aligned} \quad (25)$$

where  $\sigma = 2 - d/2$  and  $\gamma = 0.577\dots$  is Euler's constant. For  $\sigma \neq 0$ , the limit  $\Lambda_o \rightarrow \infty$  may be taken since  $\sigma$  regulates the integral. For EDC states,  $D_{edc}$  must yield consistent results for both cutoff and dimensional regularization methods. Considering the requirements used to derive (21) and employing appendix formulae in [25], we conclude

$$D_{edc} = D(\Delta_o, \sigma) . \quad (26)$$

Therefore, the net S-matrix correction computed from (15) is convergent and involves a factor

$$\delta D = D - D_{edc} = -\ln \frac{\Delta}{\Delta_o} , \quad (27)$$

where we have manually negated  $D_{edc}$  as required by (16). For examples, compare (27) with the photon and fermion self-energy expressions in (36) and (40).

#### IV. APPLICATION TO SECOND ORDER LOOP PROCESSES

Let us apply the foregoing theory with integration formulae given above to verify that the net amplitudes for radiative corrections are convergent and agree with results obtained via renormalization theory. To this end, the approximations

$$(m + \lambda M)^2 \simeq \eta^2 m^2 \quad (28)$$

$$(p + \lambda P_M)^2 \simeq P_M^2 = M^2 + \delta \simeq \eta^2 m^2 \quad (29)$$

$$(m_\gamma + \lambda M_\gamma)^2 \simeq \eta^2 m_\gamma^2 , \quad (30)$$

utilizing (17) with  $\eta \gg 1$ , will be useful for reduction of EDC corrections. They ensure that the regulated integral  $D_{edc}$  in (21) or (26) and  $\Omega_{edc}$  in (16) are independent of individual mass levels ( $\lambda = \pm 1$ ) for  $M \gg m$ . In the expansion of  $P_M^2$  about  $M^2$  on the right side of (29), the off-shell term  $\delta$  is assumed bounded and therefore negligible compared to  $M^2$ . Dimensional and cutoff regularization approaches will be used to illustrate the method.

##### A. Vacuum polarization

Fig. 3 (a) results in a photon propagator modification [21]

$$iD_F^{\alpha\beta} = iD_F^{\alpha\beta} + iD_F^{\alpha\mu} (i\Pi_{\mu\nu}) iD_F^{\nu\beta} , \quad (31)$$

where

$$\Pi_{\mu\nu} \equiv \Pi_{\mu\nu}^{core} + \Pi_{\mu\nu}^{edc}$$

is a polarization tensor generalized to include the EDC correction, and whose core mass term

$$\Pi_{\mu\nu}^{core}(k, m) = \frac{ie^2}{(2\pi)^4} \int d^4 p \text{Tr} [\gamma_\mu S_F(p, m) \gamma_\nu S_F(p - k, m)] \quad (32)$$

follows from the Feynman-Dyson rules [5, 12]. In consequence of Lorentz and gauge invariance [7] or by direct calculation, it factors into

$$\Pi_{\mu\nu}^{core}(k, m) = \Pi_{core}(k^2, m) (k_\mu k_\nu - g_{\mu\nu} k^2), \quad (33)$$

where  $\Pi_{core}(k^2, m)$  is a scalar function. As is well known, the contribution from terms  $k_\mu k_\nu$  vanishes due to current conservation upon connection to an external fermion line. After dimensional regularization, reduction using Dirac matrix algebra, and Feynman parameterization, (25) is employed to cast  $\Pi_{core}$  into a form equivalent to that given in Mandl & Shaw [16]

$$\Pi_{core}(k^2, m) = \frac{2\alpha}{\pi} \int_0^1 dz z(1-z) D(\Delta, \sigma), \quad (34)$$

where

$$\Delta = m^2 - k^2 z(1-z).$$

Applying (16) and (26) we obtain

$$\begin{aligned} \Pi_{edc} &= -\frac{1}{2} \lim_{\eta \rightarrow \infty} [\Pi_{core}(k^2, m + \eta m) + \Pi_{core}(k^2, m - \eta m)] \\ &= -\frac{2\alpha}{\pi} \int_0^1 dz z(1-z) D(\Delta_\circ, \sigma), \end{aligned} \quad (35)$$

where

$$\Delta_\circ = m^2$$

follows from (22) using (28). We see that (35) is equivalent to the subtracted core amplitude evaluated on the light cone

$$\Pi_{edc} = -\Pi_{core}(k^2 = 0, m),$$

which is associated with a correction to the bare charge in renormalization theory, but here the correction represents an interaction between the observed core electron charge associated with a transient EDC state and a polarization current that is required for charge stability in the intermediate state. Combining (34), (35), and using (25), we obtain

$$\begin{aligned} \Pi &= \Pi_{core} + \Pi_{edc} \\ &= -\frac{2\alpha}{\pi} \int_0^1 dz z(1-z) \ln \left[ 1 - \frac{k^2 z(1-z)}{m^2} \right] \end{aligned} \quad (36)$$

in agreement with the renormalization result [16]. For a free photon,  $\Pi(k^2 = 0) = 0$  as required.

## B. Fermion self-energy

The fermion self-energy operator for the core mass corresponding to the Feynman diagram in Fig. 3 (b) with  $\lambda = 0$  is given by

$$\Sigma_{core}(p, m) = \frac{-ie^2}{(2\pi)^4} \int d^4k \gamma_\mu S_F(p-k, m) \gamma^\mu \frac{1}{k^2 - m_\gamma^2}. \quad (37)$$

Employing dimensional regularization,  $\Sigma_{core}$  simplifies to

$$\Sigma_{core}(p, m) = \frac{\alpha}{2\pi} \left\{ S_1 + \int_0^1 dx [2m - \not{p}x + \sigma(\not{p}x - m)] D(\Delta, \sigma) \right\}, \quad (38)$$

where  $D(\Delta, \sigma)$  is given by (25) with

$$\Delta = (1-x)(m^2 - xp^2) + xm_\gamma^2.$$

The integral expression in (38) is equivalent to a form given in Peskin & Schroeder [26], while the term

$$S_1 = -\frac{1-\sigma}{4}\not{p}$$

follows from appendix formulae in [24] and represents a surface contribution arising from a term linear in  $k$  during reduction of (37).

Evaluation of  $\Sigma_{edc}$  using (16) reduces to negating (38) and replacing  $\Delta \rightarrow \Delta_\circ$  according to (26); we obtain

$$\Sigma_{edc}(p, m) = -\frac{\alpha}{2\pi} \left\{ S_1 + \int_0^1 dx [2m - \not{p}x + \sigma(\not{p}x - m)] D(\Delta_\circ, \sigma) \right\}, \quad (39)$$

where

$$\Delta_\circ = m^2(1-x)^2 + xm_\gamma^2$$

follows from (22) using (28), (29), and (30). Terms involving  $(\lambda P_M, \lambda M)$  have canceled in the average over EDC mass levels yielding a function of the core mass and momentum only. The net correction, including all three mass levels in Fig. 3 (b), is given by (cf. [12])

$$\begin{aligned} \Sigma &= \Sigma_{core} + \Sigma_{edc} \\ &= \frac{\alpha}{2\pi} \int_0^1 dx (2m - \not{p}x) \ln \frac{m^2(1-x)^2 + xm_\gamma^2}{(m^2 - xp^2)(1-x) + xm_\gamma^2}, \end{aligned} \quad (40)$$

where the limit  $\sigma \rightarrow 0$  has been taken to recover four-dimensional space-time. With a change of variables  $x = 1 - z$ , (40) is seen to be identical to the renormalized result given in Bjorken & Drell [27].

The processes in Fig. 3 (b), including iterations, results in a modified propagator [5, 21]

$$\begin{aligned} iS'_F &= iS_F + iS_F (-i\Sigma) iS'_F \\ &= \frac{i}{\not{p} - m - \Sigma + i\varepsilon}, \end{aligned} \quad (41)$$

which has the desired pole at  $\not{p} = m$  since (40) vanishes on the mass shell

$$\Sigma(p^2 = m^2) = 0. \quad (42)$$

Upon identifying

$$m_{em}^+ = \Sigma_{core}(\not{p} = m, m_\gamma = 0) \quad (43a)$$

$$m_{em}^- = \Sigma_{edc}(\not{p} = m, m_\gamma = 0), \quad (43b)$$

we see that (42) is equivalent to the stability principle (4). Setting  $\sigma = 0$  and using (19) with cutoff  $\Lambda_\circ$ , it follows that (43a) reduces to Feynman's result (1); for derivation, see [24]. In the language of renormalization theory, the bare mass in the propagator [16]

$$iS'_F(p, m) = \frac{i}{\not{p} - m_b - \Sigma_{core} + i\varepsilon} \quad (44)$$

must be renormalized using (6) with (43a).

### C. Vertex

Since the scattering amplitude is a complex analytic function, it follows from Cauchy's formula that the real and imaginary parts are related by a dispersion relation; for applications in quantum theory, see [28]. The imaginary part is divergence free; it may be obtained by replacing Feynman propagators with cut propagators on the mass shell according to Cutkosky's cutting rule [29] or, alternatively, via calculation in the Heisenberg representation as shown in Källén [30].

In particular, the second-order correction to a corner involves a replacement

$$ie\gamma^\mu \rightarrow i\Gamma^\mu = ie(\gamma^\mu + \Lambda^\mu) , \quad (45)$$

where the vertex function  $\Lambda^\mu$  for the core mass corresponding to Fig. 3 (c) with  $\lambda = 0$  is given by

$$\Lambda_{core}^\mu(q, m) = \gamma^\mu F_1(q^2, m) + \frac{i}{2m} \sigma^{\mu\nu} q_\nu F_2(q^2, m) , \quad (46)$$

where

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

are spin matrices, and the form factors  $\{F_1, F_2\}$  are defined by Hilbert transforms [31]

$$F_i = \int_{4m^2}^{4\Lambda_0^2} ds \frac{\Lambda_i\left(\frac{4m^2}{s}\right)}{s - q^2} \quad (47)$$

with imaginary parts given by

$$\Lambda_1(w) = \frac{\alpha}{4\pi} \frac{1}{\sqrt{1-w}} \left\{ (w-2) \ln \left[ 1 + 4 \left( \frac{m}{m_\gamma} \right)^2 \left( \frac{1-w}{w} \right) \right] + 3 - 4w \right\} \quad (48)$$

$$\Lambda_2(w) = \frac{\alpha}{4\pi} \frac{w}{\sqrt{1-w}} . \quad (49)$$

For evaluation of the EDC correction using (16), only the divergent term  $F_1$  needs to be considered since  $F_2$  is convergent and its coefficient vanishes as  $M \rightarrow \infty$ . Upon substituting  $\{(11),(12),(13)\}$ , using  $\{(28), (30)\}$ , and performing a change of variables  $s = \eta^2 t$  in  $F_1$ , we have

$$\Lambda_{edc}^\mu = -\gamma^\mu \frac{1}{2} \sum_{\lambda=\pm 1} \lim_{\eta \rightarrow \infty} \int_{4m^2}^{4\Lambda_0^2} dt \frac{\Lambda_1\left(\frac{4m^2}{t}\right)}{t - \eta^{-2} Q_\lambda^2} , \quad (50)$$

where  $Q_\lambda = q + \lambda(P'_M - P_M)$ . Assuming  $Q_\lambda^2$  is bounded, we find

$$\Lambda_{edc}^\mu = -\gamma^\mu F_1(q^2 = 0, m) ; \quad (51)$$

therefore, the total vertex function, including the stability correction (51), is convergent and given by

$$\begin{aligned} \Lambda^\mu(q) &= \Lambda_{core}^\mu + \Lambda_{edc}^\mu \\ &= \gamma^\mu [F_1(q^2) - F_1(0)] + \frac{i}{2m} \sigma^{\mu\nu} q_\nu F_2(q^2) , \end{aligned} \quad (52)$$

where the term in brackets reduces to a once-subtracted dispersion relation in agreement with renormalized QED, and the functional dependence on  $m$  is now omitted for simplicity. This completes verification that lowest-order S-matrix corrections are finite without renormalization.

Historically, the identification of the last term in (52) marked a milestone in QED's development – substituting (49) into (47) yields

$$F_2(q^2 = 0) = \frac{\alpha}{2\pi} \equiv a^{(2)}$$

as the leading correction to the anomalous magnetic moment  $a$  of the electron first derived by Schwinger [32] and verified experimentally by Foley & Kusch [33].

## V. GENERALIZATION TO HIGHER ORDERS

The remaining task is to show that higher-order radiative corrections are convergent. The proof closely follows methods in the references; therefore, we keep our remarks brief highlighting required modifications.

Irreducible (skeleton) diagrams include second-order self-energy (SE) and vertex (V) parts discussed in Sec. IV plus infinitely many higher-order primitively divergent V-parts. Using Dyson's expansion method [21], second-order SE- and V-part operators for the core mass are

$$\Sigma_{core} = mA + (\not{p} - m)B + \Sigma, \quad (53)$$

$$\Pi_{core} = C + \Pi, \quad (54)$$

$$A_{core}^\mu = \gamma^\mu L + A^\mu, \quad (55)$$

where  $\{A, B, C, L = B\}$  are logarithmically divergent coefficients depending on  $D_\circ$ . For example,

$$A = \frac{3\alpha}{4\pi} \left( D_\circ + \frac{3}{2} \right)$$

may be obtained using (1), where  $m_{em} \equiv mA$ , and vanishingly small terms in (23) are dropped. Refer to Jauch & Rohrlich [24] for the remaining coefficients, or derive from expressions given in Sec. IV noting that insignificant finite terms can depend on the regularization method used. Higher-order primitively divergent V-parts are also of the form (55) since  $K = 0$  in the convergence condition

$$K = 4 - \frac{3}{2}f_e - b_e,$$

where  $f_e(b_e)$  are the number of external fermion (boson) lines; in this case,  $L(D_\circ)$  is a power series in  $\alpha$ .

To determine the interaction of an electromagnetically dressed core with the polarized vacuum, we apply (16) using the invariance of  $D_\circ$  under the scaling rules (17) and (18) [see Eq. (24)] to obtain

$$\Sigma_{edc} = -[mA + (\not{p} - m)B], \quad (56)$$

$$\Pi_{edc} = -C, \quad (57)$$

$$A_{edc}^\mu = -\gamma^\mu L, \quad (58)$$

where the vanishing of the finite parts  $\{\Sigma, \Pi, A^\mu\}$  as  $M \rightarrow \infty$  is both a physical requirement and a consequence of their convergent integrals. In this way, (14) yields convergent results

$$\Sigma = \Sigma_{core} + \Sigma_{edc} \quad (59)$$

$$\Pi = \Pi_{core} + \Pi_{edc} \quad (60)$$

$$A^\mu = A_{core}^\mu + A_{edc}^\mu \quad (61)$$

for all irreducible diagrams; therefore, SE-part insertions

$$\begin{aligned} iS_F &\rightarrow iS_F (-i\Sigma) iS_F \\ iD_F^{\alpha\beta} &\rightarrow iD_F^{\alpha\mu} (-ig_{\mu\nu}k^2\Pi) iD_F^{\mu\beta} \end{aligned}$$

into lines, and V-part insertions

$$\gamma^\mu \rightarrow \Lambda^\mu$$

into corners of a skeleton diagram yield well-defined higher-order corrections to the S-matrix.

For reducible vertex diagrams, the V-part resolves into a skeleton along with SE- and V-part insertions. Working from inside out and applying (59), (60), and (61) to the irreducible subdiagrams, we see that the insertions, the skeleton, and therefore the complete reducible V-part are convergent.

Similarly, reducible SE-parts involve a second-order skeleton along with SE- and V-part insertions. Self-energy insertions and the skeleton are handled in the same way as for reducible vertex diagrams. However, vertex insertions into a SE skeleton involve overlapping divergences that have been addressed by Salam [34] and Ward [35]; in the following, we employ Ward's method. For vertex insertions into fermion and photon SE skeletons, Ward's identities

$$\begin{aligned} \frac{\partial \Sigma^*(p, m)}{\partial p_\mu} &= \Lambda^\mu(p, m) = \gamma^\mu L + \Lambda_f^\mu(p, m) \\ -\frac{\partial \Pi^*(k, m)}{\partial k_\mu} &= \Delta^\mu(k, m) = 2k^\mu C + \Delta_f^\mu(k, m) \end{aligned}$$

are integrated to derive expressions of the same form as (53) and (54), respectively. Self-energy operators  $\Sigma_{core} \equiv \Sigma^*$  and  $\Pi_{core} \equiv -k^{-2} \Pi^*$  denote sums over all proper SE-parts, and  $\Lambda^\mu(p' = p, m)$  and  $\Delta^\mu(k, m)$  are corresponding sums over all V-parts connected to a zero-momentum photon line generated by the differentiation. In this case, the coefficients  $\{A, B, C\}$  are all power series in  $\alpha$  depending on  $D_o$ , and vertex insertions in SE-parts are convergent upon including EDC corrections (56) and (57). This result is consistent with our assumption that infinite field actions excite mass levels  $m \pm M$  in all internal fermion lines of a loop for EDC corrections; for a specific example, apply (15) to calculate the real part of the fourth-order vacuum polarization kernel [36] using the method given in Sec. IV for the vertex. Therefore, a diagram with overlapping divergences is not a special case for implementation of EDC corrections for fermion stability.

The complete fermion and photon propagation functions are given by

$$iS'_F(p) = \frac{i}{\not{p} - m - \Sigma(p) + i\varepsilon}, \quad (62)$$

$$iD'^{\alpha\beta}_F(k) = \frac{-ig^{\alpha\beta}}{k^2 [1 + \Pi(k)] + i\varepsilon}, \quad (63)$$

where  $\{\Sigma, \Pi\}$  are given by sums over all proper self-energy parts. Self-energy insertions (62) and (63) replace fermion and photon lines in a skeleton diagram. Similarly, the most general vertex replacing a corner in a skeleton diagram is given by a sum over all proper V-parts. Since both core and EDC contributions are included for each divergent subdiagram as outlined above, the complete propagators and vertices are well defined (convergent).

## VI. CONCLUDING REMARKS

In this paper, we developed a model for a stable elementary charge wherein a hidden interaction between the electromagnetically dressed charge and an opposing polarization current offsets the positive electromagnetic (EM) field energy. Concise rules for constructing S-matrix corrections for the electromagnetically dressed core were developed and applied to resolve divergence issues to all orders, and we maintained the observed fermion mass and charge as fundamental constants throughout. Model predictions agree precisely with renormalized QED and therefore are consistent with current experiments also. Our findings provide compelling evidence for negative and positive EM mass components in virtual intermediate states of infinitesimally short duration. Since there is no renormalization in this approach, the EM coupling is a constant independent of the energy scale; therefore, QED is scale-invariant. This prediction is in sharp contrast to renormalization group arguments which contend that the coupling constant scales with energy; see review given in [37]. The stability approach for computing finite amplitudes in QED is simpler compared to renormalization, and it more accurately characterizes the physics involved in radiative processes since it includes the reaction (16) of the vacuum back on a core charge.

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